

2009 - 2010 Log1 Contest Round 1
Theta Probability

Name: _____

4 points each		
1	In order to go to the Fair, Leia must have finished all her homework which happens with probability $\frac{2}{3}$; and independently cleaned her room which has probability $\frac{1}{4}$. What is the probability that Leia can go to the Fair?	
2	Leia and Luke go to the concession stand for lunch. For lunch, they each can choose to have a salad or not, pick a peanut butter sandwich, turkey sandwich or hamburger; then choose potato chips, french fries or carrot sticks. If they both randomly choose lunch, what is the probability they get the same lunch?	
3	The first game that Luke plays is "Pick a Marble". There are 20 red marbles that earn you a gumball, 7 blue marbles for which you get a teddy bear and 5 white marbles that gets you a doll. If Luke draws a marble, what is the probability that he does not get a gumball?	
4	Leia can win a special prize if the three marbles (from problem 3) she draws are red, white and blue in any order. What is the probability of Leia winning the special prize?	
5	In one game, the player is given three fair coins and flips them. The player wins if no heads are showing. What is the probability of winning the game?	

5 points each		
6	Leia has 2 quarters, 3 dimes and 2 nickels in her pocket. If it costs 35 cents to buy some cotton candy and she randomly draws two coins, what is the probability that she will have drawn <u>enough</u> money?	
7	I have a weighted 6-sided die so that instead of equal probabilities, the probabilities of getting 1, 2, 3, 4, 5 or 6 are in the ratio 1:2:3:4:5:6 respectively. What is the probability of rolling a 3?	
8	Luke rolls a fair six-sided die three times. He wins if the three values are in strictly increasing order, such as: 1, 3, 6. What is the probability that he wins?	
9	Beginning at 4:00pm, Lando and Han Solo play one of four games at the fair. They play for exactly 15 minutes and then "instantly" go to another game so that after an hour, they each have played all the games. If Lando plays the games in order, and Han Solo randomly plays them; what is the probability that they will play at least one game together at the same time?	
10	Luke challenges Leia to a game rolling the high number on a fair 6-sided die. Since Luke challenged Leia, he must roll a larger number than Leia does on a single roll. What is the probability that Luke wins?	

6 points each

11	Now Luke gets 2 dice and only needs the maximum of his two dice to beat Leia's roll of a single die. What is Luke's chance of winning this game?	
12	Luke and Leia walk over to the petting zoo. There are a total of 42 groundhogs. 18 groundhogs have ebola, 14 are bearded, 27 have an extra nose, and there is one lucky groundhog with none of the above. Three groundhogs are bearded and have ebola, eight are bearded and have an extra nose, eight are infected with ebola and have an extra nose, and one is bearded, infected with ebola, and has an extra nose. Leia wants to pet a groundhog, but only wants to touch one that only has one problem: ebola, beard, or extra nose. If Luke throws a random groundhog at Leia, what is the probability that she will want to pet it?	
13	Luke and Leia are going to play the "Ring Toss" game. From past experience, Luke has a .3, and Leia a .5 probability of winning. The chance that at least one of them wins is .6. Leia goes first and wins, what is the (conditional) probability that Luke loses? Express your answer as a decimal.	
14	Leia is rolling a fair 4-sided die (tetrahedron) labeled 1, 2, 3 and 4. In four rolls of the die, what is the probability of getting the numbers 1, 2, 3, and 4 in some order?	
15	Leia wins the "Guess a Number" game if the digit she names is in a randomly chosen integer from 000 to 999 inclusive. What is the probability that Leia wins if she selects 7?	

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4	Leia can win a special prize if the three marbles (from problem 3) she draws are red, white and blue in any order. What is the probability of Leia winning the special prize?	
5	Before Leia picks a marble (again using the marbles from problem 3), the operator secretly removes a non-red marble. What is the probability that when Leia picks her marble, she gets a blue one?	

