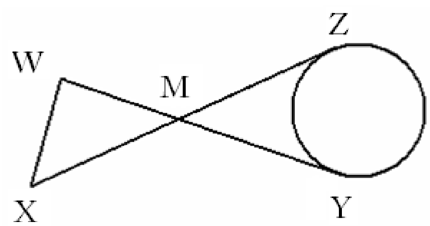
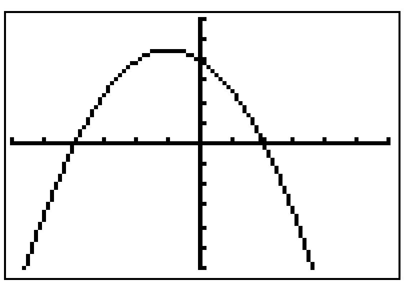
**2009 – 2010 Log1 Contest Round 3**
**Theta Individual**

Name: ___________________

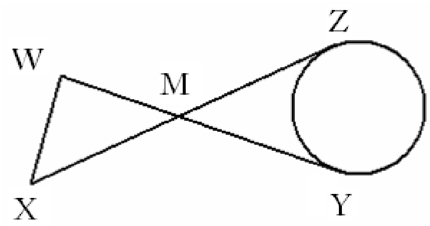
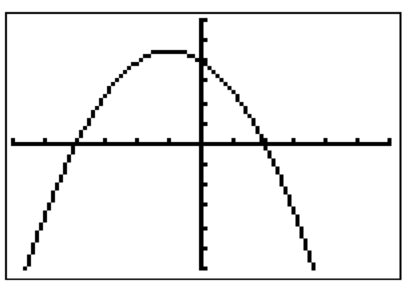
| 4 points each | | |
|---|---|---|
| 1 | $\overline{XZ}$ is tangent to a circle at Z, and $\overline{WY}$ is tangent to the same circle at Y, as shown in the diagram. $m\angle W = 3(m\angle X)$. If $\overline{ZM} = 32$ and $\overline{XM} = 18$, then find the length of $\overline{MY}$.   Note: Picture is not drawn to scale. | |
| 2 | What are the coordinates of the point on the curve $(x-4)^2 + (y+6)^2 = 64$ with the minimum y-value? | |
| 3 | The country of Ozz has two types of coins; a zog has a value of 7 cents and a dag which is worth 11 cents. When I visited there, I got exact change for my $20.00 bill in coins. In how many distinct ways can this be done? | |
| 4 | Determine the equation of the following quadratic in the form $y = ax^2 + bx + c$.  Note: Increments = 1 unit | |
| 5 | Find the product ab if $x^4 + 9 = (x^2 + ax + 3)(x^2 + bx + 3)$ for all $x$. | |

| 5 points each | | |
|---|---|---|
| 6 | A committee of 4 is chosen from 6 firemen and 6 policemen. Determine the number of ways of selecting the committee if it is to contain at least one of each profession. | |
| 7 | What is the coefficient of the term containing $x^3$ in the expansion of $(2-x)(3x+1)^9$? | |
| 8 | Solve for all ordered pairs (x,y) that satisfy $(x+i)(3-iy)=1+13i$. | |
| 9 | What is the sum of the integer values of n, such that $\frac{2n^2-3n+1}{n-2}$ is also an integer? | |
| 10 | Carl the Critter is a creature that resides within the Cartesian plane. He starts on the point $(-3,0)$ and begins crawling eastbound along the function $f(x)=\frac{x^2+3x}{x+1}$. As he glances up ahead, Carl realizes that the path he has chosen has "invisible divides" that go up to infinity. Determine the equations of the asymptotes of the function $f(x)=\frac{x^2+3x}{x+1}$. | |

| 6 points each | | |
|---|---|---|
| 11 | Simplify: $2\left[\frac{4n-8}{4}\left[\frac{n!(n-3)}{(n-2)(n-2)!(n-1)}+\frac{n!}{2(n-2)!}-\frac{1}{2n-4}\right]+1\right]$ | |
| 12 | What is the area of the convex quadrilateral that is formed by connecting the intersections of the curves whose equations are $-2y^2-8x^2=-12x-12y-190$ and $-6y=6x-y^2+31$? | |
| 13 | Using the digits 1, 2, 3, 4, 5, and 6, three two-digit numbers are constructed where each digit is used only once. How many sets (order does not matter) of three two-digit numbers can be made? | |
| 14 | If $\sum_{j=0}^{n} 4^j = 1365$, solve for *n*. | |
| 15 | If $x^2-x+1=0$ what is $x^3-2x^2+2x+1$? | |

