2009 – 2010 Log1 Contest Round 2
Theta Sequences and Series

Name: _____

4 points each		
1	What is the next term in the arithmetic sequence: 3, 7, 11… ?	
2	What is the sum of the first 10 terms of the arithmetic sequence 2, 5, 8…	
3	If $a_1 = 3x + 17$ and $a_4 = 73 - 4x$ are elements of an arithmetic sequence, then what is the common difference in terms of x?	
4	Let $a_n = \frac{1}{n}$. What is $a_1 + a_2 + a_3$ expressed as an improper fraction?	
5	Evaluate: $\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+\ldots}}}}$	

5 points each		
6	Let S_n be the sum of the first n positive perfect squares. What is $S_3 + S_5 + S_7$?	
7	What is the positive difference between the harmonic mean and the arithmetic mean of 4 and 8?	
8	What is the sum of the following infinite geometric series: $54 + 36 + 24 + \ldots$?	
9	A square is inscribed in a circle of diameter 2. Another circle is then inscribed in the square and then a square is inscribed into that circle. If this process continues infinitely, what is the total area of the squares?	
10	If $0.8\overline{4}\ldots$ is expressed as a reduced fraction $\frac{n}{m}$, then what is n+m?	

6 points each		
11	A sequence is defined recursively by: $a_0 = 4,\ a_1 = 1,\ a_{n+2} = \frac{2a_{n+1} + a_n}{3}$. What is a_4 , expressed as an improper fraction?	
12	Evaluate the sum of the following infinite series: $\frac{1}{4} - \frac{3}{16} + \frac{1}{64} - \frac{3}{256} + \ldots$	
13	Let a_n be an arithmetic sequence such that a_1 , a_4 , and a_{13} creates a geometric sequence. If $a_{14} = 58$, then what is the common difference of a_n ?	
14	Find the sum and express your answer as an improper fraction: $\sum_{n=2}^{10}\left(\frac{2}{n^2-1}\right)$	
15	A set of lines in the first quadrant has a special quality. The x-intercepts of the lines form an arithmetic sequence with common difference of 1 and the y-intercepts of the lines form a geometric sequence with a common ratio of 2. What is the point of intersection between the third line of this set and the line $y = x$, if the first line has a standard equation of $x + y = 1$?	

2009 – 2010 Log1 Contest Round 2
Alpha Sequences and Series

Name: _____

	4 points each	
1	What is the next term in the arithmetic sequence: 3, 7, 11… ?	
2	What is the sum of the first 10 terms of the arithmetic sequence 2, 5, 8…	
3	If $a_1 = 3x + 17$ and $a_4 = 73 - 4x$ are elements of an arithmetic sequence, then what is the common difference in terms of x?	
4	Let $a_n = \frac{1}{n}$. What is $a_1 + a_2 + a_3$ expressed as an improper fraction?	
5	Evaluate: $\cfrac{13}{12 + \cfrac{13}{12 + \cfrac{13}{12 + \cfrac{13}{12 + \ldots}}}}$	

	5 points each	
6	Let S_n be the sum of the first n positive perfect squares. What is $S_3 + S_5 + S_7$?	
7	What is the positive difference between the harmonic mean and the arithmetic mean of 4 and 8?	
8	What is the sum of the following infinite geometric series: $54 + 36 + 24 + \ldots$?	
9	A square is inscribed in a circle of diameter 2. Another circle is then inscribed in the square and then a square is inscribed into that circle. If this process continues infinitely, what is the total area of the squares?	
10	If $0.89\overline{772}\ldots$ is expressed as a reduced fraction $\frac{n}{m}$, then what is n+m?	

	6 points each	
11	A sequence is defined recursively by: $a_0 = 4,\ a_1 = 1,\ a_{n+2} = \frac{2a_{n+1} + a_n}{3}$. What is a_4 , expressed as an improper fraction?	
12	Evaluate the sum of the following infinite series: $\frac{1}{4} - \frac{3}{16} + \frac{1}{64} - \frac{3}{256} + \ldots$	
13	Let a_n be an arithmetic sequence such that a_1 , a_4 , and a_{13} creates a geometric sequence. If $a_{14} = 58$, then what is the common difference of a_n ?	
14	Find the sum and express your answer as an improper fraction: $\sum_{n=2}^{10}\left(\frac{2}{n^2 - 1}\right)$	
15	Evaluate: $\sum_{x=0^\circ}^{180^\circ} \sin^2 x$.	