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6	Leia has 2 quarters, 3 dimes and 2 nickels in her pocket. If it costs 35 cents to buy some cotton candy and she randomly draws two coins, what is the probability that she will have drawn <u>enough</u> money?	
7	The possible outcomes of an experiment are the positive integers: 1, 2, 3, ... with the probability of each number being $\frac{1}{3}$ of the probability of getting the number before it. What is the probability of getting a 2?	
8	Luke rolls a fair six-sided die three times. He wins if the three values are in strictly increasing order, such as: 1, 3, 6. What is the probability that he wins?	
9	Beginning at 4:00pm, Lando and Han Solo play one of four games at the fair. They play for exactly 15 minutes and then "instantly" go to another game so that after an hour, they each have played all the games. If Lando plays the games in order, and Han Solo randomly plays them; what is the probability that they will play at least one game together at the same time?	
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11	Now Luke gets 2 dice and only needs the maximum of his two dice to beat Leia's roll of a single die. What is Luke's chance of winning this game?	
12	Luke and Leia walk over to the petting zoo. There are a total of 42 groundhogs. 18 groundhogs have ebola, 14 are bearded, 27 have an extra nose, and there is one lucky groundhog with none of the above. Three groundhogs are bearded and have ebola, eight are bearded and have an extra nose, eight are infected with ebola and have an extra nose, and one is bearded, infected with ebola, and has an extra nose. Leia wants to pet a groundhog, but only wants to touch one that only has one problem: ebola, beard, or extra nose. If Luke throws a random groundhog at Leia, what is the probability that she will want to pet it?	
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14	Leia is rolling a fair 4-sided die (tetrahedron) labeled 1, 2, 3 and 4. In four rolls of the die, what is the probability of getting the numbers 1, 2, 3, and 4 in some order?	
15	Luke is given a bag with 3 coins in it. One coin is black on both sides, another is red on both sides, the third is red on one side and black on the other; otherwise the coins are identical. Luke randomly takes a coin from the bag and lays it flat on the table. If the top side shows red, what is the probability that the bottom is also red?	

2009 - 2010 Log1 Contest Round 1

Mu Probability

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4 points each		
1	In order to go to the Fair, Leia must have finished all her homework which happens with probability $\frac{2}{3}$; and independently cleaned her room which has probability $\frac{1}{4}$. What is the probability that Leia can go to the Fair?	
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3	The first game that Luke plays is "Pick a Marble". There are 20 red marbles that earn you a gumball, 7 blue marbles for which you get a teddy bear and 5 white marbles that gets you a doll. If Luke draws a marble, what is the probability that he does not get a gumball?	
4	In the dart game, the board is 3 feet by 5 feet with many non-overlapping colored circles. There are 8 red circles with radius 4 inches, 12 green circles of radius 3 inches, and 20 yellow circles of radius 2 inches. Hitting any circle gets a prize. If Luke throws a dart at random and hits the board, what is the probability that he wins a prize?	
5	Luke flips two fair coins and gets \$1 for each head that comes up. What are his expected (average) winnings each time he plays the game?	

5 points each		
6	Leia has 2 quarters, 3 dimes and 2 nickels in her pocket. If it costs 35 cents to buy some cotton candy and she randomly draws two coins, what is the probability that she will have drawn <u>enough</u> money?	
7	The possible outcomes of an experiment are the positive integers: 1, 2, 3, ... with the probability of each number being $\frac{1}{3}$ of the probability of getting the number before it. What is the probability of getting a 2?	
8	Luke rolls a fair six-sided die three times. He wins if the three values are in strictly increasing order, such as: 1, 3, 6. What is the probability that he wins?	
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11	Now Luke gets 2 dice and only needs the maximum of his two dice to beat Leia's roll of a single die. What is Luke's chance of winning this game?	
12	Luke and Leia walk over to the petting zoo. There are a total of 42 groundhogs. 18 groundhogs have ebola, 14 are bearded, 27 have an extra nose, and there is one lucky groundhog with none of the above. Three groundhogs are bearded and have ebola, eight are bearded and have an extra nose, eight are infected with ebola and have an extra nose, and one is bearded, infected with ebola, and has an extra nose. Leia wants to pet a groundhog, but only wants to touch one that only has one problem: ebola, beard, or extra nose. If Luke throws a random groundhog at Leia, what is the probability that she will want to pet it?	
13	A continuous probability density is defined as $p(x)=kx$ on the interval $[0,8]$ and 0 elsewhere. What is k ?	
14	Luke and Leia are playing the "Ring Toss" game. From past experience, Luke has a .3 probability of winning and Leia has a .5 chance of winning. Han Solo is standing by and did not get to see who (Luke or Leia) tossed the ring, but he noticed that they won. What is the probability that Luke tossed the winning ring? Express your answer as a fraction.	
15	Luke is given a bag with 3 coins in it. One coin is black on both sides, another is red on both sides, the third is red on one side and black on the other; otherwise the coins are identical. Luke randomly takes a coin from the bag and lays it flat on the table. If the top side shows red, what is the probability that the bottom is also red?	

