2007 – 2008 Log1 Contest Round 3
Theta Individual

Name: _____

	4 points each	
1	If Bill's favorite number is 3 more than the result of multiplying the number of integers from 10 to 20 by the interior degree measure of a regular convex decagon, what is his favorite number?	
2	If Eric can write a math test in 1 hour and Trevor can write one in two hours, how long in hours will it take them to write 6 math tests if they work together?	
3	A square is inscribed in a circle of diameter 4, what is the area inside the circle but outside the square?	
4	What number is halfway between $\frac{1}{3}$ and $\frac{1}{4}$ on the number line?	
5	What is $\frac{1}{2}+\frac{2}{3}+\frac{3}{5}$? Leave answer as an improper fraction.	

	5 points each	
6	If x+y=4 and xy=3, then what is $3x^2+3y^2$?	
7	What the largest base 10 number that can be expressed in 3 digits of base 16?	
8	If $f(x)=x^2+3x-4$, then what is $f(\pi+1)$?	
9	What is the largest positive difference between 2 roots of the equation $y=5x^3+x^2-5x-1$?	
10	Eric's route from his home to school has 5 stoplights, each with a 25% chance of delaying him 2 minutes, and the drive takes him 12 minutes with no stoplights slowing him. Every day he leaves for school 15 minutes before his first period class. What is Eric's expected amount of (average) time between when he arrives at school and when his first class starts?	

	6 points each	
11	If the length of a rectangle is increased 30% and the width is decreased 40%, then the area of the new rectangle is what percentage of the original rectangle's area?	
12	How many times do the graphs of $3x-y=-10$ and $y=x^2+14x+50$ intersect?	
13	A 60° sector is cut from a circle with radius 6 and rolled to form a cone. What is the volume of this cone?	
14	Given two positive odd numbers: j and k with j > k; what is the largest integer that divides (no remainder) the quantity: $j^2 - k^2$?	
15	An isosceles triangle has a perimeter of 50 and an altitude to the base (between the equal angles) of 5. What is the area of the triangle?	

2007 – 2008 Log1 Contest Round 3
Alpha Individual

Name: _____

	4 points each	
1	Evaluate $\cos\left(\sin\left(\tan x\right)\right)$, if x=0.	
2	If Eric can write a math test in 1 hour and Trevor can write one in two hours, how long in hours will it take them to write 6 math tests if they work together?	
3	A square is inscribed in a circle of diameter 4, what is the area inside the circle but outside the square?	
4	If the angle between two points on a circle of radius 4 is 20°, then what is the length of the minor arc defined by these two points?	
5	What are the roots of the equation $y = 3x^2 + 13x + 4$?	

	5 points each	
6	If x+y=4 and xy=3, then what is $3x^2 + 3y^2$?	
7	What the largest base 10 number that can be expressed in 3 digits of base 16?	
8	If $\sin\theta = \frac{12}{13}$, then what is $\cos 2\theta$?	
9	If the letters of the word INDIVIDUAL are rearranged, then how many distinct possibilities are there?	
10	Eric's route from his home to school has 5 stoplights, each with a 25% chance of delaying him 2 minutes, and the drive takes him 12 minutes with no stoplights slowing him. Every day he leaves for school 15 minutes before his first period class. What is Eric's expected amount of (average) time between when he arrives at school and when his first class starts?	

	6 points each	
11	If the points $(-1,17)$, $(2,8)$, and $(3,17)$ are on the parabola $y = Ax^2 + Bx + C$, then what is $(A + B + C)$?	
12	What is the sum of the following infinite series: $1 - \frac{1}{2} - \frac{1}{4} + \frac{1}{8} - \frac{1}{16} - \frac{1}{32} + \frac{1}{16} - \ldots$	
13	A 60° sector is cut from a circle with radius 6 and rolled to form a cone. What is the volume of this cone?	
14	Given two positive odd numbers: j and k with j > k; what is the largest integer that divides (no remainder) the quantity: $j^2 - k^2$?	
15	An isosceles triangle has a perimeter of 50 and an altitude to the base (between the equal angles) of 5. What is the area of the triangle?	

