Name: _____

	4 points each	
1	I ate a hemispherical scoop of ice cream with diameter 6 inches. If three cubic inches of ice cream contain two grams of fat, how many grams of fat were in the ice cream I ate?	
2	Anna runs one mile every Monday, two miles every Tuesday, four miles every Wednesday, and so on, doubling the length of her run every consecutive day. She does not run on Sundays. How many miles does she run in one week?	
3	Fifteen years ago, my brother was twice my age. Eleven years ago, I was three-fourths of his age. How old is my brother now?	
4	A ball launched in the air follows the path of a parabola with its height given by $h = -10x^2+15x+5$. How high is the ball at its highest point?	
5	If $\log_b a = 10$ and $\log_c b = \frac{3}{2}$, what is $\log_a c$?	

	5 points each	
6	Roque forgot to study for his 50-question True/False test. He guessed True on every third question and False on the rest. The answers to the test were False on every fifth question and True on the rest. What percent did Roque get on his test?	
7	I have two identical charms and three different keys. How many distinguishable ways can I arrange them on a keyring?	
8	I drop a ball from a height of 6 feet. Every time it bounces, it bounces to one-third of its previous height. How far does the ball travel before it comes to rest?	
9	One day Athena got to school late, so on her way home she decided to drive 10 miles per hour faster. That day, she averaged 24 miles per hour. How fast did she drive on her way to school, assuming she drives at constant speed?	
10	The measure of each angle in a regular polygon is 179°. How many sides does it have?	

	6 points each	
11	The equation $y=x^3-12x^2+44x+k$ has integer coefficients. If its roots form an arithmetic sequence, what is k?	
12	There are 12 students in Esther's calculus class but there are only 10 chairs. What is the probability that both Esther and her friend get seats, assuming that everyone gets to class in random order?	
13	Find the coordinates of the reflection of the point (1,1) about the line $y = 2x$.	
14	What is the probability of being dealt a full house from a standard deck of 52 cards? A full house consists of one pair and one three of a kind.	
15	Ashley's drink contains 8 ounces of orange juice and 12 ounces of pineapple juice. How many ounces of orange juice must she add to make her drink 60% orange juice?	

Name: _____

	4 points each	
1	I ate a hemispherical scoop of ice cream with diameter 6 inches. If three cubic inches of ice cream contain two grams of fat, how many grams of fat were in the ice cream I ate?	
2	Anna runs one mile every Monday, two miles every Tuesday, four miles every Wednesday, and so on, doubling the length of her run every consecutive day. She does not run on Sundays. How many miles does she run in one week?	
3	Fifteen years ago, my brother was twice my age. Eleven years ago, I was three-fourths of his age. How old is my brother now?	
4	A ball launched in the air follows the path of a parabola with its height given by $h = -10x^2+15x+5$. How high is the ball at its highest point?	
5	If $\tan\theta=12/5$, what is $\sin(2\theta)$?	

	5 points each	
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8	I drop a ball from a height of 6 feet. Every time it bounces, it bounces to one-third of its previous height. How far does the ball travel before it comes to rest?	
9	One day Athena got to school late, so on her way home she decided to drive 10 miles per hour faster. That day, she averaged 24 miles per hour. How fast did she drive on her way to school, assuming she drives at constant speed?	
10	An elliptical swimming pool defined by the equation $9x^2+16y^2-72x-96y+144=0$ is 5 feet deep. How many cubic feet of water are needed to fill the pool?	

	6 points each	
11	The equation $y=x^3-12x^2+44x+k$ has integer coefficients. If its roots form an arithmetic sequence, what is k?	
12	There are 12 students in Esther's calculus class but there are only 10 chairs. What is the probability that both Esther and her friend get seats, assuming that everyone gets to class in random order?	
13	Find the coordinates of the reflection of the point (1,1) about the line y = 2x.	
14	What is the probability of being dealt a full house from a standard deck of 52 cards? A full house consists of one pair and one three of a kind.	
15	Evaluate: $\lim\limits_{n \to 0} \dfrac{\sqrt{n+2} - \sqrt{2}}{n}$	

2007 – 2008 Log1 Contest Round 1
Mu Applications

Name: _____

	4 points each	
1	I ate a hemispherical scoop of ice cream with diameter 6 inches. If three cubic inches of ice cream contain two grams of fat, how many grams of fat were in the ice cream I ate?	
2	Anna runs one mile every Monday, two miles every Tuesday, four miles every Wednesday, and so on, doubling the length of her run every consecutive day. She does not run on Sundays. How many miles does she run in one week?	
3	Fifteen years ago, my brother was twice my age. Eleven years ago, I was three-fourths of his age. How old is my brother now?	
4	There are 4 pink marbles and 2 green marbles in Urn A and 5 pink marbles and 6 green marbles in Urn B. Andrew randomly moves 2 marbles from Urn A into Urn B, then draws a third marble from Urn A. What is the probability that the last marble he draws is pink?	
5	If $\tan\theta=12/5$, what is $\sin(2\theta)$?	

