2010 – 2011 Log1 Contest Round 1
Theta Sequences and Series

Name: _____

4 points each		
1	What is the next term in the following arithmetic sequence -4, 19, 42, … ?	
2	How many terms does the following arithmetic sequence contain -2, 11, …, 310 ?	
3	What is the sum of the digits of the 195th term of the arithmetic sequence -41, -26, … ?	
4	What is the term, a_5 of the recursive sequence defined by $a_{n+1} = 2a_n - a_{n-1} + 3$ and given $a_0 = 1$, $a_1 = 3$?	
5	The first five terms of a geometric sequence are: $a, 2, b, \frac{9}{2}, c$. What is the product bc?	

5 points each		
6	Find the sum of the terms of the infinite geometric series 5, 2, $\frac{4}{5}$, …	
7	Harry Potter drops a rubber ball from a height of 20 feet. Each time the ball drops it rebounds 80% of the height from which it is falling. What is the total distance (in feet) traveled by the bouncing rubber ball?	
8	What is the sum of the series $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \frac{1}{16} - \cdots$?	
9	Given an arithmetic sequence with $a_1 = 2x-11$, $a_3 = x+4$, $a_5 = 3x-2$, find the value of a_8.	
10	Given a square with side length 4, connect the midpoints of the sides to form a second square inscribed within the first. Using the inscribed square, connect its midpoints to form another inscribed square. If this process is continued, what is the area of the 11th square?	

6 points each		
11	The sum of an infinite geometric series is $\frac{32}{5}$. The second term of the series is -2. What is the value of the first term of the series?	
12	Find the coefficient of x^4 in the expansion of $\left(\frac{1}{4x^2} + 2x\right)^{10}$	
13	Johnny begins with 10 gallons of water in the morning of the first day. Each day, he uses one-third of his water and at the end of the day; gets 5 more gallons. If this continues indefinitely (in the limit), how many gallons of water will he have after his nightly allotment?	
14	A circle is inscribed within an equilateral triangle of side length 12. A second equilateral triangle is inscribed within the circle and a second circle is inscribed within the second equilateral triangle. If this process continues, what is the area of the 6th inscribed circle?	
15	Given $(x + 3)$ and $(x^3 + 11x^2 + 40x + 48)$ form the 1st and 3rd terms of an infinite geometric sequence respectively, for what values of x will the common ratio be between -1 and 1 exclusive?	

2010 – 2011 Log1 Contest Round 1
Alpha Sequences and Series

Name: _____

4 points each		
1	What is the next term in the following arithmetic sequence -4, 19, 42, … ?	
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4	What is the term, a_5 of the recursive sequence defined by $a_{n+1} = 2a_n - a_{n-1} + 3$ and given $a_0 = 1$, $a_1 = 3$?	
5	Evaluate $\sum_{n=1}^{14}(5n-7)$	

5 points each		
6	Find the sum of the terms of the infinite geometric series $5, 2, \frac{4}{5}, \ldots$	
7	Harry Potter drops a rubber ball from a height of 20 feet. Each time the ball drops it rebounds 80% of the height from which it is falling. What is the total distance (in feet) traveled by the bouncing rubber ball?	
8	What is the sum of the series $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \frac{1}{16} - \cdots$?	
9	Given an arithmetic sequence with $a_1 = 2x-11$, $a_3 = x+4$, $a_5 = 3x-2$, find the value of a_8.	
10	Evaluate $\prod_{i=0}^{6} 18\left(\frac{1}{3}\right)^i$	

6 points each		
11	The sum of an infinite geometric series is $\frac{32}{5}$. The second term of the series is -2. What is the value of the first term of the series?	
12	Find the coefficient of x^4 in the expansion of $\left(\frac{1}{4x^2} + 2x\right)^{10}$	
13	Johnny begins with 10 gallons of water in the morning of the first day. Each day, he uses one-third of his water and at the end of the day; gets 5 more gallons. If this continues indefinitely (in the limit), how many gallons of water will he have after his nightly allotment?	
14	A circle is inscribed within an equilateral triangle of side length 12. A second equilateral triangle is inscribed within the circle and a second circle is inscribed within the second equilateral triangle. If this process continues, what is the area of the 6th inscribed circle?	
15	Evaluate $\frac{2}{5} + \frac{3}{25} + \frac{4}{125} + \cdots$	