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Probability Answers

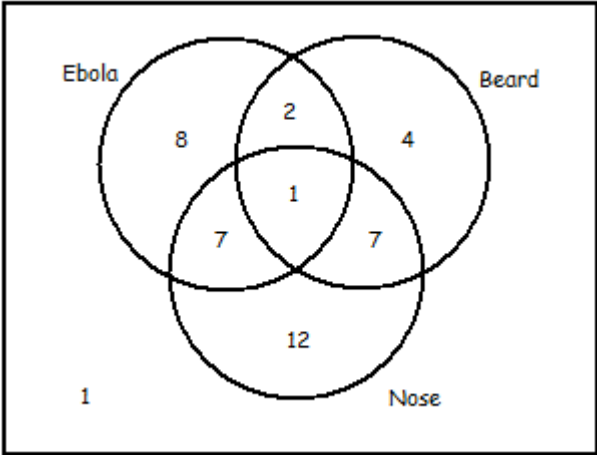
Theta Answers	
1	$\frac{1}{6}$
2	$\frac{1}{18}$
3	$\frac{3}{8}$
4	$\frac{35}{248}$
5	$\frac{1}{8}$
6	$\frac{1}{3}$
7	$\frac{1}{7}$
8	$\frac{5}{54}$
9	$\frac{5}{8}$
10	$\frac{5}{12}$
11	$\frac{125}{216}$
12	$\frac{4}{7}$
13	0.6
14	$\frac{3}{32}$
15	$\frac{271}{1000}$

Alpha Answers	
1	$\frac{1}{6}$
2	$\frac{1}{18}$
3	$\frac{3}{8}$
4	$\frac{35}{248}$
5	$\frac{77}{372}$
6	$\frac{1}{3}$
7	$\frac{2}{9}$
8	$\frac{5}{54}$
9	$\frac{5}{8}$
10	$\frac{5}{12}$
11	$\frac{125}{216}$
12	$\frac{4}{7}$
13	0.6
14	$\frac{3}{32}$
15	$\frac{2}{3}$

Mu Answers	
1	$\frac{1}{6}$
2	$\frac{1}{18}$
3	$\frac{3}{8}$
4	$\frac{79\pi}{540}$
5	\$1
6	$\frac{1}{3}$
7	$\frac{2}{9}$
8	$\frac{5}{54}$
9	$\frac{5}{8}$
10	$\frac{5}{12}$
11	$\frac{125}{216}$
12	$\frac{4}{7}$
13	$\frac{1}{32}$
14	$\frac{3}{8}$
15	$\frac{2}{3}$

