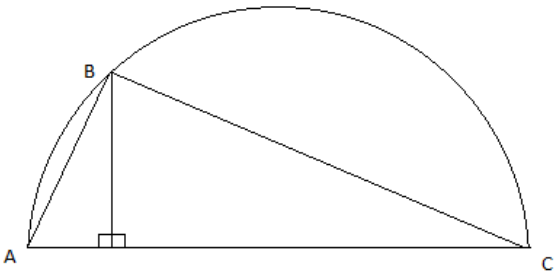
2011 – 2012 Log1 Contest Round 1
Theta Polygons, Circles and Pi

Name: _____
Units do not have to be included.

4 points each		
1	How many diagonals are there in a regular hexagon?	
2	The diagonals in a regular pentagon divide the interior angle into three smaller angles, what is the measure of the smallest of these angles, in degrees?	
3	Find the diameter of a circle that has an area numerically double its circumference.	
4	A rectangle is constructed such that its diagonals each measure 100 inches. Given that the side lengths are integers, find the rectangle's perimeter in inches.	
5	Two chords AB and CD drawn in a circle and intersect at point P. If AP = 2, CP = 3, and PB = 9, what is the length of PD?	

5 points each		
6	A triangle is inscribed in a circle such that one of its sides forms a diameter of the circle. If the other two sides are in the ratio of 1:2, what is the measure of the largest angle of the triangle in degrees?	
7	How many non-congruent triangles can be constructed from 6 evenly spaced points on a circle?	
8	A cube of white foam is painted blue on the outside and then cut in half by a plane through opposite face diagonals. What is the ratio of white to blue surface area?	
9	What is the area, in square units, enclosed by a regular hexagon inscribed in a circle with a circumference of 16π?	
10	A goat is tethered to one corner of the Apple Store, shaped like a rectangle with dimensions 56 by 75 feet. The store is completely surrounded by grass. The goat is tethered by a 64 foot long rope. How much, in square feet, grass can the goat graze?	

6 points each		
11	An acute triangle is constructed such that it has two sides measuring 5 inches and 13 inches, respectively. How many integral values are possible for the third side's length in inches?	
12	The sides of a triangle are of length 4, 8 and 10. What is the length of the median to the longest side?	
13	 Triangle ΔABC is circumscribed by the semicircular arc ABC. If the segment BC measures $3\sqrt{10}$ and the height of ΔABC is 3, what is the area of the semicircle?	
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15	Quadrilateral ABCD is inscribed in a circle, and the measure of angle A is 60 degrees and the measure of angle B is 80 degrees. What is the positive difference between the measures of angles C and D, in degrees?	

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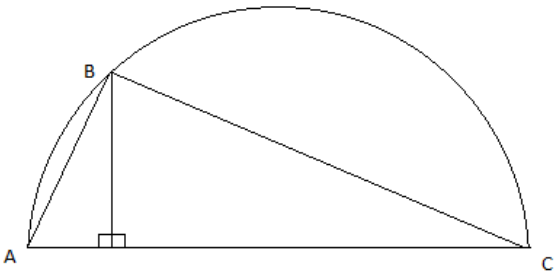
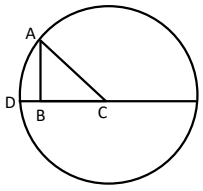
13	 Triangle ΔABC is circumscribed by the semicircular arc ABC. If the segment BC measures $3\sqrt{10}$ and the height of ΔABC is 3, what is the area of the semicircle?	
14	The vertex at point C of ΔABC is the center of a circle. Segment BC lies on a diameter of the circle, $\angle ABC = 90°$ and $\angle ACB = 15°$. The radius of the circle is AC = 1. What is the area, in square units, of the region bounded by the points A, B and D? 	
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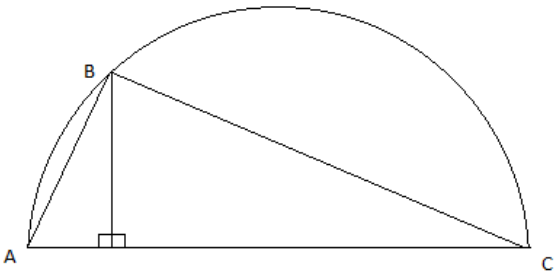
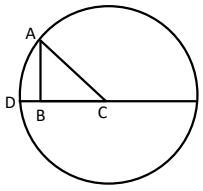
13	 Triangle ΔABC is circumscribed by the semicircular arc ABC. If the segment BC measures $3\sqrt{10}$ and the height of ΔABC is 3, what is the area of the semicircle?	
14	The vertex at point C of ΔABC is the center of a circle. Segment BC lies on a diameter of the circle, $\angle ABC = 90°$ and $\angle ACB = 15°$. The radius of the circle is AC = 1. What is the area, in square units, of the region bounded by the points A, B and D? 	
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2	The diagonals in a regular pentagon divide the interior angle into three smaller angles, what is the measure of the smallest of these angles, in degrees?	36 [degrees]
3	Find the diameter of a circle that has an area numerically double its circumference.	8
4	A rectangle is constructed such that its diagonals each measure 100 inches. Given that the side lengths are integers, find the rectangle's perimeter in inches.	280 [inches]
5	Two chords AB and CD drawn in a circle and intersect at point P. If AP = 2, CP = 3, and PB = 9, what is the length of PD?	6