	5 points each	
6	Roque forgot to study for his 50-question True/False test. He guessed True on every third question and False on the rest. The answers to the test were False on every fifth question and True on the rest. What percent did Roque get on his test?	
7	I have two identical charms and three different keys. How many distinguishable ways can I arrange them on a keyring?	
8	I drop a ball from a height of 6 feet. Every time it bounces, it bounces to one-third of its previous height. How far does the ball travel before it comes to rest?	
9	There are 70 students in the marching band, and all of them are either tall or short. Seven students like to run and sing. Seven tall students and thirteen short students like to sing but not run. Thirteen students like to run but not sing. Twelve tall students don't like running or singing. If half of the students in the marching band are tall, how many short students don't like to run or sing?	
10	An elliptical swimming pool defined by the equation $9x^2+16y^2-72x-96y+144=0$ is 5 feet deep. How many cubic feet of water are needed to fill the pool?	

	6 points each	
11	The equation $y=x^3-12x^2+44x+k$ has integer coefficients. If its roots form an arithmetic sequence, what is k?	
12	There are 12 students in Esther's calculus class but there are only 10 chairs. What is the probability that both Esther and her friend get seats, assuming that everyone gets to class in random order?	
13	Find the coordinates of the reflection of the point (1,1) about the line $y = 2x$.	
14	What is the area of the largest rectangle that can be drawn bounded by the x-axis and the curve $y=9-x^2$?	
15	Evaluate: $\lim\limits_{n\to 0}\dfrac{\sqrt{n+2}-\sqrt{2}}{n}$	

2007 – 2008 Log1 Contest Round 1
Applications Answers

	Theta Answers
1	12π [grams]
2	63 [miles]
3	19
4	85/8
5	1/15
6	40%
7	6 [ways]
8	12 [feet]
9	20 [mph]
10	360
11	-48
12	15/22
13	$\left(\frac{1}{5}, \frac{7}{5}\right)$
14	6/4165
15	10 [ounces]

	Alpha Answers
1	12π [grams]
2	63 [miles]
3	19
4	85/8
5	120/169
6	40%
7	6 [ways]
8	12 [feet]
9	20 [mph]
10	60π [ft³]
11	-48
12	15/22
13	$\left(\frac{1}{5}, \frac{7}{5}\right)$
14	6/4165
15	$\frac{\sqrt{2}}{4} \, or \, \frac{1}{2\sqrt{2}}$

	Mu Answers
1	12π [grams]
2	63 [miles]
3	19
4	2/3
5	120/169
6	40%
7	6 [ways]
8	12 [feet]
9	18
10	60π [ft³]
11	-48
12	15/22
13	$\left(\frac{1}{5}, \frac{7}{5}\right)$
14	$12\sqrt{3}$
15	$\frac{\sqrt{2}}{4} \, or \, \frac{1}{2\sqrt{2}}$