2007 – 2008 Log1 Contest Round 3
Mu Individual

Name: _____

	4 points each	
1	Evaluate $\cos\left(\sin\left(\tan x\right)\right)$, if x=0.	
2	Evaluate: $\frac{d^2y}{dx^2}(4x^3+7x^2+11x+1)$.	
3	A square is inscribed in a circle of diameter 4, what is the area inside the circle but outside the square?	
4	If the angle between two points on a circle of radius 4 is 20°, then what is the length of the minor arc defined by these two points?	
5	What are the roots of the equation $y=3x^2+13x+4$?	

	5 points each	
6	Find the determinant of the 3x3 matrix: $\begin{bmatrix} 6 & -8 & 5 \\ -6 & 2 & -4 \\ 9 & 3 & 8 \end{bmatrix}$	
7	What the largest base 10 number that can be expressed in 3 digits of base 16?	
8	If $\sin\theta=\frac{12}{13}$, then what is $\cos 2\theta$?	
9	If the letters of the word INDIVIDUAL are rearranged, then how many distinct possibilities are there?	
10	What is the sum of the complex (ignore real roots) fifth roots of 1?	

	6 points each	
11	If the points $(-1,17)$, $(2,8)$, and $(3,17)$ are on the parabola $y=Ax^2+Bx+C$, then what is $(A+B+C)$?	
12	What is the sum of the following infinite series: $1-\frac{1}{2}-\frac{1}{4}+\frac{1}{8}-\frac{1}{16}-\frac{1}{32}+\frac{1}{16}-\ldots$	
13	A 60° sector is cut from a circle with radius 6 and rolled to form a cone. What is the volume of this cone?	
14	A guardsman wants to build a rectangular containment center along a 200m wall. If he has 1000m of electric fencing and has to use the entire wall to form part of one side of the enclosure, then what is the maximum area of such a center?	
15	A sphere of radius 2 is centered at the origin. Planes determined by the equations z=1 and z=-1 slice the sphere into three pieces. What is the volume of the sphere piece that is between the two planes?	

2007 – 2008 Log1 Contest Round 3
Individual Answers

Theta Answers	
1	1587
2	4 [hours]
3	$4\pi - 8$
4	$\frac{7}{24}$
5	$\frac{53}{30}$
6	30
7	FFF_{16}=4095
8	$= \pi^2 + 5\pi$
9	2
10	0.5 [minutes] or 30 seconds
11	78%
12	0
13	$\pi\frac{\sqrt{35}}{3}$
14	8
15	60

Alpha Answers	
1	1
2	4 [hours]
3	$4\pi - 8$
4	$\frac{4\pi}{9}$
5	$x = -4$ *and* $-\frac{1}{3}$
6	30
7	FFF_{16}=4095
8	$-\frac{119}{169}$
9	302400
10	0.5 [minutes] or 30 seconds
11	5
12	$\frac{1}{3}$
13	$\pi\frac{\sqrt{35}}{3}$
14	8
15	60

Mu Answers	
1	1
2	24x+14
3	$4\pi - 8$
4	$\frac{4\pi}{9}$
5	$x = -4$ *and* $-\frac{1}{3}$
6	-108
7	FFF_{16}=4095
8	$-\frac{119}{169}$
9	302400
10	-1
11	5
12	$\frac{1}{3}$
13	$\pi\frac{\sqrt{35}}{3}$
14	90000
15	$\frac{22\pi}{3}$