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6	A triangle is inscribed in a circle such that one of its sides forms a diameter of the circle. If the other two sides are in the ratio of 1:2, what is the measure of the largest angle of the triangle in degrees?	90 [degrees]
7	How many non-congruent triangles can be constructed from 6 evenly spaced points on a circle?	3
8	A cube of white foam is painted blue on the outside and then cut in half by a plane through opposite face diagonals. What is the ratio of white to blue surface area?	$\sqrt{2}:3$ or $\frac{\sqrt{2}}{3}$
9	What is the area, in square units, enclosed by a regular hexagon inscribed in a circle with a circumference of 16 π?	$96\sqrt{3}$
10	A goat is tethered to one corner of the Apple Store, shaped like a rectangle with dimensions 56 by 75 feet. The store is completely surrounded by grass. The goat is tethered by a 64 foot long rope. How much, in square feet, grass can the goat graze?	3088π

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11	An acute triangle is constructed such that it has two sides measuring 5 inches and 13 inches, respectively. How many integral values are possible for the third side's length in inches?	4
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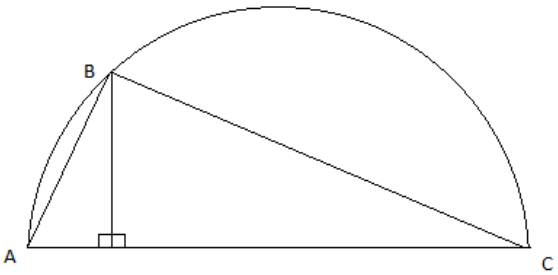
13	 Triangle ΔABC is circumscribed by the semicircular arc ABC. If the segment BC measures $3\sqrt{10}$ and the height of ΔABC is 3, what is the area of the semicircle?	$\frac{25\pi}{2}$
14	A square is inscribed in a circle that is inscribed in a square that is inscribed in a circle. The innermost square has area 1. What is the radius of the outermost circle?	1
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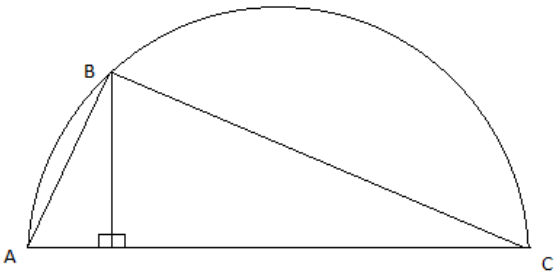
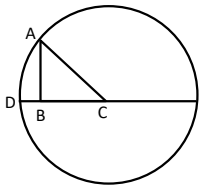
13	 Triangle ΔABC is circumscribed by the semicircular arc ABC. If the segment BC measures $3\sqrt{10}$ and the height of ΔABC is 3, what is the area of the semicircle?	$\frac{25\pi}{2}$
14	The vertex at point C of ΔABC is the center of a circle. Segment BC lies on a diameter of the circle, $\angle ABC = 90°$ and $\angle ACB = 15°$. The radius of the circle is AC = 1. What is the area, in square units, of the region bounded by the points A, B and D? 	$\frac{\pi}{24} - \frac{1}{8}$ [sq un]
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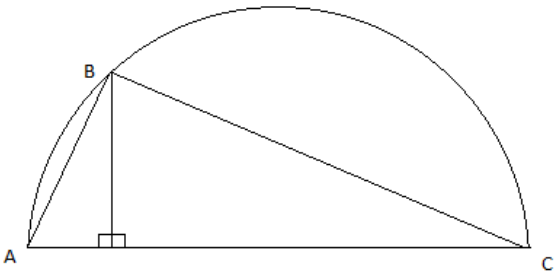
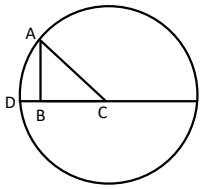
13	 Triangle ΔABC is circumscribed by the semicircular arc ABC. If the segment BC measures $3\sqrt{10}$ and the height of ΔABC is 3, what is the area of the semicircle?	$\frac{25\pi}{2}$
14	The vertex at point C of ΔABC is the center of a circle. Segment BC lies on a diameter of the circle, $\angle ABC = 90°$ and $\angle ACB = 15°$. The radius of the circle is AC = 1. What is the area, in square units, of the region bounded by the points A, B and D? 	$\frac{\pi}{24} - \frac{1}{8}$ [sq un]
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Polygons, Circles and Pi Solutions