2010 – 2011 Log1 Contest Round 1
Mu Sequences and Series

Name: _____

	4 points each	
1	What is the next term in the following arithmetic sequence -4, 19, 42, … ?	
2	How many terms does the following arithmetic sequence contain -2, 11, …, 310 ?	
3	What is the sum of the digits of the 195th term of the arithmetic sequence -41, -26, … ?	
4	Evaluate $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$	
5	Evaluate $\sum_{n=1}^{14} (5n-7)$	

	5 points each	
6	Find the sum of the terms of the infinite geometric series $5, 2, \frac{4}{5}, \ldots$	
7	Harry Potter drops a rubber ball from a height of 20 feet. Each time the ball drops it rebounds 80% of the height from which it is falling. What is the total distance (in feet) traveled by the bouncing rubber ball?	
8	What is the sum of the series $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \frac{1}{16} - \cdots$?	
9	State whether the following series is absolutely convergent, divergent, or conditionally convergent $\sum_{k=1}^{\infty} (-1)^{k+1} \frac{lnk}{k}$	
10	Evaluate $\prod_{i=0}^{6} 18\left(\frac{1}{3}\right)^{i}$	

	6 points each	
11	The sum of an infinite geometric series is $\frac{32}{5}$. The second term of the series is -2. What is the value of the first term of the series?	
12	Find the coefficient of x^4 in the expansion of $\left(\frac{1}{4x^2} + 2x\right)^{10}$	
13	Johnny begins with 10 gallons of water in the morning of the first day. Each day, he uses one-third of his water and at the end of the day; gets 5 more gallons. If this continues indefinitely (in the limit), how many gallons of water will he have after his nightly allotment?	
14	Find the interval of convergence for the series $\sum_{n=1}^{\infty} (-1)^n \frac{(7x)^{n+1}}{n+1}$	
15	Evaluate $\frac{2}{5} + \frac{3}{25} + \frac{4}{125} + \cdots$	

2010 – 2011 Log1 Contest Round 1
Theta Sequences and Series

Name: _____

	4 points each	
1	What is the next term in the following arithmetic sequence -4, 19, 42, … ?	65
2	How many terms does the following arithmetic sequence contain -2, 11, …, 310 ?	25
3	What is the sum of the digits of the 195th term of the arithmetic sequence -41, -26, … ?	25
4	What is the term, a_5 of the recursive sequence defined by $a_{n+1} = 2a_n - a_{n-1} + 3$ and given $a_0 = 1$, $a_1 = 3$?	41
5	The first five terms of a geometric sequence are: $a, 2, b, \frac{9}{2}, c$. What is the product bc?	$\frac{81}{4}$

	5 points each	
6	Find the sum of the terms of the infinite geometric series 5, 2, $\frac{4}{5}$, …	25/3
7	Harry Potter drops a rubber ball from a height of 20 feet. Each time the ball drops it rebounds 80% of the height from which it is falling. What is the total distance (in feet) traveled by the bouncing rubber ball?	180
8	What is the sum of the series $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \frac{1}{16} - \cdots$?	$\frac{2}{3}$
9	Given an arithmetic sequence with $a_1 = 2x-11$, $a_3 = x+4$, $a_5 = 3x-2$, find the value of a_8.	31
10	Given a square with side length 4, connect the midpoints of the sides to form a second square inscribed within the first. Using the inscribed square, connect its midpoints to form another inscribed square. If this process is continued, what is the area of the 11th square?	1/64

	6 points each	
11	The sum of an infinite geometric series is $\frac{32}{5}$. The second term of the series is -2. What is the value of the first term of the series?	8
12	Find the coefficient of x^4 in the expansion of $\left(\frac{1}{4x^2} + 2x\right)^{10}$	720
13	Johnny begins with 10 gallons of water in the morning of the first day. Each day, he uses one-third of his water and at the end of the day; gets 5 more gallons. If this continues indefinitely (in the limit), how many gallons of water will he have after his nightly allotment?	15 [gallons]
14	A circle is inscribed within an equilateral triangle of side length 12. A second equilateral triangle is inscribed within the circle and a second circle is inscribed within the second equilateral triangle. If this process continues, what is the area of the 6th inscribed circle?	$3\pi/256$
15	Given $(x + 3)$ and $(x^3 + 11x^2 + 40x + 48)$ form the 1st and 3rd terms of an infinite geometric sequence respectively, for what values of x will the common ratio be between -1 and 1 exclusive?	(-5, -3)

