2007 – 2008 Log1 Contest Round 2
Ciphering Question 10

Name: ___________________ 1 minute=9 points; 2 minutes=7 points, 3 minutes=5 points

MU
What is the $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{n+i}$?
Answer

ALPHA
If $\log_a x = \frac{1}{2}$ and $\log_b x = \frac{3}{8}$ then what is $\log_{ab} x$?
Answer

THETA
If $\log_a x = \frac{1}{2}$ and $\log_b x = \frac{3}{8}$ then what is $\log_{ab} x$?
Answer

2007 – 2008 Log1 Contest Round 2
Ciphering Question 9

Name: ___________________ 1 minute=9 points; 2 minutes=7 points, 3 minutes=5 points

MU
Find the exact value of the sine of 195 degrees expressed in the form: $\frac{\sqrt{a}-\sqrt{b}}{c}$, where a, b, and c are integers.
Answer

ALPHA
Find the exact value of the sine of 195 degrees expressed in the form: $\frac{\sqrt{a}-\sqrt{b}}{c}$, where a, b, and c are integers.
Answer

THETA
Find the smallest positive number, c, so that: $cx-\frac{1}{x}=2$ has a root that is a rational number.
Answer

2007 – 2008 Log1 Contest Round 2
Ciphering Question 8

Name: ___________________ 1 minute=9 points; 2 minutes=7 points, 3 minutes=5 points

MU
I flip a fair coin 8 times and obtain 3 heads. What is the probability that no two of the heads occur consecutively?
Answer

ALPHA
I flip a fair coin 8 times and obtain 3 heads. What is the probability that no two of the heads occur consecutively?
Answer

THETA
I flip a fair coin 8 times and obtain 3 heads. What is the probability that no two of the heads occur consecutively?
Answer

2007 – 2008 Log1 Contest Round 2
Ciphering Question 7

Name: ____________________ 1 minute=9 points; 2 minutes=7 points, 3 minutes=5 points

MU
Suppose x is a complex number that satisfies: $x^2 - x + 1 = 0$. What is the value of x^3?
Answer

ALPHA
Suppose x is a complex number that satisfies: $x^2 - x + 1 = 0$. What is the value of x^3?
Answer

THETA
Suppose x is a complex number that satisfies: $x^2 - x + 1 = 0$. What is the value of x^3?
Answer

2007 – 2008 Log1 Contest Round 2
Ciphering Question 6

Name: ___________________ 1 minute=9 points; 2 minutes=7 points, 3 minutes=5 points

MU
The numbers 1, 3, 4, 7 and 9 can be arranged to form 120 five-digit numbers. How many of these are divisible by 11?
Answer

ALPHA
The numbers 1, 3, 4, 7 and 9 can be arranged to form 120 five-digit numbers. How many of these are divisible by 11?
Answer

THETA
The numbers 1, 3, 4, 7 and 9 can be arranged to form 120 five-digit numbers. How many of these are divisible by 11?
Answer

2007 – 2008 Log1 Contest Round 2
Ciphering Question 5

Name: ____________________ 1 minute=9 points; 2 minutes=7 points, 3 minutes=5 points

MU
How many real solutions, x, are there such that $x > \frac{2}{3}$ and $\log_x(6x-4) = 3$?
Answer

ALPHA
How many real solutions, x, are there such that $x > \frac{2}{3}$ and $\log_x(6x-4) = 3$?
Answer

THETA
Bob and Jane are playing a game. They take turns flipping a fair coin and the first to get a head wins. If Bob goes first, what is the probability that he will win?
Answer

2007 – 2008 Log1 Contest Round 2
Ciphering Question 4

Name: ___________________ 1 minute=9 points; 2 minutes=7 points, 3 minutes=5 points

MU
If $\sin x + \cos x = 1.3$, what is $\sin 2x$?
Answer

ALPHA
If $\sin x + \cos x = 1.3$, what is $\sin 2x$?
Answer

THETA
The interior angles of a convex octagon form an arithmetic sequence with a common difference of 4 degrees. What is the measure of the smallest interior angle in degrees?
Answer

2007 – 2008 Log1 Contest Round 2
Ciphering Question 3

Name: ___________________ 1 minute=9 points; 2 minutes=7 points, 3 minutes=5 points

MU

If $f(x) = x^3 + 3x^2 + x + 2$, find a value of c on the interval [0,1] which satisfies the conclusion of the Mean Value Theorem for differentiation.

Answer

ALPHA

What is the largest prime divisor of $2^{16} - 16$?

Answer

THETA

What is the largest prime divisor of $2^{16} - 16$?

Answer

2007 – 2008 Log1 Contest Round 2
Ciphering Question 2

Name: ___________________ 1 minute=9 points; 2 minutes=7 points, 3 minutes=5 points

MU
Twelve inches of string is equally divided in two and used to make up the perimeter of an equilateral triangle and a regular hexagon. What is the area of the hexagon divided by the area of the triangle?
Answer

ALPHA
Twelve inches of string is equally divided in two and used to make up the perimeter of an equilateral triangle and a regular hexagon. What is the area of the hexagon divided by the area of the triangle?
Answer

THETA
Twelve inches of string is equally divided in two and used to make up the perimeter of an equilateral triangle and a regular hexagon. What is the area of the hexagon divided by the area of the triangle?
Answer

2007 – 2008 Log1 Contest Round 2
Ciphering Question 1

Name: ___________________ 1 minute=9 points; 2 minutes=7 points, 3 minutes=5 points

MU
What is the smallest positive integer with exactly 12 positive factors?
Answer

