2011 – 2012 Log1 Contest Round 2
Theta Applications

Name: _____
Units do not have to be included.

4 points each		
1	It takes Richard 70 minutes to drive from Bellingham to Seattle, but the drive from Seattle to Bellingham takes 110 minutes due to traffic. He takes the same 80 mile route both ways. What is his average speed, in miles per hour, for the roundtrip?	
2	A father is now four times as old as his son. Three years ago the father was six years more than three times the son's present age. What is the sum of the father and son's current ages?	
3	A speeding car is moving with a constant velocity of v = 150 ft/sec when it passes a stationary police car. Starting from rest, the police car accelerates at a rate of a = 12 ft/sec^2. At the moment the speeding car passes the police car, their positions are given by the equations $y(t) = 150t$ and $y(t) = 6t^2$, respectively. At what time t, in seconds, will the police car catch up to the speeding car?	
4	Tom and Rich want to get a stack of math exams graded as quickly as possible. Normally, Tom takes ¾ of an hour to grade a stack of 100 tests. Rich takes 1 ½ hours to do the same stack. Together, how long will it take, in hours, to complete the entire stack of 100 tests?	
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6	How many different ways can Bertha pay for a $0.68 apple if she only has dimes, nickels, and pennies?	
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8	Trung drops a ball off the top of a building 192 feet high. After each bounce, the ball rebounds to a height exactly one-fourth of the previous drop height. If this were to continue bouncing forever, what would be the total distance, in feet, the ball travels?	
9	A hose sustaining a water flow at 1 cm^3/sec is used to fill an initially empty inverted cone of radius 5 cm and height 5 cm. The cone drains at a rate of ¼ cm^3/sec. How long, in seconds, will it take before the cone overflows at the top?	
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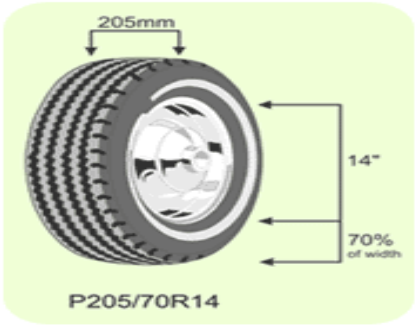
13	Your car uses a standard P205 – 70R14 tire. The numbers are defined as follows: 205 is the width of the tire in millimeters. The 14 is the inner diameter of the tire in inches. The 70 is the "height" of the tire, the aspect ratio expressed as a percentage of the tire's width. Calculate the total diameter of the tire, in millimeters, based on this information. 25.4 mm = 1 inch. 	
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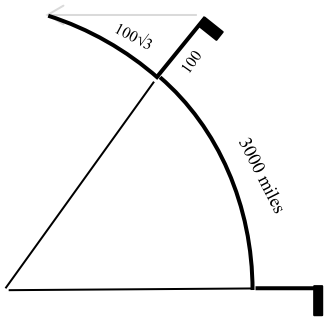
13	You are a visiting an Earth-like planet and wish to determine its circumference. You stand in an abandoned city at the planet's equator where there is a 100ft flagpole. The planet's sun does not cast a shadow since it is directly overhead. You head directly north for 3000 miles and discover another abandoned city with an identical 100ft flagpole that casts an **approximately linear shadow** equal to $100\sqrt{3}ft$ when viewed at exactly the same time as the flagpole at the equator. Assuming that the star's light rays are parallel, what is the circumference of the planet, in miles? You may assume that the flagpole and its shadow form a right triangle as shown in the diagram above. 	
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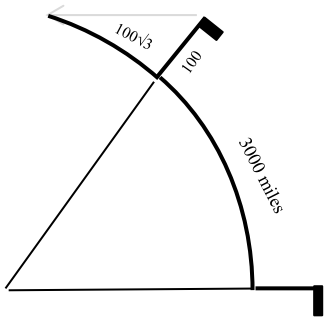
11	You are a visiting an Earth-like planet and wish to determine its circumference. You stand in an abandoned city at the planet's equator where there is a 100ft flagpole. The planet's sun does not cast a shadow since it is directly overhead. You head directly north for 3000 miles and discover another abandoned city with an identical 100ft flagpole that casts an **approximately linear shadow** equal to $100\sqrt{3}ft$ when viewed at exactly the same time as the flagpole at the equator. Assuming that the star's light rays are parallel, what is the circumference of the planet, in miles? You may assume that the flagpole and its shadow form a right triangle as shown in the diagram above. 	