2010 – 2011 Log1 Contest Round 1
Alpha Sequences and Series

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	4 points each	
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2	How many terms does the following arithmetic sequence contain -2, 11, …, 310 ?	25
3	What is the sum of the digits of the 195th term of the arithmetic sequence -41, -26, … ?	25
4	What is the term, a_5 of the recursive sequence defined by $a_{n+1} = 2a_n - a_{n-1} + 3$ and given $a_0 = 1$, $a_1 = 3$?	41
5	Evaluate $\sum_{n=1}^{14}(5n-7)$	427

	5 points each	
6	Find the sum of the terms of the infinite geometric series 5, 2, $\frac{4}{5}$, …	25/3
7	Harry Potter drops a rubber ball from a height of 20 feet. Each time the ball drops it rebounds 80% of the height from which it is falling. What is the total distance (in feet) traveled by the bouncing rubber ball?	180
8	What is the sum of the series $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \frac{1}{16} - \cdots$?	$\frac{2}{3}$
9	Given an arithmetic sequence with $a_1 = 2x-11$, $a_3 = x+4$, $a_5 = 3x-2$, find the value of a_8.	31
10	Evaluate $\prod_{i=0}^{6} 18\left(\frac{1}{3}\right)^i$	128/2187

	6 points each	
11	The sum of an infinite geometric series is $\frac{32}{5}$. The second term of the series is -2. What is the value of the first term of the series?	8
12	Find the coefficient of x^4 in the expansion of $\left(\frac{1}{4x^2} + 2x\right)^{10}$	720
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14	A circle is inscribed within an equilateral triangle of side length 12. A second equilateral triangle is inscribed within the circle and a second circle is inscribed within the second equilateral triangle. If this process continues, what is the area of the 6th inscribed circle?	$3\pi/256$
15	Evaluate $\frac{2}{5} + \frac{3}{25} + \frac{4}{125} + \cdots$	9/16

2010 – 2011 Log1 Contest Round 1
Mu Sequences and Series

Name: _____

	4 points each	
1	What is the next term in the following arithmetic sequence -4, 19, 42, … ?	65
2	How many terms does the following arithmetic sequence contain -2, 11, …, 310 ?	25
3	What is the sum of the digits of the 195th term of the arithmetic sequence -41, -26, … ?	25
4	Evaluate $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$	1
5	Evaluate $\sum_{n=1}^{14} (5n-7)$	427

	5 points each	
6	Find the sum of the terms of the infinite geometric series $5, 2, \frac{4}{5}, \ldots$	25/3
7	Harry Potter drops a rubber ball from a height of 20 feet. Each time the ball drops it rebounds 80% of the height from which it is falling. What is the total distance (in feet) traveled by the bouncing rubber ball?	180
8	What is the sum of the series $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \frac{1}{16} - \cdots$?	$\frac{2}{3}$
9	State whether the following series is absolutely convergent, divergent, or conditionally convergent $\sum_{k=1}^{\infty} (-1)^{k+1} \frac{lnk}{k}$	Conditionally convergent
10	Evaluate $\prod_{i=0}^{6} 18\left(\frac{1}{3}\right)^{i}$	128/2187

	6 points each	
11	The sum of an infinite geometric series is $\frac{32}{5}$. The second term of the series is -2. What is the value of the first term of the series?	8
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14	Find the interval of convergence for the series $\sum_{n=1}^{\infty} (-1)^n \frac{(7x)^{n+1}}{n+1}$	[-1/7, 1/7]
15	Evaluate $\frac{2}{5} + \frac{3}{25} + \frac{4}{125} + \cdots$	9/16