ALPHA
What is the smallest positive integer with exactly 12 positive factors?
Answer

THETA
What is the smallest positive integer with exactly 12 positive factors?
Answer

2007 – 2008 Log1 Contest Round 2
Ciphering Question 0

Name: ____________________ 1 minute=9 points; 2 minutes=7 points, 3 minutes=5 points

MU
The number a543b, where a and b are digits represents a five-digit number that is divisible by 72. What is a?
Answer

ALPHA
The number a543b, where a and b are digits represents a five-digit number that is divisible by 72. What is a?
Answer

THETA
The number a543b, where a and b are digits represents a five-digit number that is divisible by 72. What is a?
Answer

2007 – 2008 Log1 Contest Round 2
Ciphering Answers

Theta Answers	
0	4
1	60
2	3/2
3	13
4	121 [degrees]
5	2/3
6	12
7	-1
8	5/14
9	[c=]5/4
10	$\frac{3}{14}$

Alpha Answers	
0	4
1	60
2	3/2
3	13
4	0.69
5	2
6	12
7	-1
8	5/14
9	$\frac{\sqrt{2}-\sqrt{6}}{4}$
10	$\frac{3}{14}$

Mu Answers	
0	4
1	60
2	3/2
3	$\frac{-3+\sqrt{21}}{3}$
4	0.69
5	2
6	12
7	-1
8	5/14
9	$\frac{\sqrt{2}-\sqrt{6}}{4}$
10	ln2

2007 – 2008 Log1 Contest Round 1
Logs and Exponents Solutions

Mu	Al	Th	Solution
0	0	0	To be divisible by 72, it must be divisible by 8 and 9. For 8, the number formed by the last 3 digits must be divisible by 8, so b must be 2. Since the sum of the digits must be divisible by 9, a must be 4.
1	1	1	We look at the prime factorization: $n = p_1^{e1}\, p_2^{e2} \ldots p_k^{ek}$. The number of factors is: $(e_1+1)(e_2+1)\ldots(e_k+1)$. Trying a couple of possible factors of 12 with small prime numbers. $2^3 3^2 = 72 \text{ and } 2^2 3(5) = 60$
2	2	2	The area of the triangle is: $\frac{s^2\sqrt{3}}{4} = \frac{2^2\sqrt{3}}{4} = \sqrt{3}$ while the hexagon has side length of 1 and its area is $6\frac{s^2\sqrt{3}}{4} = 6\frac{1^2\sqrt{3}}{4} = \frac{3\sqrt{3}}{2}$
	3	3	$2^{16} - 16 = 16(2^{12} - 1)$ $= 16(2^6 - 1)(2^6 + 1)$ $= 16(63)(65) = 16(7)(9)(5)(13)$
3			f(0)=2, f(1)=7, so you must find a point on the interval whose derivative is (7-2)/1=5. This means solving: $3x^2 + 6x + 1 = 5$ using the quadratic formula.
		4	The angles will be of the form a, a+d, …, a+7d which total to 8a+28d. The interior angles add to (8-2)180=1080 degrees. Solve for a when d=4.
4	4		Squaring both sides: $(\sin x + \cos x)^2 = 1.3^2$ $\sin^2 x + \cos^2 x + 2\sin x \cos x = 1.69$ $1 + \sin 2x = 1.69$
		5	On the first round, he has a ½ chance of winning right away. If neither Bob nor Jane win on the first round (p=1/4), then he has a ½ chance on the second round. His probability is then: $\frac{1}{2} + \frac{1}{8} + \frac{1}{32} + \ldots = \frac{1}{2}\left(\frac{1}{1 - 1/4}\right) = \frac{2}{3}$
5	5		The log equation gives the polynomial equation: $x^3 = 6x - 4$. x=2 is clearly a solution. After dividing that factor out, the other solutions are: $x = -1 \pm \sqrt{3}$, only 1 of which is in the proper range.

6	6	6	To be divisible by 11, the digits in the odd places: first, third and fifth must sum to the same (or different by 11) that those in the second and fourth. Since all the digits add to 24, that total must be 12. The only way to break the digits into two groups that add to 12 are 1, 4, 7 and 3, 9. There are 6 ways of permuting the 1, 4, and 7 and two ways for 3 and 9 or 12 total. i.e. 13497 is divisible by 11.
7	7	7	Multiply both sides of the equation by x. $x^3 - x^2 + x = 0$ $x^3 - (x^2 - x + 1) + 1 = 0$ or $x^3 + 1 = 0$
8	8	8	There are ${}_8C_3$ = 56 ways of obtaining 3 heads. Consider the 5 tails and put x's in between: xTxTxTxTxTx, choosing any 3 of the x's to put a H gives 3 heads with no two in a row ${}_6C_3$ = 20. So 20/56 = 5/14.
		9	Multiply by x to get the quadratic: $cx^2 - 2x - 1 = 0$. To have a rational root, the discriminant, 4+4c must be a perfect square. Solve: 4+4c=9.
9	9		$\sin 195 = -\sin 165 = -\sin 15$ $= -\sin(45 - 30)$ $= -(\sin 45 \cos 30 - \cos 45 \sin 30)$ If you use the half-angle formula, you will get roots of roots.
	10	10	$a^{1/2} = x, a = x^2$. $b = N^{8/3}$, so $ab = x^{14/3}$, $\log_{ab} x = \dfrac{3}{14}$
10			$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{n+i} = \lim_{n\to\infty} \sum_{i=1}^{n} \frac{1/n}{1+i/n}$ $= \lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{n} \frac{1}{1+i/n} = \int_1^2 \frac{1}{x} dx = \ln 2$