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14	Stacey is trying to retrieve her mischievous cat, Lotus, from the roof of her house. She leans a 24 feet long ladder against a wall and the foot of the ladder is sliding away at a constant rate of 3 ft/sec. Meanwhile, Stacey is climbing up the ladder at a rate of 2 ft/sec, When Stacey has climbed up the ladder 6 feet it makes an angle of $\frac{\pi}{3}$ with the ground. At that time, what rate is the height of the ladder decreasing, in ft/sec?	
15	Stacey likes to cuddle with her cat, Lotus, at night. But Lotus is more active at night so the probability that Lotus will want to cuddle with Stacey on any given night is 40%. What is the probability that Lotus will cuddle with Stacey for at least two out of five nights? Express the probability of a decimal rounded to the nearest hundredth.	

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3	A speeding car is moving with a constant velocity of v = 150 ft/sec when it passes a stationary police car. Starting from rest, the police car accelerates at a rate of a = 12 ft/sec^2. At the moment the speeding car passes the police car, their positions are given by the equations y(t) = 150t and y(t) = 6t^2, respectively. At what time t, in seconds, will the police car catch up to the speeding car?	5 [seconds]
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9	A hose sustaining a water flow at 1 cm^3/sec is used to fill an initially empty inverted cone of radius 5 cm and height 5 cm. The cone drains at a rate of ¼ cm^3/sec. How long, in seconds, will it take before the cone overflows at the top?	$\frac{500\pi}{9}$ [seconds]
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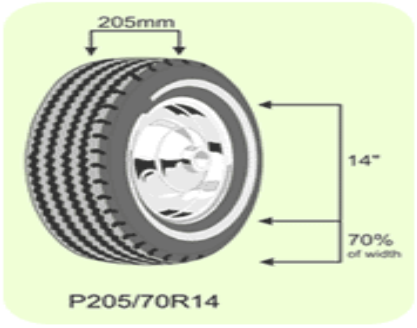
13	Your car uses a standard P205 – 70R14 tire. The numbers are defined as follows: 205 is the width of the tire in millimeters. The 14 is the inner diameter of the tire in inches. The 70 is the "height" of the tire, the aspect ratio expressed as a percentage of the tire's width. Calculate the total diameter of the tire, in millimeters, based on this information. 25.4 mm = 1 inch. 	642.6 [mm]
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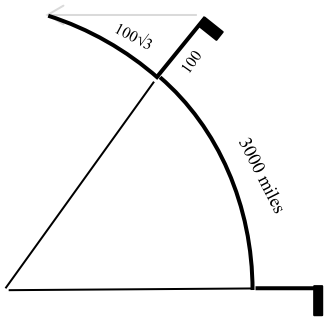
12	An ocean liner heads directly north in idle waters with a speed of 25 knots when it suddenly encounters a 10 knot ocean current directed at 30° North of East. Due to the northeasterly directed current, what is the resultant speed of the ocean liner, in knots, relative to the Earth?	$5\sqrt{39}$ [knots]
13	You are a visiting an Earth-like planet and wish to determine its circumference. You stand in an abandoned city at the planet's equator where there is a 100ft flagpole. The planet's sun does not cast a shadow since it is directly overhead. You head directly north for 3000 miles and discover another abandoned city with an identical 100ft flagpole that casts an **approximately linear shadow** equal to $100\sqrt{3}ft$ when viewed at exactly the same time as the flagpole at the equator. Assuming that the star's light rays are parallel, what is the circumference of the planet, in miles? You may assume that the flagpole and its shadow form a right triangle as shown in the diagram above. 	18,000 [miles]
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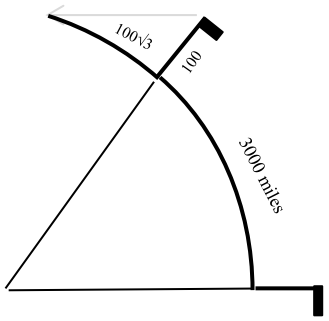
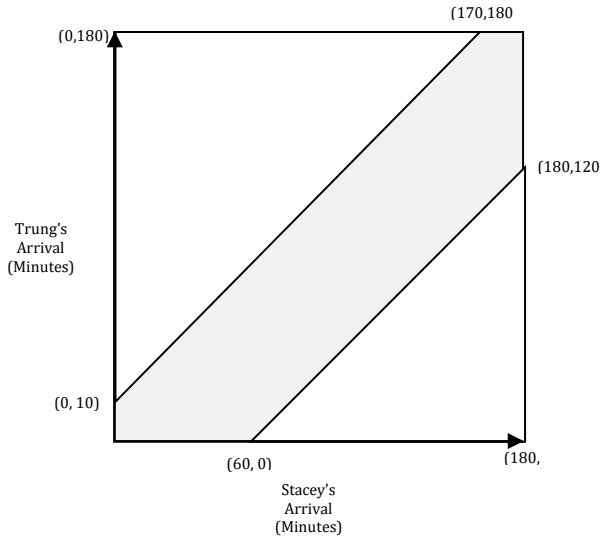