Th	Al	Mu	Solution
1	1	1	The volume of the ice cream is $$\frac{1}{2}\left(\frac{4}{3}\pi 3^3\right)=18\pi$$ Set up a ratio: $$\frac{2}{3}=\frac{x}{18\pi}$$ Solve for x: 3x = 36π So x = 12π
2	2	2	1 + 2 + 4 + 8 + 16 + 32 = 63
3	3	3	Let m = my age, and b = my brother's age: 2(m-15) = b-15 4(m-11) = 3(b-11) solve for b.
4	4		The x-coordinate of the vertex of the parabola is at –b/2a, or $\frac{3}{4}$. $-10(\frac{3}{4})^2+15(\frac{3}{4})+5 =$ 85/8.
		4	He initially can either draw 2 pink marbles, 2 green marbles, or one of each from Urn A. Partitioning into these three possibilities and multiplying by the probability of the third marble being pink in each case: $$\left(\frac{4}{6}\cdot\frac{3}{5}\right)\frac{2}{4}+\left(\frac{2}{6}\cdot\frac{1}{5}\right)\frac{4}{4}+\left(2\cdot\frac{4}{6}\cdot\frac{2}{5}\right)\frac{3}{4}=\frac{2}{3}$$ Urn B does not enter into the solution.
5			$$\log_a b=\frac{1}{\log_b a}=\frac{1}{10} \text{ and } \log_b c=\frac{2}{3}.$$ $$\log_a c=\log_a b\bullet\log_b c=\frac{1}{15}$$
	5	5	θ is the middle angle in a 5-12-13 triangle. sinθ = 12/13 and cosθ = 5/13, so sin(2θ) = 2sinθcosθ = 2(12/13)(5/13) = 120/169
6	6	6	He will answer the first 15 questions FFTFFTFFTFFTFFT and the real answers will be TTTTFTTTTFTTTTF. Out of these, he will get 6 questions right, so out of the first 45 questions he will get 18 right. He will also get questions 48 and 50 right, so 20/50 = 40%
7	7	7	Let the 5 items be labeled A, A, B, C, D. If the A's are together then anyone of the other 3 can be opposite them (AABCD, AACBD, AABDC). If they are separated by 1, then anyone of the 3 others can be in the middle (ABACD, ACABD, ADABC). All the rest are the same either by rotating the items or flipping the keychain over. Therefore, there are 6 distinct arrangments.
8	8	8	The ball will travel 6 feet plus twice the sum of the geometric series 2+2/3+2/9+…. The sum of this series is 2/(1-1/3) = 3. So, 6+2(3) = 12
9	9		If the school is x miles away and she drives y mph there, then her average will be $$\frac{2x}{\dfrac{x}{y}+\dfrac{x}{y+10}}=24mph$$ Getting rid of the denominator and dividing both sides by x, we get y(y+10)=12(2y+10), or y^2-14y-120=0. This means y=20 or -6, so she drove 20 mph on her way to school that day.

		9	Make a Venn diagram. A total of 7+13=20 students like singing but not running; 13 students like running but not singing and 7 students like both. So, a total of 40 students like running, singing or both. 12 tall students don't like either leaving 70-40-12 = 18 short students that don't like running or singing. You did not need to know how many tall students there were.
10			If each angle measures 179°, then each exterior angle is 1°. Since the sum of the exterior angles is 360°, the polygon has 360/1=360 sides.
	10	10	The equation of the ellipse is: $\dfrac{(x-4)^2}{4^2}+\dfrac{(y-3)^2}{3^2}=1$. The area is then $\pi ab=12\pi$. The volume is then $12\pi(5)=60\pi$.
11	11	11	Let r-m, r and r+m be the three roots. The sum of the roots is –b/a=12, so 3r=12, r=4. The sum of the roots taken two at a time is 44; that is, r(r-m)+r(r+m)+(r-m)(r+m)= $3r^2-m^2$=44. So, m^2 =4, m=2 or -2. The roots are then 2, 4 and 6. –k is the product of the roots, so k=-48
12	12	12	There are 12C10= 66 ways to seat the class. You want to pick Esther and her friend. Out of the 10 others you want to pick 8. 10C8=45. 45/66=15/22.
13	13	13	The equation of the line perpendicular to y=2x going through (1,1) is 2y+x=3 which intercepts y=2x at the point (3/5.6/5). The reflection is then (2(3/5)-1, 2(6/5)-1) or (1/5, 7/5).
14	14		There are $\dbinom{52}{5}$ ways to pick 5 cards from a deck, so this is the denominator. To get a full house, pick 1 of the 13 ranks to have 3 of a kind and then pick 3 of the fours cards of that rank. (13)(4C3). Now pick one of the 12 other ranks to have a pair and then 2 of that rank (12)(4C2). The probability is then: $\dfrac{\dbinom{13}{1}\dbinom{4}{3}\dbinom{12}{1}\dbinom{4}{2}}{\dbinom{52}{5}}=\dfrac{6}{4165}$
		14	The area of the rectangle will be A=2x(9-x^2). To find the maximum, take the derivative and set it equal to zero: 0=9-3x^2; $x=\sqrt{3}$. Therefore the area is $12\sqrt{3}$.
15			If she adds x ounces, then $\dfrac{8+x}{20+x}=\dfrac{60}{100}=\dfrac{3}{5}$ Solve for x: 40+5x=60+3x; x = 10.
	15	15	Multiply the top and bottom of the fraction by $\sqrt{n+2}+\sqrt{2}$ to get $\dfrac{1}{\sqrt{n+2}+\sqrt{2}}$ When n is zero this is just $\dfrac{1}{2\sqrt{2}}$.