2008 – 2009 Log1 Contest Round 1 Theta Matrices & Vectors

Name: _____

	4 points each	
1	Evaluate: $\begin{bmatrix} 17 & 23 \\ 41 & 0 \end{bmatrix} + \begin{bmatrix} 16 & 9 \\ 11 & 52 \end{bmatrix}$	
2	Evaluate: $\begin{bmatrix} 7 & 9 \\ 3 & -12 \end{bmatrix} \begin{bmatrix} 6 & 5 \\ -11 & 4 \end{bmatrix}$	
3	Evaluate: $\begin{bmatrix} 3 & 0 & 2 \\ 1 & 2 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 2 & 0 \\ 4 & 1 \end{bmatrix}$	
4	If $\begin{vmatrix} 15 & x \\ 5 & 1 \end{vmatrix} = \begin{vmatrix} 10 & 5 \\ 2 & 3 \end{vmatrix}$, what is x?	
5	Is the statement AB = BA for all matrices A and B true or false?	

	5 points each	
6	If $A = \begin{bmatrix} 3 & 5 \\ 7 & 0 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 4 \\ 2 & 4 \\ 6 & 3 \end{bmatrix}$, find 4A – 3B.	
7	Find the sum of entries of A. $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}^2$	
8	Evaluate: $\begin{vmatrix} 3 & 1 & 2 \\ 6 & 3 & 4 \\ 9 & 7 & 5 \end{vmatrix}$	
9	Find the area of a triangle with vertices (2, 6), (3, 1), and (4, 6).	
10	What is the determinant of A^{-1}? $A = \begin{bmatrix} x & x \\ 4 & x+2 \end{bmatrix}$	

	6 points each	
11	Solve for x and y: $\begin{bmatrix} 2x & 1 & 5 \\ 9 & 12 & y \end{bmatrix} + \begin{bmatrix} 3y & 0 & 7 \\ 2 & 4 & 6x \end{bmatrix} = \begin{bmatrix} 11 & 1 & 12 \\ 11 & 16 & 9 \end{bmatrix}$	
12	What is the transpose of $\begin{bmatrix} 2 & 7 \\ 7 & 3 \\ 7 & 8 \end{bmatrix}$?	
13	Find A^{-1}, if it exists, where $A = \begin{bmatrix} 2 & 4 \\ 3 & 6 \end{bmatrix}$	
14	Give the trace of an n x n identity matrix.	
15	What value of x makes A singular? $A = \begin{bmatrix} 7 & 101 & 91 \\ x & 97 & 65 \\ 3 & 52 & 39 \end{bmatrix}$	

2008 – 2009 Log1 Contest Round 1
Alpha Matrices & Vectors

Name: _____

	4 points each	
1	Evaluate: $\begin{bmatrix} 17 & 23 \\ 41 & 0 \end{bmatrix} + \begin{bmatrix} 16 & 9 \\ 11 & 52 \end{bmatrix}$	
2	Evaluate: $\begin{bmatrix} 3 & 0 & 2 \\ 1 & 2 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 2 & 0 \\ 4 & 1 \end{bmatrix}$	
3	If $\begin{vmatrix} 15 & x \\ 5 & 1 \end{vmatrix} = \begin{vmatrix} 10 & 5 \\ 2 & 3 \end{vmatrix}$, what is x?	
4	Is the statement AB = BA for all matrices A and B true or false?	
5	Evaluate: $\begin{vmatrix} 3 & 1 & 2 \\ 6 & 3 & 4 \\ 9 & 7 & 5 \end{vmatrix}$	

	5 points each	
6	Find the area of a triangle with vertices (2, 6), (3, 1), and (4, 6).	
7	What is the determinant of A^{-1}? $A = \begin{bmatrix} x & x \\ 4 & x+2 \end{bmatrix}$	
8	Solve for x and y: $\begin{bmatrix} 2x & 1 & 5 \\ 9 & 12 & y \end{bmatrix} + \begin{bmatrix} 3y & 0 & 7 \\ 2 & 4 & 6x \end{bmatrix} = \begin{bmatrix} 11 & 1 & 12 \\ 11 & 16 & 9 \end{bmatrix}$	
9	What is the transpose of $\begin{bmatrix} 2 & 7 \\ 7 & 3 \\ 7 & 8 \end{bmatrix}$?	
10	Find A^{-1}, if it exists, where $A = \begin{bmatrix} 2 & 4 \\ 3 & 6 \end{bmatrix}$	