2007 – 2008 Log1 Contest Round 3
Individual Solutions

Mu	Al	Th	Solution
		1	The interior measure of a angle of a regular decagon using the exterior angle = (180 – 360/10)=144. 144x11+3=1587.
1	1		tan(0)=0 and sin(0)=0 and cos(0)=1.
	2	2	Eric works at a rate of 1/1 and Trevor at a rate of 1/2 so together they work at the rate of 3/2, so to do 6 tests it takes 6 / (3/2) = 4 hours.
3	3	3	The area of the circle is $\pi 2^2 = 4\pi$. The diameter 4 is the diagonal of the square. The area of the square is ½ the product of the two diagonals = ½ (4)(4)=8. Now subtract $4\pi - 8$
		4	
4	4		The circumference of the whole circle is $2\pi(4) = 8\pi$. The portion included in the arc is $\frac{20}{360}8\pi = \frac{4\pi}{9}$
		5	A common denominator is 30. $\frac{1}{2}+\frac{2}{3}+\frac{3}{5} = \frac{15}{30}+\frac{20}{30}+\frac{18}{30} = \frac{53}{30}$
5	5		One can either use the quadratic formula or use grouping to factor: (3x+1)(x+4) giving the roots -4 and -1/3
	6	6	$(x+y)^2 = x^2+2xy+y^2$, so $4^2 = x^2+y^2+2(3)$ and $x^2+y^2 = 10$. $3x^2+3y^2 = 30$
2			The first derivative is $12x^2+14x+11$ and the second derivative is $24x+14$
6			Add the first row to the second getting: $\begin{bmatrix} 6 & -8 & 5 \\ 0 & -6 & 1 \\ 9 & 3 & 8 \end{bmatrix}$ and expand along the first column: 6[(-6)(8)-(3)(1)] – 0[] + 9[(-8)(1)-(-6)(5)] = -108.
7	7	7	$FFF_{16} = 1000_{16} - 1 = 16^3 - 1 = 4095$
		8	$f(\pi+1) = (\pi+1)^2 + 3(\pi+1) - 4$ $= \pi^2 + 2\pi + 1 + 3\pi + 3 - 4$ $= \pi^2 + 5\pi$
8	8		$\cos 2\theta = 1 - 2\sin^2\theta = 1 - 2\frac{144}{169} = -\frac{119}{169}$
		9	This can be factored by grouping: $x^2(5x+1) - 1(5x+1) =$ $(x^2-1)(5x+1) = (x-1)(x+1)(5x+1)$ so the roots -1 and +1 have the largest difference.
9	9		There are 10 letters with 3 I's, and 2 D's so the number of distinct permutations is $\frac{10!}{3!2!} = 302400$
	10	10	The expected wait = (5 lights)*(2 min) * (.25 prob) = 2.5 minutes. Adding the 12 min. gives a .5 minute cushion.
10			The fifth roots of 1 are the roots of the polynomial $x^5 - 1 = 0$. The sum of all the roots is 0. Since 1 is the only real root, the complex ones must add to -1.

Mu	Al	Th	Solution
		11	$LW = (1.3/)(.6w) = .78/w$
11	11		One can substitute each of the points into the parabola and get three equations in A, B and C. Or one can realize that two of the points have the same y-coord. and since a parabola is symmetric the vertex must be at x=1 so the equation must be $y = a(x-1)^2 + b$. Plug two of points to get equations for a and b giving a=3 and b=5. Multiply this out to get A+B+C=5.
		12	One could try graphing but to be sure set the y-coordinates equal to each other and solve. $3x - 10 = x^2 + 14x + 50$ $x^2 + 11x + 60 = 0$ which has a negative discriminant and only complex roots.
12	12		Taking the terms three at a time, the new series is: $\frac{1}{4} + \frac{1}{32} + \frac{1}{256} + \ldots$ and the sum is $\frac{1/4}{1 - 1/8} = \frac{2}{7}$
13	13	13	60° is 1/6 of the circle so the arc which will become the circumference of the base is $\frac{1}{6}(2\pi)(6) = 2\pi$ so the radius of the base is 1. The slant height is 6, so by Pythagoras the height is $\sqrt{35}$ and the volume is $\pi\frac{\sqrt{35}}{3}$
	14	14	Let j=2n+1 and k = 2m+1. $j^2 - k^2 = 4(n^2+n) - 4(m^2+m)$. n^2 and n are either both even or both odd making their sum even; so $4(n^2+n)$ is divisible by 8. The difference between two multiples of 8 is also a multiple of 8.
14			Let x be the additional length along the wall and y the width. The perimeter of fencing is then y+x+y+(x+200)=1000 so x+y=400, y=400-x and the area is (400-x)(200+x) which is maximized when x = 100. The maximum area is then (100+200)(400-100) = 90000.
	15	15	Let the two equal sides be equal to x and the base equal to 2y. The perimeter 2x+2y=50 or x+y=25. By Pythagoras, $25 = x^2 - y^2 = (x+y)(x-y)$. So x-y=1 and y = 12. The area is then 5x12=60.
15			Using the disc method and using horizontal discs, the volume is then $\int_{-1}^{1} \pi\left(\sqrt{4-y^2}\right)^2 dy = \pi\left(4y - \frac{y^3}{3}\right)\Big|_{-1}^{1} = \frac{22}{3}\pi$