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Probability Solutions

Mu	Al	Th	Solution
1	1	1	Since the events are independent, the probability that they both occur is the product of the probabilities. $(2/3)(1/4)=1/6$.
2	2	2	There are 18 possible lunches, (2 choices for salad)(3 choices of sandwich)(3 choices of sides). What Leia gets, there is a 1/18 chance that Luke matches it.
3	3	3	There are 12 marbles that will not get him a gumball. $12/32 = 3/8$.
	4	4	There are 20 ways to get a red, 5 ways for white and 7 ways for blue. There are 32 choose 3 ways of picking three marbles. The answer is then $(20 \times 5 \times 7)/32C3 = 35/248$.
4			The board is 36×60 square inches and the circles are $8(4 \times 4) + 12(3 \times 3) + 20(2 \times 2)\pi = 316\pi$ square inches. Dividing, the probability of hitting a circle is $\frac{79\pi}{540}$
5			To find expected value multiply each outcome by its probability and add. So $0(1/4) + 1(1/2) + 2(1/4) = \1 .
	5		Use the notation B1 to be a blue marble on the first draw, etc. Then given that a non-red (blue or white) marble was taken out first: $P(B2) = P(B2 B1)P(B1) + P(B2 W1)P(W1) = (6/31)(7/12) + (7/31)(5/12) = 77/372$. $P(B1) = 7/12$ since you know it was either blue or white, 12 marbles total with 7 blues.
		5	In order to win, the player must get three tails, each having probability 1/2; so the chance of winning is 1/8.
6	6	6	There are 7 coins total and $7C2 = 21$ ways of drawing 2 coins. Either quarter and any dime will be enough makes 6 ways; both quarters make 7 ways. $7/21 = 1/3$
		7	There are 21 total parts, so each part is worth 1/21. Getting a 3 then has a probability of $3/21 = 1/7$.
7	7		The probabilities for a geometric sequence with common ratio 1/3. The sum is the first term a divided by $(1 - 1/3)$. So $\frac{a}{1 - 1/3} = 1$ and $a = 2/3$. The probability of a 2 is then $(2/3)(1/3) = 2/9$.
8	8	8	There are $6 \times 6 \times 6 = 216$ total possibilities. The desirable outcomes can be enumerated. 1: 2 (3,4,5,6) 3(4,5,6) 4(5,6) 5(6) 10 ways 2: 3(4,5,6) 4(5,6) 5(6) 6 ways 3: 4(5,6) 5(1) 3 ways 4: 5(6) 1 way $20/216 = 5/54$
9	9	9	There are $4! = 24$ ways of permuting ABCD. If any of the 4 letters are in the right place, the other 3 can be interchanged in any way $4(3!) = 24$. This double counts those that with 2 letters the same, so we need to subtract those $(4C2)2! = 12$. Now those with 3 letters the same need to be added back $(4C3)1!$, etc. $(4C1)3! - (4C2)2! + (4C3)1! - (4C4)0! = 24 - 12 + 4 - 1 = 15$. So $15/24 = 5/8$.
10	10	10	There are 36 possible combinations for Luke and Leia's rolls. Only in 15 of them is Luke higher than Leia. $15/36 = 5/12$.
11	11	11	There are now 216 possible combinations of the 3 dice. If Leia rolls a 1, 2, 3, 4, 5 or 6; there are respectively 35, 32, 27, 20, 11 and 0 possible rolls for Luke to beat her. Therefore the probability is $125/216$. Students may be tempted to consider $(5/12) + (5/12) - (5/12)(5/12)$ as the probability that either of Luke's tosses beats Leia. This however, assumes that the probability of Luke's second roll beating Leia is <u>independent</u> of her first roll winning. It is not - if Leia rolls a 6, it does not matter how many times Luke rolls, he will lose.

12	12	12	 <p style="text-align: center;"> $\frac{8 + 4 + 12}{42} = \frac{4}{7}$ </p>
	13	13	<p> $P(\text{Luke or Leia}) = P(\text{Luke}) + P(\text{Leia}) - P(\text{Luke and Leia})$ so $P(\text{Luke and Leia}) = .2$. So, $P(\text{not Luke and Leia}) = .5 - .2 = .3$. $P(\sim\text{Luke} \text{Leia}) = .3/.5 = .6$ </p>
13			<p> $\int_0^8 kx dx = \frac{k}{2} x^2 \Big _0^8 = 32k$ so $k=1/32$. This could also be done by just finding the area of a right triangle. </p>
	14	14	<p>There are $4! = 24$ ways to reorder the numbers 1, 2, 3 and 4. There are $4^4 = 256$ possible rolls. $24/256 = 3/32$.</p>
14			<p> $P(\text{Lu} \text{Win}) = P(\text{Lu and W}) / P(\text{W})$ $= P(\text{W} \text{Lu})P(\text{Lu}) / [P(\text{W} \text{Lu})P(\text{Lu}) + P(\text{W} \text{Le})P(\text{Le})] =$ $(.3)(.5) / [(.3)(.5) + (.5)(.5)] = .3/.8 = 3/8$ </p>
		15	<p>There are 1000 numbers that could be chosen. Say, Leia chooses 7, there are 9 choices that are not 7 for each of the three digits for 729 total. Thus, 271 do contain a 7 and she wins with probability $271/1000$.</p>
15	15		<p>If we consider each of the six sides as equally likely, then there are 3 sides that could be showing. Two of them have red on the other side as well.</p>