**2009 – 2010 Log1 Contest Round 3**
**Alpha Individual**

Name: ____________________

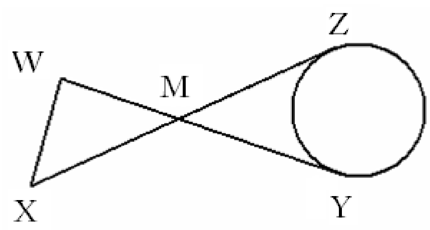
| | 4 points each | |
|---|---|---|
| 1 | $\overline{XZ}$ is tangent to a circle at Z, and $\overline{WY}$ is tangent to the same circle at Y, as shown in the diagram. $m\angle W = 3(m\angle X)$. If $\overline{ZM} = 32$ and $\overline{XM} = 18$, then find the length of $\overline{MY}$.   Note: Picture is not drawn to scale. | |
| 2 | What are the coordinates of the point on the curve $(x-4)^2 + (y+6)^2 = 64$ with the minimum y-value? | |
| 3 | The country of Ozz has two types of coins; a zog has a value of 7 cents and a dag which is worth 11 cents. When I visited there, I got exact change for my $20.00 bill in coins. In how many distinct ways can this be done? | |
| 4 | Determine the equation of the following quadratic in the form $y = ax^2 + bx + c$.  Note: Increments = 1 unit | |
| 5 | Simplify <u>completely</u>: $(2\sin\frac{5\pi}{12} + 3\cos\frac{5\pi}{12})^2 + (3\sin\frac{5\pi}{12} - 2\cos\frac{5\pi}{12})^2$ | |

| 5 points each | | |
|---|---|---|
| 6 | A committee of 4 is chosen from 6 firemen and 6 policemen. Determine the number of ways of selecting the committee if it is to contain at least one of each profession. | |
| 7 | What is the coefficient of the term containing $x^3$ in the expansion of $(2-x)(3x+1)^9$? | |
| 8 | Solve for all ordered pairs (x,y) that satisfy $(x+i)(3-iy)=1+13i$. | |
| 9 | What is the sum of the integer values of n, such that $\frac{2n^2-3n+1}{n-2}$ is also an integer? | |
| 10 | There are 6 complex sixth-roots of 1. What is the product of the real parts of these roots? | |

| 6 points each | | |
|---|---|---|
| 11 | Simplify: $2\left[\frac{4n-8}{4}\left[\frac{n!(n-3)}{(n-2)(n-2)!(n-1)}+\frac{n!}{2(n-2)!}-\frac{1}{2n-4}\right]+1\right]$ | |
| 12 | What is the area of the convex quadrilateral that is formed by connecting the intersections of the curves whose equations are $-2y^2-8x^2=-12x-12y-190$ and $-6y=6x-y^2+31$? | |
| 13 | Using the digits 1, 2, 3, 4, 5, and 6, three two-digit numbers are constructed where each digit is used only once. How many sets (order does not matter) of three two-digit numbers can be made? | |
| 14 | If $\sum_{j=0}^{n}4^j=1365$, solve for *n*. | |
| 15 | What is the equation of the plane through the points: (2, 1, 0), (1, 3, 2) and (3, 3, 1) in the form Ax+By+Cz+D=0? | |

**2009 – 2010 Log1 Contest Round 3**
**Mu Individual**

Name: ___________________

| | **4 points each** | |
|---|---|---|
| 1 | $\overline{XZ}$ is tangent to a circle at Z, and $\overline{WY}$ is tangent to the same circle at Y, as shown in the diagram. $m\angle W = 3(m\angle X)$. If $\overline{ZM} = 32$ and $\overline{XM} = 18$, then find the length of $\overline{MY}$.   Note: Picture is not drawn to scale. | |
| 2 | What are the coordinates of the point on the curve $(x-4)^2 + (y+6)^2 = 64$ with the minimum y-value? | |
| 3 | The country of Ozz has two types of coins; a zog has a value of 7 cents and a dag which is worth 11 cents. When I visited there, I got exact change for my $20.00 bill in coins. In how many distinct ways can this be done? | |
| 4 | Using linear differential approximation, compute an approximation to the solution of $x^2 - 10 = 0$ starting from x=3. Express your answer as an improper fraction. | |
| 5 | Simplify completely: $(2\sin\frac{5\pi}{12} + 3\cos\frac{5\pi}{12})^2 + (3\sin\frac{5\pi}{12} - 2\cos\frac{5\pi}{12})^2$ | |