11	You are a visiting an Earth-like planet and wish to determine its circumference. You stand in an abandoned city at the planet's equator where there is a 100ft flagpole. The planet's sun does not cast a shadow since it is directly overhead. You head directly north for 3000 miles and discover another abandoned city with an identical 100ft flagpole that casts an **approximately linear shadow** equal to $100\sqrt{3}ft$ when viewed at exactly the same time as the flagpole at the equator. Assuming that the star's light rays are parallel, what is the circumference of the planet, in miles? You may assume that the flagpole and its shadow form a right triangle as shown in the diagram above. 	18,000 [miles]
12	Four expectant grandparents all possess brown eyes and the recessive gene for blue eyes. The blue-eyed gene passes from parents at a likelihood of 50%, but a child will only have blue eyes if both parents pass on the recessive gene. If the parents (children of the above grandparents) both have brown eyes, what is the likelihood of their child having blue eyes?	$\frac{1}{9}$
13	A toy plane is flying in a horizontal circular spiral pattern with initial radius 5 m, making one full revolution each second. The plane decreases its radius of motion 1 m every revolution. What total distance, in meters, has it flown after 5 revolutions on the spiral path?	25π[m]
14	Stacey is trying to retrieve her mischievous cat, Lotus, from the roof of her house. She leans a 24 feet long ladder against a wall and the foot of the ladder is sliding away at a constant rate of 3 ft/sec. Meanwhile, Stacey is climbing up the ladder at a rate of 2 ft/sec, When Stacey has climbed up the ladder 6 feet it makes an angle of $\frac{\pi}{3}$ with the ground. At that time, what rate is the height of the ladder decreasing, in ft/sec?	$-\frac{3}{\sqrt{3}}$ [ft/sec]
15	Stacey likes to cuddle with her cat, Lotus, at night. But Lotus is more active at night so the probability that Lotus will want to cuddle with Stacey on any given night is 40%. What is the probability that Lotus will cuddle with Stacey for at least two out of five nights? Express the probability of a decimal rounded to the nearest hundredth.	0.66

2011 – 2012 Log1 Contest Round 2
Applications Solutions

Mu	Al	Th	Solution
1	**1**	**1**	Average Speed = Total Distance / Total Time 160 / 180 = **160/3** with optional units of [miles / hours, mph, etc.] Alternatively, take the harmonic mean of the two average velocities. $V_{ave} = \frac{2}{\frac{1}{80}+\frac{1}{80}} = \frac{160\ miles}{180\ minutes} = \frac{160\ miles}{3\ hours}$
2	**2**	**2**	Let F = father's present age and S = son's present age. F = 4S and F – 3 = 3S +6 or F = 3S +9. Subtracting the two equations for F, 0 = -S + 9 or S = 9. Since F = 4S, F = 36. Therefore, **F + S = 45.**
	3	**3**	The speeding car and police car will meet when their distances traveled are equal. Therefore, $150t = 6t^2$. Solving for t, the answer is t = ±5 seconds. The positive root is the only viable answer.
3	**4**	**4**	Let the work done for grading 100 tests be arbitrarily set at 1. The rate at which Tom does the work of grading 100 tests is $\frac{4}{3}$. Rich's rate of doing the same work is $\frac{2}{3}$. The amount of work Tom and Rich do in time, t, is $\frac{4}{3}t$ and $\frac{2}{3}t$, respectively. The total work is $1 = \frac{4}{3}t + \frac{2}{3}t$. t = ½ hour.