	6 points each	
11	Give the trace of an n x n identity matrix.	
12	What value of x makes A singular? $A = \begin{bmatrix} 7 & 101 & 91 \\ x & 97 & 65 \\ 3 & 52 & 39 \end{bmatrix}$	
13	What is the larger eigenvalue of $\begin{bmatrix} 6 & -5 \\ 3 & -2 \end{bmatrix}$?	
14	Calculate the magnitude of the vector <2, -2, 4>.	
15	If u = <3, -1, 7> and v = <-2, -4, 3>, what is $u \bullet v$?	

2008 – 2009 Log1 Contest Round 1 Mu Matrices & Vectors

Name: _____

	4 points each	
1	Evaluate: $\begin{bmatrix} 17 & 23 \\ 41 & 0 \end{bmatrix} + \begin{bmatrix} 16 & 9 \\ 11 & 52 \end{bmatrix}$	
2	Evaluate: $\begin{bmatrix} 3 & 0 & 2 \\ 1 & 2 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 2 & 0 \\ 4 & 1 \end{bmatrix}$	
3	If $\begin{vmatrix} 15 & x \\ 5 & 1 \end{vmatrix} = \begin{vmatrix} 10 & 5 \\ 2 & 3 \end{vmatrix}$, what is x?	
4	If $A = \begin{bmatrix} 3 & 5 \\ 7 & 0 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 4 \\ 2 & 4 \\ 6 & 3 \end{bmatrix}$, find $4A - 3B$.	
5	Find the sum of entries of A. $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}^2$	

	5 points each	
6	Evaluate: $\begin{vmatrix} 3 & 1 & 2 \\ 6 & 3 & 4 \\ 9 & 7 & 5 \end{vmatrix}$	
7	Find the area of a triangle with vertices (2, 6), (3, 1), and (4, 6).	
8	What is the determinant of A^{-1}? $A = \begin{bmatrix} x & x \\ 4 & x+2 \end{bmatrix}$	
9	Find A^{-1}, if it exists, where $A = \begin{bmatrix} 2 & 4 \\ 3 & 6 \end{bmatrix}$	
10	What value of x makes A singular? $A = \begin{bmatrix} 7 & 101 & 91 \\ x & 97 & 65 \\ 3 & 52 & 39 \end{bmatrix}$	

	6 points each	
11	What is the larger eigenvalue of $\begin{bmatrix} 6 & -5 \\ 3 & -2 \end{bmatrix}$?	
12	Calculate the magnitude of the vector <2, -2, 4>.	
13	What is the cofactor of 2 in matrix A? $A = \begin{bmatrix} 7 & 4 & 9 \\ 2 & 5 & -3 \\ 0 & 1 & -2 \end{bmatrix}$	
14	If u = <6, 2, -1> and v = <-1, -2, 5>, find the value of v x u. Use angle bracket notation.	
15	What is the cosine of the angle between the vectors u and v, where u = <-1, -2, 2> and v = <-8, 0, 6> ?	

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Applications Answers

Theta Answers	
1	$\begin{bmatrix} 33 & 32 \\ 52 & 52 \end{bmatrix}$
2	$\begin{bmatrix} -57 & 71 \\ 150 & -33 \end{bmatrix}$
3	$\begin{bmatrix} 11 & 8 \\ 21 & 6 \end{bmatrix}$
4	x = -1
5	False
6	$\begin{bmatrix} 9 & 8 \\ 22 & -12 \\ -14 & -5 \end{bmatrix}$
7	30
8	-3
9	5
10	$\frac{1}{x^2 - 2x}$ or $\frac{1}{x(x-2)}$
11	x = 1 y = 3
12	$\begin{bmatrix} 2 & 7 & 7 \\ 7 & 3 & 8 \end{bmatrix}$
13	No inverse
14	n
15	5