| | **5 points each** | |
|---|---|---|
| 6 | A committee of 4 is chosen from 6 firemen and 6 policemen. Determine the number of ways of selecting the committee if it is to contain at least one of each profession. | |
| 7 | What is the coefficient of the term containing $x^3$ in the expansion of $(2-x)(3x+1)^9$? | |
| 8 | Solve for all ordered pairs (x,y) that satisfy $(x+i)(3-iy) = 1+13i$. | |
| 9 | Taylor is being chased by Mr. Fast! In the frantic pursuit Taylor arrives at a broad river bank, where the river itself is 100 meters wide. There is an ice cream shop across the river and 200 meters downstream from her standing point. She swims at a rate of 2 meters/second and runs at a rate of 4 meters/second. How far should she swim to get across the river and to the ice cream shop in the least amount of time? | |
| 10 | There are 6 complex sixth-roots of 1. What is the product of the real parts of these roots? | |

| 6 points each | | |
|---|---|---|
| 11 | Simplify: $2\left[\frac{4n-8}{4}\left[\frac{n!(n-3)}{(n-2)(n-2)!(n-1)}+\frac{n!}{2(n-2)!}-\frac{1}{2n-4}\right]+1\right]$ | |
| 12 | What is the area of the convex quadrilateral that is formed by connecting the intersections of the curves whose equations are $-2y^2-8x^2=-12x-12y-190$ and $-6y=6x-y^2+31$? | |
| 13 | Using the digits 1, 2, 3, 4, 5, and 6, three two-digit numbers are constructed where each digit is used only once. How many sets (order does not matter) of three two-digit numbers can be made? | |
| 14 | Evaluate $\int_0^1 \frac{dx}{\sqrt{x}\left(1+\sqrt[3]{x}\right)}$. | |
| 15 | What is the equation of the plane through the points: (2, 1, 0), (1, 3, 2) and (3, 3, 1) in the form Ax+By+Cz+D=0? | |

**2009 – 2010 Log1 Contest Round 3**
**Individual Answers**

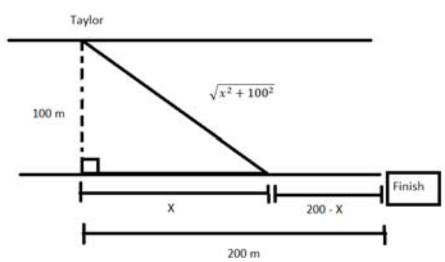
| Theta Answers | |
|---|---|
| 1 | 32 |
| 2 | (4,-14) |
| 3 | 26 |
| 4 | $y=-\frac{1}{2}x^2-x+4$ |
| 5 | -6 |
| 6 | 465 |
| 7 | 4212 |
| 8 | $(2,-5)$; $(-\frac{5}{3},6)$ |
| 9 | 8 |
| 10 | x = -1<br>y = x + 2 |
| 11 | $n^3-n^2-4n+1$ |
| 12 | 96 units$^2$ |
| 13 | 120 |
| 14 | n = 5 |
| 15 | 2 |

| Alpha Answers | |
|---|---|
| 1 | 32 |
| 2 | (4,-14) |
| 3 | 26 |
| 4 | $y=-\frac{1}{2}x^2-x+4$ |
| 5 | 13 |
| 6 | 465 |
| 7 | 4212 |
| 8 | $(2,-5)$; $(-\frac{5}{3},6)$ |
| 9 | 8 |
| 10 | $-\frac{1}{16}$ |
| 11 | $n^3-n^2-4n+1$ |
| 12 | 96 units$^2$ |
| 13 | 120 |
| 14 | n = 5 |
| 15 | $2x-3y+4z-1=0$ |

| Mu Answers | |
|---|---|
| 1 | 32 |
| 2 | (4,-14) |
| 3 | 26 |
| 4 | $\frac{19}{6}$ |
| 5 | 13 |
| 6 | 465 |
| 7 | 4212 |
| 8 | $(2,-5)$; $(-\frac{5}{3},6)$ |
| 9 | $\frac{200\sqrt{3}}{3}$ [meters] |
| 10 | $-\frac{1}{16}$ |
| 11 | $n^3-n^2-4n+1$ |
| 12 | 96 units$^2$ |
| 13 | 120 |
| 14 | $6-\frac{3\pi}{2}$ |
| 15 | $2x-3y+4z-1=0$ |