	5	**5**	The number of distinct arrangements is 26x26x25x9x10x10x10 = 152,100,000. Adding up all the digits, 1+5+2+1+0+0+0+0+0 = 9
4	**6**	**6**	Let the ordered triple (d,n,p) represent the combination of dimes, nickels and pennies that add up to \$0.68. For d=6, the possibilities are (6,1,3) and (6,0,8). Taking away 1 dime requires that you replace it with either 1 nickel and 5 pennies or 10 pennies. Thus each additional dime taken away increases the possible combinations by 2. The pattern down to 0 dimes is C = 2+4+6+8+10+12+14 = 56
5	**7**	**7**	Let V_N = Nancy's average speed and let V_S = Sid's average speed. Sid's average speed may be determined from the harmonic mean of his going average (V_N – 5) and coming average (V_N +5) speeds. $V_{Sid} = \frac{2}{\frac{1}{V_N-5}+\frac{1}{V_N+5}} = V_N - 1$. Rearranging, $V_N - 1 = \frac{2(V_N^2-25)}{2(V_N)} \rightarrow V_N^2 - V_N = V_N^2 - 25$. Thus, V_N = 25 and V_S = 24
6	**8**	**8**	The ball travels 192 ft for the first drop. The first rebound travels a roundtrip of 2x48 ft. The infinite series is $192+2*192(1/4)^1+2*192(1/4)^2+\ldots$. Equivalently, $H = 192 - 384\left(\frac{1}{4}\right)^0 + \sum_0^\infty 384\left(\frac{1}{4}\right)^n = -192 + \frac{384}{1-\frac{1}{4}} = 320$ with optional units of [feet].
7	**9**	**9**	Volume of the cone = 1/3 π r^2h = 125π/3 cm^3. The net flow is ¾cm^3/sec. Total time is T = 125π/3 cm^3/(¾ cm^3/sec) = 500 π/9 seconds.
8	**10**	**10**	A torus is equivalent to a circular cylinder bent so that its ends are attached. Given that the volume of the torus is 1500 m^3, we can find the height of the equivalent cylinder with radius = 5m. $1500 = \pi 5^2 h$. $h = \frac{60}{\pi}$. The surface area of the torus is also equivalent to the lateral surface area of the circular cylinder. Thus $A = 2\pi rh = 2\pi 5\frac{60}{\pi} = 600m^2$. Thus the minimum cost to build the tower with sheet metal is \$4,200

9	**11**	**11**	Let E_1 = Elisa's earnings in her first year of working. Let S = Elisa's savings from her first year of working. We wish to solve for S/E_1. If E_2 = the amount that Elisa has to spend during the second year, then $E_2 = S(1+r)$. However, E_2 = half of what Elisa spent during the first year = ½ $(E_1 - S)$. Thus, ½ $(E_1 - S) = S(1 + r)$. Rearranging yields, $\frac{E_1 - S}{S} = \frac{E_1}{S} - 1 = 2(1 + r) \rightarrow \frac{E_1}{S} = 3 + 2r \rightarrow \mathbf{\frac{S}{E_1} = \frac{1}{3+2r}}$
		12	Setting I_0 =1 the decibel equation yields $40 = 10log(I)$. For a sound that is twice the original intensity, $dB = 10log(2I) = 10log2 + 10log(I) = 10log2 + 40$; the amount above 40 is then $10\log 2$.
		13	Twice the height of the tire must be added to the inner diameter to get the overall diameter of the tire. The height is 205 x 0.70 = 143.5 mm. Twice the height is 287 mm. The inner diameter is 14 in x 25.4 mm/in = 355.6 mm. Overall diameter = 642.6 with optional units of [mm]
		14	Solve for t. $2000 = 5000(2^{-t}) \rightarrow \frac{2}{5} = 2^{-t} \rightarrow log_2\left(\frac{2}{5}\right) = -t \rightarrow t = log_2 5 - 1.$ Alternatively, using the definition of radioactive decay: Solve for t. $2000 = 5000e^{-t\,ln2}, \rightarrow \frac{2}{5} = e^{-t\,ln2} \rightarrow ln\left(\frac{2}{5}\right) = -t\,ln2 \rightarrow -\frac{ln\left(\frac{2}{5}\right)}{ln2} = t \rightarrow -\left(\frac{ln2 - ln5}{ln2}\right) = t \rightarrow t = \frac{ln5}{ln2} - 1$ Both answers are acceptable, albeit equivalent.