2009 – 2010 Log1 Contest Round 2
Mu Sequences and Series

Name: _____

	4 points each	
1	What is the next term in the arithmetic sequence: 3, 7, 11… ?	
2	What is the sum of the first 10 terms of the arithmetic sequence 2, 5, 8…	
3	If $a_1 = 3x + 17$ and $a_4 = 73 - 4x$ are elements of an arithmetic sequence, then what is the common difference in terms of x?	
4	Let $a_n = \frac{1}{n}$. What is $\sum_{n=1}^{4} a_n$ expressed as an improper fraction?	
5	Evaluate: $\cfrac{13}{12+\cfrac{13}{12+\cfrac{13}{12+\cfrac{13}{12+\ldots}}}}$	

	5 points each	
6	Let S_n be the sum of the first n positive perfect squares. What is $S_3 + S_5 + S_7$?	
7	What is the positive difference between the harmonic mean and the arithmetic mean of 4 and 8?	
8	What is the sum of the following infinite geometric series: $54 + 36 + 24 + \ldots$?	
9	An equilateral triangle is inscribed in a circle of diameter 2. Another circle is then inscribed in the triangle and then an equilateral triangle is inscribed into that circle. If this process continues infinitely, what is the total area of the triangles?	
10	If $0.89\overline{772}$ … is expressed as a reduced fraction $\frac{n}{m}$, then what is n+m?	

	6 points each	
11	A sequence is defined recursively by: $a_0 = 4,\ a_1 = 1,\ a_{n+2} = \frac{2a_{n+1} + a_n}{3}$. What is $\lim_{n\to\infty} a_n$, expressed as an improper fraction?	
12	Evaluate the sum of the following infinite series: $\frac{1}{4} - \frac{3}{16} + \frac{1}{64} - \frac{3}{256} + \ldots$	
13	Let a_n be an arithmetic sequence such that a_1 , a_4 , and a_{13} creates a geometric sequence. If $a_{14} = 58$, then what is the common difference of a_n ?	
14	Find the sum and express your answer as an improper fraction: $\sum_{n=2}^{10}\left(\frac{2}{n^2-1}\right)$	
15	Evaluate: $\sum_{x=0^\circ}^{180^\circ} \sin^2 x$.	

2009 – 2010 Log1 Contest Round 2
Sequences and Series Answers

Theta Answers	
1	15
2	155
3	$\frac{56-7x}{3}$
4	$\frac{11}{6}$
5	3
6	209
7	$\frac{2}{3}$
8	162
9	4
10	83
11	$\frac{16}{9}$
12	$\frac{1}{15}$
13	4
14	$\frac{72}{55}$
15	$\left(\frac{12}{7}, \frac{12}{7}\right)$

Alpha Answers	
1	15
2	155
3	$\frac{56-7x}{3}$
4	$\frac{11}{6}$
5	1
6	209
7	$\frac{2}{3}$
8	162
9	4
10	167
11	$\frac{16}{9}$
12	$\frac{1}{15}$
13	4
14	$\frac{72}{55}$
15	90

Mu Answers	
1	15
2	155
3	$\frac{56-7x}{3}$
4	$\frac{25}{12}$
5	1
6	209
7	$\frac{2}{3}$
8	162
9	$\sqrt{3}$
10	167
11	$\frac{7}{4}$
12	$\frac{1}{15}$
13	4
14	$\frac{72}{55}$
15	90

2009 – 2010 Log1 Contest Round 2
Sequences and Series Solutions

Mu	Al	Th	Solution
1	1	1	Common difference = 4 $\therefore 11+4=15$
2	2	2	$t_1 = 2$ $t_{10} = 2 + 9*3 = 29$ $10(\frac{2+29}{2}) = 155$
3	3	3	Common difference = r $3x+17+3r = 73-4x$ $r = \frac{56-7x}{3}$
	4	4	$\frac{1}{1}+\frac{1}{2}+\frac{1}{3}=\frac{11}{6}$
4			$\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}=\frac{25}{12}$
		5	$x=\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+...}}}}$ $x=\sqrt{6+x}$ $x^2=6+x$ $x^2-x-6=0$ $x=-2,3$ $x=3$
5	5		$x=\frac{13}{12+\frac{13}{12+\frac{13}{12+\frac{13}{12+...}}}}$ $x=\frac{13}{12+x}$ $x(x+12)=13$ $x^2+12x-13=0$ $x=1,-13$ $x=1$
6	6	6	$S_3=(1+4+9)=14$ $S_5=(1+4+9+16+25)=55$ $S_7=(1+4+9+16+25+36+49)=140$ $S_3+S_5+S_7=209$
7	7	7	Harmonic mean $=\frac{2ab}{a+b}$ Arithmetic mean $=\frac{a+b}{2}$ $\left\lvert\frac{2(4)(8)}{4+8}-\frac{4+8}{2}\right\rvert=\frac{2}{3}$
8	8	8	$r=\frac{36}{54}=\frac{2}{3}$ $\frac{t_1}{1-r}=\frac{54}{1-\frac{2}{3}}=162$