**2009 – 2010 Log1 Contest Round 3**
**Individual Solutions**

| Mu | Al | Th | Solution |
|---|---|---|---|
| 1 | 1 | 1 | Segments tangent to a circle from the same point are congruent.<br>Therefore $\overline{ZM} = \overline{MY} = 32$ |
| 2 | 2 | 2 | The circle has a radius of 8 units and is centered at the point (4,-6). Hence the minimum y-value is 8 units below the center of the circle, located at (4,-14). |
| 3 | 3 | 3 | Starting with 2000 successively subtract multiples of 7 until a multiple of 11 is reached. 2000=6(7)+178(11). Since 7 and 11 are relatively prime, the next way to make 2000 will be (6+11)(7)+(178-7)(11) and so on to 281(7)+3(11). There are 26 ways to do this. |
| | 4 | 4 | The quadratic has factors y=a(x+4)(x-2). Plugging in the y-intercept (0,4) yields a=-1/2.<br>Thus : $y = -\frac{1}{2}(x+4)(x-2) = -\frac{1}{2}x^2 - x + 4$ |
| 4 | | | Linear approximation is determined by<br>$f(x) \approx f(a) + f'(a)(x-a)$. We rewrite the quadratic so that $x^2 = 10$<br>and let $f(x) = x^2$.<br>This follows with $f'(x) = 2x$.<br>Take a=3. Letting f(x) = 10, our linear approximation becomes<br><br>$10 \approx f(3) + f'(3)(x-3)$. Solving this approximation gives us x ≈ 19/6. |
| | | 5 | Expand the RHS of the equation and equate the coefficients to the LHS to get 3 equations,<br>a + b = 0, ab + 6 = 0, and<br>3a + 3b = 0. Clearly then ab =-6 |
| 5 | 5 | | $(2\sin\theta + 3\cos\theta)^2 + (3\sin\theta - 2\cos\theta)^2 =$<br>$4\sin^2\theta + 9\cos^2\theta + 9\sin^2\theta + 4\cos^2\theta = 13$ |
| 6 | 6 | 6 | For the committee to have one of each profession, we eliminate the total number of possibilities of a committee composed entirely of policemen or firemen. Hence the number of ways of selecting the committee is determined by<br><br>$\binom{12}{4} - \binom{6}{4}\binom{6}{0} - \binom{6}{4}\binom{6}{0} = 495 - 30 = 465$ |

| | | | |
|---|---|---|---|
| 7 | 7 | 7 | The expression can be rewritten using the binomial theorem as<br>$$(2-x)\left[(3x)^9+\binom{9}{1}(3x)^8(1)+\binom{9}{2}(3x)^7(1)^2+\ldots\right]$$<br>So then the terms containing $x^3$ are: $2\binom{9}{6}3^3x^3-\binom{9}{7}3^2x^3=4212x^3$ |
| 8 | 8 | 8 | Expand the LHS of the equation and equate the real parts and imaginary parts on both sides of the equation. With two equations 3x + y = 1 and 3 - xy = 13, use a method of solving a system of equations to find x and y, which result in<br>$(2,-5);\ (-\frac{5}{3},6)$. |
| | 9 | 9 | $\frac{2n^2-3n+1}{n-2}=\frac{(2n+1)(n-2)+3}{n-2}=2n+1+\frac{3}{n-2}$<br>So, n-2 divides 3 and n must be -1, 1, 3 or 5 which sum to 8. |
| 9 | | | <br><br>Setting up the problem results in a diagram similar to the one above. Hence the shortest minimal distance that Taylor must travel to the ice cream shop is:<br>$\sqrt{x^2+100^2}+(200-x)$, where $\sqrt{x^2+100^2}$ is the distance that Taylor swims and $(200-x)$ is the distance that she runs.<br><br>Given that this is true, the time it takes her to travel to the ice cream shop is determined by the distance she needs to travel times the rate at which she travels. Hence we see that time is determined by:<br>$t=\frac{1}{2}\sqrt{x^2+100^2}+\frac{1}{4}(200-x)$<br>Recalling that she swims at 2 m/s and runs at 4 m/s, we take the reciprocal of these rates to get s/m and multiply that by distance (m) to obtain time.<br>To minimize the time, we take the derivative of the time function and set it equal to 0:<br>$0=\frac{x}{2}(x^2+100^2)^{-\frac{1}{2}}-\frac{1}{4}$<br>Solving for x yields<br>$x=\frac{100\sqrt{3}}{3}$, which is used to find<br>$\left(\frac{200\sqrt{3}}{3},\frac{600-100\sqrt{3}}{3}\right)$. |
| | | 10 | Because f(x) does not exist when x = 1 is in the denominator, there is a vertical asymptote at x = -1. Simplifying f(x) using long division yields the oblique or slant asymptote y = x + 2. |