Mu	Al	Th	Solution
1	1	1	Each of the n vertices can be connected to n-3 others but this double counts each diagonal. The formula is n(n-3)/2 = 6(3)/2 = 9.
2	2	2	Actually, the diagonals divide the interior angle into equal angles. The measure of the interior angle is 180 – 360/5 = 108 and 108/3 = 36 degrees.
3	3	3	Area of circle is $\pi\frac{d^2}{4}$. This must be equal to twice its circumference of πd. Therefore $\pi\frac{d^2}{4} = 2\pi d$. Simplifying and solving, d=8.
4	4		The hypotenuse will be of length 17. The area of the triangle can be calculated using any base and height so: $\frac{8(15)}{2} = \frac{17x}{2}, x = \frac{120}{17}$.
		4	Two sides of the rectangle and its diagonal form a right triangle. The Pythagorean triple 60-80-100 determines sides of the rectangle. Its perimeter is 2(60+80) = 280 inches.
5			$A = s^2,\ \frac{dA}{dt} = 2s\frac{ds}{dt},\ \frac{dA}{dt} = 2(5)(-0.2) = -2$
	5	5	Any two intersecting chords have the property that (AP)(PB) = (CP)(PD), so 2(9) = 3x, x = 6.
6	6	6	The angle opposite the diameter is a right angle as the arc it determines has a central angle of 180 degrees.
7	7	7	There are three non-congruent triangles. One uses 3 consecutive points, another has 2 consecutive points and another separated by a point. The third uses 3 points each separated by an unused point.
8	8	8	Let x represent the length of one side of the cube. The total blue surface area is $6x^2$. Cutting along the diagonal, you get one side on each half that has an area of $x^2\sqrt{2}$. Thus the total white surface area is $2x^2\sqrt{2}$. The ratio is $2x^2\sqrt{2}:6x^2$, which equates to $\sqrt{2}:3$
9	9		The third angle must have measure 30 degrees, so the area will be $\frac{1}{2}ab\sin\theta = \frac{1}{2}(6)(6)\left(\frac{1}{2}\right) = 9$.
		9	The radius of the circle is 8. Cutting the hexagon into six equal sectors produces six equilateral triangles with side length equal to 8. Each triangle has area $\frac{s^2\sqrt{3}}{4} = 16\sqrt{3}$. Multiply by 6 to get the entire area of the regular hexagon.
10			Two ways of doing this problem. (1) Let $\frac{n}{2} = \frac{1}{x}$ and the limit becomes $\lim_{x\to 0}\frac{\sin\pi x}{x}$ and after applying L'Hopitals rule, $\lim_{x\to 0}\frac{\pi\cos\pi x}{1} = \pi$ or (2) Realize that $\frac{n}{2}\sin\frac{2\pi}{n}$ is the area of a regular polygon with n sides and where the distance from each vertex to the center is 1. In the limit, this polygon becomes a circle of radius 1 with area π.
	10	10	The goat can graze for ¾ of a circle of radius 64 feet. The last quarter circle is cut to a radius of 8 feet because the rope wraps around the other corner on the 56 foot long side of the rectangle. ¾ of a circle of radius 64 feet has an area of 3072π and the last ¼ of a circle with radius 8 feet has an area of 16 π.

11	11	11	If these two sides formed a right triangle, its hypotenuse would be between 13 and 14 (25+169<196) inches. Keeping the angle acute, the third side must be greater than 9 inches otherwise it wouldn't be long enough to form a triangle. The only integers less than 14 and greater than 9 are 10, 11, 12 and 13.
12	12	12	The formula for the length of the median, d, is $$d = \sqrt{\frac{a^2}{2} + \frac{b^2}{2} - \frac{c^2}{4}} = \sqrt{8 + 32 - 25} = \sqrt{15}$$ This is a special case of Stewart's Theorem where d is the length of an arbitrary cevian that divides the side length c into lengths m (on the 'a' side) and n. $a^2n + b^2m = c(d^2 + mn)$.
13	13	13	Let the base point of the height of ΔABC be labeled D. Then ΔABC is similar to ΔBDC. Thus, AC:BC = BC:CD. $\frac{AC}{3\sqrt{10}} = \frac{3\sqrt{10}}{CD}$. From the Pythagorean Theorem, CD = 9. So AC = 10. This is the diameter of the semicircle ABC. Its radius is therefore equal to 5 and its area is $\frac{25\pi}{2}$.
14	14		The area of the entire circle is π square units so the area of the sector ACD = $\frac{\pi}{24}$. You will obtain the area of the region bounded by A, B and D by subtracting out the area of ΔABC. $$BC = cos15° \; and \; AB = sin15°,$$ $$Area = \frac{sin15°cos15°}{2} = \frac{2sin15°cos15°}{4} = \frac{sin30°}{4} = \frac{1}{8}$$
		14	The innermost square has a diagonal length of √2. Thus its circumscribed circle has a radius of √2/2. The square that circumscribes this circle has sides equal to √2. Its diagonal would be √2√2, or just 2. This diagonal is the diameter of the outermost circle so its radius is therefore 1.
15			The area of the square is 16π. The formula for the area of a quarter-circle is ¼ πr^2. Equating these two equations gives r = 8 units. To find out the time when r = 8 units, integrate dr/dt. $r(t)=2t^2$. The constant of integration is defined to be 0 in this problem. t must be equal to 2 seconds.
	15	15	Quadrilateral ABCD is a cyclic quadrilateral. One of its properties is that the measures of the angles follow: $A + C = B + D = 180$, this means C=120 and D=100 and the difference between them is 20 degrees.