2010 – 2011 Log1 Contest Round 1
Sequences and Series Solutions

Mu	Al	Th	Solution
1	1	1	d =23, 42+23 = 65
2	2	2	$a_n = 13n - 15$; $a_{25} = 310$, $\frac{310-(-2)}{13} + 1 = 25$
3	3	3	$a_n = 15n - 56$; $a_{195} = 2869$ 2+8+6+9 = 25
	4	4	$a_2 = 2(3)-1+3=8$, $a_3 = 2(8)-3+3=16$; $a_4 = 27$; $a_5 = 41$
4			Partial fraction decomp. simplifies this to $\sum_{n=1}^{\infty}\left(\frac{1}{n}-\frac{1}{n+1}\right)$ which telescopes down to the first term of 1
		5	One can solve for the first term (4/3) and common ration (3/2) but it is easier to note that the middle term (9/2) is the geometric mean of b and c so that bc=(9/2) squared = 81/4.
5	5		$\sum_{n=1}^{14}(5n-7) = 5\sum_{n=1}^{14} n - 14(7)$ $= 5\frac{14(15)}{2} - 14(7) = 427$
6	6	6	5 / (1 - (2/5)) = 5 * 5/3 = 25/3
7	7	7	The drops are counted by the infinite geometric series sum, $\frac{20}{1-\frac{4}{5}} = 100$, and the rebounds are given by $\frac{16}{1-\frac{4}{5}} = 80$. 100+80=180
8	8	8	Combine the terms into pairs to get the geometric series: $\frac{1}{2}+\frac{1}{8}+\cdots = \frac{\frac{1}{2}}{1-\frac{1}{4}} = \frac{1}{2}\cdot\frac{4}{3} = \frac{2}{3}$
	9	9	x + 4 – (2x – 11) = -x + 15 3x - 2 – (x + 4) = 2x - 6 Set these equal to each other to obtain x = 7, common difference is 4 with first term 3, so a_8 is 31
9			Diverges when you apply the absolute value to the terms and use direct comparison against a form of the divergent harmonic series; converges by alternating series test
		10	The first square has area 16. Each successive square's area is half the preceding term. 16 * ½^10 = 1/64
10	10		$\prod_{i=0}^{6} 18\left(\frac{1}{3}\right)^{i} = 18^7 \cdot \left(\frac{1}{3}\right)^{0+1+\cdots+6}$ $= 2^7 \cdot 3^{14}/3^{21} = 128/2187$
11	11	11	$\frac{\frac{-2}{r}}{1-r} = \frac{32}{5}$, solve for r; r = -1/4. The solution 5/4 does not converge. a_1 = -2 * (-4) = 8
12	12	12	In the binomial expansion, the x^4 term is $\binom{10}{2}\left(\frac{1}{4x^2}\right)^2 (2x)^8$. 10C2 * (1/4)^2 * 2^8 = 720
13	13	13	$w_{n+1} = \frac{2}{3}w_n + 5$. In the limit, one can substitute w for w_n. Solve for w=15.

	14	14	The first inscribed circle has radius $2\sqrt{3}$, and each successive circle's radius is half the preceding value. The 6th circle has radius $\frac{\sqrt{3}}{16}$
14			Apply ratio test to obtain $$\lim_{n\to\infty}\left|\frac{(-1)^{n+1}\frac{(7x)^{n+2}}{n+2}}{(-1)^{n}\frac{(7x)^{n+1}}{n+1}}\right|$$ which simplifies to $\lim_{n\to\infty}\left\|\frac{n+1}{n+2}(7x)\right\| = \|7x\|$ $\|7x\| < 1 \Rightarrow \|x\| < 1/7$ Test endpoints which both converge as alternating harmonic series
		15	Divide the third term by the first and factor to get the common ratio of (x+4). x+4 < 1 and x+4 > -1
15	15		Call the sum S. $$\frac{S}{5} = \frac{2}{25} + \frac{3}{125} + \cdots$$ $$\frac{4S}{5} = \frac{2}{5} + \frac{1}{25} + \frac{1}{125} \cdots$$ Sum the infinite geometric series with first term 1/25 and r = 1/5 along with 2/5 to obtain 9/20. Thus 4/5 S = 9/20 and S = 9/16.