| | | | |
|---|---|---|---|
| 10 | 10 | | The complex sixth-roots of 1 are:<br>$cis(0), cis(\frac{\pi}{3}), cis(\frac{2\pi}{3}), cis(\pi), cis(\frac{4\pi}{3}), cis(\frac{5\pi}{3})$. The real parts are<br>$\cos(0), \cos(\frac{\pi}{3}), \cos(\frac{2\pi}{3}), \cos(\pi), \cos(\frac{4\pi}{3}), \cos(\frac{5\pi}{3})$ and the product is:<br>$(1)(\frac{1}{2})(-\frac{1}{2})(-1)(-\frac{1}{2})(\frac{1}{2}) = -\frac{1}{16}$ |
| 11 | 11 | 11 | The expression inside the outer brackets simplifies to $\frac{n(n^2 - n - 4) + 1}{2}$. Multiplying this by 2 yields $n(n^2 - n - 4) + 1$, or<br>$n^3 - n^2 - 4n + 1$. |
| 12 | 12 | 12 | The first equation can be reduced by dividing both the RHS and LHS by 2. Through substitution, both equations can be solved simultaneously for x, where $x = \pm 4$. Plugging values of x back into either equation yields the solution pairs (4,-5), (4, 11), (-4,-1), and (-4,7). The quadrilateral formed by connecting these points is a trapezoid with bases of length 8 and 16, and a height of 8. Thus the area of the trapezoid is 96. |
| 13 | 13 | 13 | There are 6! ways of ordering the 6 digits which when partitioned into 2-digit pieces give the possible triples. This counts different orders (12,34,56) different from (34,12,56) so we need to divide by 6 to get the possible sets. 6!/6 = 5! = 120. |
| | 14 | 14 | Consider the sum of a geometric series, $S_n = a\frac{r^{n+1} - 1}{r - 1}$.<br>Substitute in the given values and simplify to get<br>$4^{n+1} = 4096$ . Solving for n results in n = 5. |
| 14 | | | Using Substitution x=u$^6$,<br>$x = u^6$, $dx = 6u^5 du$, limits stay the same<br>$\int_0^1 \frac{dx}{\sqrt{x}\left(1 + \sqrt[3]{x}\right)} = \int_0^1 \frac{6u^5 du}{u^3\left(1 + u^2\right)}$<br>$= 6\int_0^1 \frac{u^2 du}{\left(1 + u^2\right)} = 6\int_0^1 1 - \frac{1}{\left(1 + u^2\right)} du$<br>$= 6[u - \arctan(u)]_c^1 = 6(1 - \frac{\pi}{4})$<br>$= 6 - \frac{3\pi}{2}$ |
| | | 15 | Divide one polynomial by the other to get: $x^3 - 2x^2 + 2x + 1 = (x - 1)(x^2 - x + 1) + 2$<br>Therefore the answer is 2. |
| 15 | 15 | | One can solve a system of equations but the most straight forward method is to solve:<br>$\begin{vmatrix} x-2 & y-1 & z-0 \\ 1-2 & 3-1 & 2-0 \\ 3-2 & 3-3 & 1-2 \end{vmatrix} = 0$<br>Simplifying: $2x - 3y + 4z - 1 = 0$ |