	9	9	The diameter of the circle is the diagonal of the square. The first square has area then of (2)(2)/2 = 2. The diameter of the second circle is the side length of the first square equals $\sqrt{2}$ so that the area of the second square is 1. The area of all the squares is: $2+1+\frac{1}{2}+\frac{1}{4}+\cdots=4$
9			The center of the circle is the centroid of the triangle which means the diameter of the second circle is half the diameter of the first circle and the area will be $\frac{1}{4}$ of the first. The first triangle will have area $t_1=\frac{3\sqrt{3}}{4}$ $TA=\frac{\frac{3\sqrt{3}}{4}}{1-\frac{1}{4}}=\sqrt{3}$
		10	$\frac{n}{m}=0.8\overline{4}$ $\frac{10n}{m}=8\frac{4}{9}=\frac{76}{9}$ $\frac{n}{m}=\frac{76}{90}=\frac{38}{45}$ $n+m=38+45=83$
10	10		$\frac{n}{m}=0.89\overline{72}$ $1000\frac{n}{m}=897.\overline{72}=897+\frac{72}{99}$ $=897+\frac{8}{11}$ $11000\frac{n}{m}=9875$ $\frac{n}{m}=\frac{9875}{11000}=\frac{79}{88}$ $79+88=167$
	11	11	$a_2=\frac{2(1)+4}{3}=2$ $a_3=\frac{2(2)+1}{3}=\frac{5}{3}$ $a_4=\frac{2(5/3)+2}{3}=\frac{16}{9}$
11			The sequence may be written: $3a_{n+2}-2a_{n+1}-a_n=0$. The solution will be of the form $ax_1^n+bx_2^n$ where x_1,x_2 solve $3x^2-2x-1=0$. The solution $a(-\frac{1}{3})^n+b(1)^n$ can be solved using the first two values. The solution is: $a_n=\frac{9}{4}(-\frac{1}{3})^n+\frac{7}{4}(1)^n$ which goes to 7/4.
12	12	12	Group the series two terms at a time and simplify: $\frac{1}{16}+\frac{1}{256}+\cdots=\frac{1/16}{1-1/16}=\frac{1}{16}\left(\frac{16}{15}\right)=\frac{1}{15}$

13	13	13	Let x be the common difference of a_n: $a_{14} = 58$ $a_{13} = 58 - x$ $a_4 = 58 - 10x$ $a_1 = 58 - 13x$ $\frac{a_{13}}{a_4} = \frac{a_4}{a_1} \rightarrow \frac{58-x}{58-10x} = \frac{58-10x}{58-13x}$ Cross-multiplying and solving for x yields $x = 4$.
14	14	14	$\sum_{n=2}^{10}\left(\frac{2}{n^2-1}\right) = \sum_{n=2}^{10}\left(\frac{1}{n-1} - \frac{1}{n+1}\right)$ $= \left(1 - \frac{1}{3}\right) + \left(\frac{1}{2} - \frac{1}{4}\right) + \left(\frac{1}{3} - \frac{1}{5}\right) + \left(\frac{1}{4} - \frac{1}{6}\right) + \cdots + \left(\frac{1}{8} - \frac{1}{10}\right) + \left(\frac{1}{9} - \frac{1}{11}\right)$ $= 1 + \frac{1}{2} - \frac{1}{10} - \frac{1}{11} = \frac{72}{55}$
		15	line x-int. y-int. $x + y = 1$ (1,0) (0,1) #2 (2,0) (0,2) #3 (3,0) (0,4) Slope of #3 $= -\frac{4}{3}$ $\therefore y = -\frac{4}{3}x + 4$ Solving the system for line #3 and $y = x$ yields the intersection point of $\left(\frac{12}{7}, \frac{12}{7}\right)$.
15	15		Use the identity: $\cos^2(180 - x) = \sin^2(x)$ and the sum becomes: $\sum_{x=0^\circ}^{89^\circ}\left(\sin^2 x + \cos^2 x\right) + \sin^2 90^\circ = 90$