Alpha Answers	
1	$\begin{bmatrix} 33 & 32 \\ 52 & 52 \end{bmatrix}$
2	$\begin{bmatrix} 11 & 8 \\ 21 & 6 \end{bmatrix}$
3	x = -1
4	False
5	-3
6	5
7	$\frac{1}{x^2 - 2x}$ or $\frac{1}{x(x-2)}$
8	x = 1 y = 3
9	$\begin{bmatrix} 2 & 7 & 7 \\ 7 & 3 & 8 \end{bmatrix}$
10	No inverse
11	n
12	5
13	3
14	$2\sqrt{6}$
15	19

Mu Answers	
1	$\begin{bmatrix} 33 & 32 \\ 52 & 52 \end{bmatrix}$
2	$\begin{bmatrix} 11 & 8 \\ 21 & 6 \end{bmatrix}$
3	x = -1
4	$\begin{bmatrix} 9 & 8 \\ 22 & -12 \\ -14 & -5 \end{bmatrix}$
5	30
6	-3
7	5
8	$\frac{1}{x^2 - 2x}$ or $\frac{1}{x(x-2)}$
9	No inverse
10	5
11	3
12	$2\sqrt{6}$
13	17
14	<-8, 29, 10>
15	$\frac{2}{3}$

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Applications Solutions

Th	Al	Mu	Solution
1	1	1	$\begin{bmatrix} 17+16 & 23+9 \\ 41+11 & 0+52 \end{bmatrix} = \begin{bmatrix} 33 & 32 \\ 52 & 52 \end{bmatrix}$
2			$\begin{bmatrix} (7)(6)+(9)(-11) & (7)(5)+(9)(4) \\ (3)(6)+(-12)(-11) & (3)(5)+(-12)(4) \end{bmatrix} = \begin{bmatrix} 42+-99 & 35+36 \\ 18+132 & 15-48 \end{bmatrix} = \begin{bmatrix} -57 & 71 \\ 150 & -33 \end{bmatrix}$
3	2	2	$\begin{bmatrix} (3)(1)+(0)(2)+(2)(4) & (3)(2)+(0)(0)+(2)(1) \\ (1)(1)+(2)(2)+(4)(4) & (1)(2)+(2)(0)+(4)(1) \end{bmatrix} =$ $\begin{bmatrix} 3+0+8 & 6+0+2 \\ 1+4+8 & 2+0+4 \end{bmatrix} = \begin{bmatrix} 11 & 8 \\ 13 & 6 \end{bmatrix}$
4	3	3	(15)(1) – 5x = (10)(3) – (5)(2) 15 - 5x = 30 – 10 -5x = 5 x = -1
5	4		Even if AB exists, BA may not because for example if A is a 3 x 2 and B is a 2 x 4, BA does not exist.
6		4	$4\begin{bmatrix} 3 & 5 \\ 7 & 0 \\ 1 & 1 \end{bmatrix} - 3\begin{bmatrix} 1 & 4 \\ 2 & 4 \\ 6 & 3 \end{bmatrix} =$ $\begin{bmatrix} 12 & 20 \\ 28 & 0 \\ 4 & 4 \end{bmatrix} - \begin{bmatrix} 3 & 12 \\ 6 & 12 \\ 18 & 9 \end{bmatrix} = \begin{bmatrix} 9 & 8 \\ 22 & -12 \\ -14 & -5 \end{bmatrix}$
7		5	$\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 4+3 & 6+6 \\ 2+2 & 3+4 \end{bmatrix} = \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix}$ 7 + 12 + 4 + 7 = 30
8	5	6	$3\begin{vmatrix} 3 & 4 \\ 7 & 5 \end{vmatrix} - 1\begin{vmatrix} 6 & 4 \\ 9 & 5 \end{vmatrix} + 2\begin{vmatrix} 6 & 3 \\ 9 & 7 \end{vmatrix} =$ 3(15–28) – 1(30-36) + 2(42–27) = -39 + 6 +30 = -3
9	6	7	$\frac{1}{2}\begin{vmatrix} 2 & 6 & 1 \\ 3 & 1 & 1 \\ 4 & 6 & 1 \end{vmatrix} = \frac{1}{2}(2\begin{vmatrix} 1 & 1 \\ 6 & 1 \end{vmatrix} - 6\begin{vmatrix} 3 & 1 \\ 4 & 1 \end{vmatrix} + 1\begin{vmatrix} 3 & 1 \\ 4 & 6 \end{vmatrix}) =$ $\frac{1}{2}[2(1-6) - 6(3-4) + 1(18-4)] =$ $\frac{1}{2}(-10 + 6 + 14) = 5$ Alternatively, you could easily plot the points, and find the area (1/2)bh = (1/2)(2)(5)
10	7	8	$\left\|A^{-1}\right\| = \frac{1}{\|A\|} = \frac{1}{x^2+2x-4x} = \frac{1}{x^2-2x}$