		15	Solve for t. $10 = 43 + 32t - 16t^2 \rightarrow 0 = 33 + 32t - 16t^2 \rightarrow$ $0 = 16t^2 - 32t - 33 \rightarrow 0 = (4t + 3)(4t - 11)$ $t = \left(-\frac{3}{4}, \frac{11}{4}\right)$. Only t=11/4 is a viable answer.
10	**12**		The speeds, when directed as vectors, of the ocean liner and ocean current add like vectors. Using the tip-to-toe method, the Law of Cosines may be used to find the resultant. The angle between the two vectors is 120. Therefore, $c^2 = a^2 + b^2 - 2abCosC = 25^2 + 10^2 - 2(25)(10)Cos120° = 625 + 100 - 500\left(-\frac{1}{2}\right) = 725 + 250 = 975$. $c = 5\sqrt{39}$.
11	**13**		The shadow of the flagpole 3000 miles north of the planet's equator forms a triangle with the shadow and the flagpole being its legs. Given that the approximately linear shadow has a length of $100\sqrt{3}$ft and the flagpole has a length of 100ft, the top angle of the triangle is defined by the inverse tangent function $\theta = tan^{-1}\frac{100\sqrt{3}}{100} = 60°$. Assuming that ray of light on the flagpole at 3000 miles north and the ray of light on the flagpole at the equator are parallel, the radial vector from the center of the planet to the flagpole at 3000 miles north of the equator is a transversal cutting two parallel light rays. The $60°$ angle is also the angle subtended by the 3000 mile arc of distance between the two cities referenced in this problem. Since $60°$ is 1/6 of a circle, the circumference of the planet is 18,000 miles.

	14		The solution is done by drawing a square with 180 minutes in both directions. Put Trung's arrival on one axis and Stacey's on the other. If Trung arrives at 0 (11am), Stacey must arrive by 11:10 or they miss. For every minute later that Trung arrives, Stacey can arrive a minute later. However, if Stacey arrives at 11am, Trung can arrive any time before noon, etc. This gives you a square with a "ribbon" in the middle that represents the arrival times where they meet. The probability of meeting is represented by the area of the shaded region as a fraction of the whole square. $P = \frac{180^2 - \left(\frac{1}{2}170^2 + \frac{1}{2}120^2\right)}{180^2}$ $= \frac{10750}{32400} = \frac{215}{648}$ 
12	**15**		The four grandparents all have the brown and blue eye alleles. Therefore, their children, if they are to have brown eyes, could have the following allele combinations: (Br,Bl), (Bl,Br) or (Br,Br). Let's assume that the four grandparents are the parents of a man-woman couple that have brown eyes according to the possible allele combinations given above. In order for the grandchild to have blue eyes, both of its parents must pass the blue eye gene to them. Both mother and father have a 2/6 chance of doing this since each gene has an equal chance of being passed on. For both to pass the blue-eye gene to their child, the probability is 2/6 x 2/6 = 4/36 = 1/9
13			Algebra Solution: The "average" radius is (5+0)/2 = 5/2, one revolution being 5π and five revolutions equals 25π meters. Calculus Solution: Since arc length is s = rθ, the differential equation for small increments of arc is ds = r dθ. We wish to sum up these differentials over 5 revolutions, 10π, where r(θ)=5 – θ/2π. Therefore $s = \int_0^{10\pi} 5 - \frac{\theta}{2\pi} d\theta = 5\theta - \frac{\theta^2}{4\pi} = 25\pi$. Units of [meters] are optional.
14			$x^2 + y^2 = 24^2$. Implicitly differentiating we get $2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$. When $\theta = \frac{\pi}{3}, we\ get\ x = 12\ and\ y = 12\sqrt{3}$. Solving for dy/dt, $12(3) + 12\sqrt{3}\frac{dy}{dt} = 0 \rightarrow \frac{dy}{dt} = -\frac{3}{\sqrt{3}}$ ft/sec.
15			P(Cuddle) = 1 – [P(0of5) + P(1of5)] $1 - \left[\binom{5}{5}0.6^5 + \binom{5}{4}0.6^4 0.4\right] = \frac{3125 - 243 - 810}{3125} \approx 0.66$