11	8		By matrix addition: $2x + 3y = 11$ $y + 6x = 9$ $2x + 3(9 - 6x) = 11$ $2x + 27 - 18x = 11$ $-16x = -16$ $x = 1$ $y = 9 - 6(1) = 3$
12	9		Interchange rows and columns.
13	10	9	There is no inverse, because the determinant of A is 0. $(2)(6) - (3)(4) = 0$
14	11		The trace is a sum of all diagonal elements. Those are all 1, so the sum would be n.
15	12	10	Normally, you would solve for x when the determinant of A is 0. However, note that the 3rd column is 13 times the 1st column if we let x=5. Anytime one column is a multiple of another, the determinant is 0.
	13	11	$\lvert A - \lambda I \rvert = 0$ $\begin{vmatrix} 6-\lambda & -5 \\ 3 & -2-\lambda \end{vmatrix} = 0$ $(6 - \lambda)(-2 - \lambda) - (-5)(3) = 0$ $-12 - 6\lambda + 2\lambda + \lambda^2 + 15 = 0$ $\lambda^2 - 4\lambda + 3 = 0$ $(\lambda - 1)(\lambda - 3) = 0$ Larger eigenvalue is 3.
	14	12	$\sqrt{(2)^2 + (-2)^2 + (4)^2} = \sqrt{4 + 4 + 16} = \sqrt{24} = 2\sqrt{6}$
	15		$u_1v_1 + u_2v_2 + u_3v_3$ $(3)(-2) + (-1)(-4) + (7)(3) = -6 + 4 + 21 = 19$
		13	To find the cofactor of 2, cross out the row and column containing the element 2 and find the determinant of the remaining elements. However, the sign is opposite because it is a determinant and in C_{ij}, i + j is an odd number. $C_{21} = -1\begin{vmatrix} 4 & 9 \\ 1 & -2 \end{vmatrix} = -1(-8 - 9) = 17$
		14	$\begin{vmatrix} i & j & k \\ -1 & -2 & 5 \\ 6 & 2 & -1 \end{vmatrix} = i\begin{vmatrix} -2 & 5 \\ 2 & -1 \end{vmatrix} - j\begin{vmatrix} -1 & 5 \\ 6 & -1 \end{vmatrix} + k\begin{vmatrix} -1 & -2 \\ 6 & 2 \end{vmatrix} =$ $i(2 - 10) - j(1 - 30) + k(-2 + 12) = -8i + 29j + 10k =$ $\langle -8, 29, 10 \rangle$
		15	The equation is: $\cos\theta = \frac{u \bullet v}{\|u\|\|v\|}$ $\cos\theta = \frac{(-1)(-8) + (-2)(0) + (2)(6)}{\sqrt{(-1)^2 + (-2)^2 + (2)^2}\sqrt{(-8)^2 + (0)^2 + (6)^2}}$ $\cos\theta = \frac{8 + 0 + 12}{\sqrt{1 + 4 + 4}\sqrt{64 + 0 + 36}}$ $\cos\theta = \frac{20}{\sqrt{9}\sqrt{100}} = \frac{20}{(3)(10)} = \frac{2}{3}$