1. A
2. B
3. B
4. B
5. C
6. A
7. D
8. B
9. A
10. C
11. D
12. B
13. A
14. A
15. B
16. B
17. C
18. D
19. B
20. D
21. C
22. D
23. A
24. A
25. C
26. B
27. A
28. A
29. A
30. D

1. A The formula for the area of a regular octagon with side length s is $2s^2(1+\sqrt{2})$. Plugging in 8 for s gives us $128+128\sqrt{2}$.

2. B Put the points in order and then use the determinant method for solving the area:

1	4	2	-6	1	
2	5	7	1	2	$=\frac{1}{2}\left[5+28+2-12-(8+10-42+1)\right]=23$

3. B Diagonal ZX is equal to 10, so it is a diameter of the circle. This means arc ZWX is 180 degrees.

4. B Set up a system of 3 equations and 3 variables: $x^2+y^2+cx+dy+f=0$

substitute the 3 point sin to get:
$$c-2d+f=-5$$
$$5c+4d+f=-41$$
$$10c+5d+f=-125$$
If you solve this system you get c=-18, d=6, and f=25; Complete some squares to transform the equation and you get: $x^2-18x+81+y^2+6y+9=-25+81+9$

The radius squared equals 65

5. C The distance from one center to the other center is R, and the distance from one center to one of the intersection points is also R. So, the triangle formed by both centers and one of the intersection points is an equilateral triangle. What we are looking for is the distance between the two intersection points, which is twice the height of this triangle. so our answer is R$\sqrt{3}$.

6. A Draw a picture: angle GIJ =30. Triangle IGJ is isosceles so angles GJI and IGJ are equal. x + x + 30 = 180. x = 75

7. D Draw yourself a picture and then set up similar triangles with "r" equal to the radius.
$$\frac{8-r}{10}=\frac{r}{6}\rightarrow 10r=48-6r\rightarrow 16r=48 \qquad r=3\rightarrow V=\frac{4}{3}\pi r^3=36\pi$$

8. B If the legs are a and b, then $\frac{ab}{2}=180$, and $\sqrt{a^2+b^2}=36$. If we add 4 times the first equation to the square of the second equation, we get $a^2+b^2+2ab=2016$. Taking the square root of both sides, we get $a+b=12\sqrt{14}$.

9. A The circumcenter is the intersection of the perpendicular bisectors of the sides of the triangle. The perpendicular bisector between (2, 6) and (6, 6) is $x=4$, and between (0, 0) and (6, 6) is $x+y=6$, so the circumcenter is $(4,2)$.

The smallest circle that pass through 2 points has the 2 points as endpoints of a diameter. So the diameter of the circle is $\sqrt{(8-4)^2+(8-2)^2}=2\sqrt{13}$. So the area of the circle is 13π.

10. C Because DE is parallel to BC, then triangle ABC and triangle ADE have all the same angles and are therefore similar. The ratio of DE to BC 1:$\sqrt{2}$, so the ratio of the area of ADE to ABC is 1:2. Therefore, ADE has an area of 24.

11. D To find the area of this hexagon, extend the sides of length 6 until they meet. Now, we have a large equilateral triangle of side length 12, which has an area of $\frac{12^2\sqrt{3}}{4}$. We

added 3 equilateral triangles of 3, which have a total area of $\frac{27\sqrt{3}}{4}$. Subtracting these we get $\frac{117\sqrt{3}}{4}$.

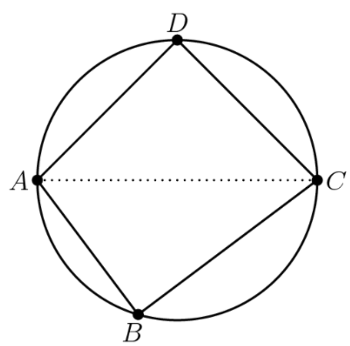
12. B We know that the ratio of an inscribed circle's area to a circumscribed circle's area is 1:2 from question number 7. The area of the first circle is 9π, and each subsequent circle has an area of half the previous one. This is an infinite geometric sequence with first term 9π and common ratio ½, which has a sum of 18π.

13. A This wheel has a circumference of 14π, so in one second it travels 70π, and in 3 minutes (180 seconds) it travels 12600π.

14. A Let $x = CD$ and h be the common height of $ABCD$ and ABP. We have that

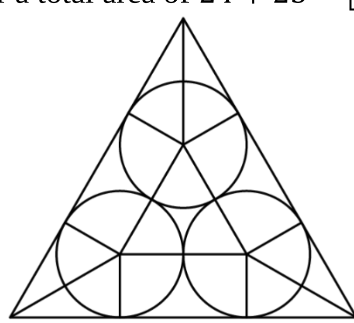
$$\frac{1}{2} \cdot 1 \cdot h = \frac{20}{21}\left(\frac{1}{2}(1+x) \cdot h\right) \Rightarrow \frac{1}{2} = \frac{10}{21} + \frac{10}{21}x \Rightarrow x = \boxed{\frac{1}{20}}.$$

15. B



Draw diagonal $\overline{AC}$, splitting $ABCD$ into ΔABC and ΔADC. Since angle B is right, the area of ΔABC is $\frac{1}{2}(6)(8) = 24$. We also know that $\overline{AC}$ is a diameter of the circle because angle B is right. To maximize the area of ΔADC, we place D as far away from $\overline{AC}$ as possible, which is the midpoint of semicircle AC. This gives an isosceles right triangle, the circumradius of which is $\frac{1}{2}\sqrt{6^2 + 8^2} = 5$. This gives the area of ΔADC as 25, for a total area of $24 + 25 = \boxed{49}$.

16. B



The equilateral triangle can be partitioned into three rectangles with length 2 and height 1, an equilateral triangle of side length 2, and six 1-$\sqrt{3}$-2 right triangles. This gives a total area of $3(2 \cdot 1) + \frac{2^2\sqrt{3}}{4} + 6\left(\frac{1}{2} \cdot \sqrt{3} \cdot 1\right) = \boxed{6 + 4\sqrt{3}}$.

17. C Call the three points A, B, and C. Choose A arbitrarily and draw a diameter of the circle through A that divides the circle into two semicircles. The angle at C is obtuse if and only if:

- B and C lie on the same semicircle.
- C is between A and B on that semicircle.

The probability of the first bullet point is clearly $\frac{1}{2}$. Given that B and C lie on the same semicircle, each configuration that has C between A and B can be matched up one-to-one with a configuration that has B between A and C by switching B and C, so the probability of the second bullet point is also $\frac{1}{2}$. This gives a total probability of $\frac{1}{4}$ that angle C is obtuse. The same reasoning can be applied to the other two vertices, and since at most one angle can be obtuse, our events are mutually exclusive, and the probability of an acute triangle is $3 \cdot \frac{1}{4} = \boxed{\frac{3}{4}}$.

18. D First, draw the line between the centers of the two circles. Then, draw a line from the center of the smaller circle parallel to the internal tangent. The radius of the larger circle that is perpendicular to the internal tangent can be extended to intersect this line at a right angle. We have a rectangle with lengths equal to the length of the internal tangent and the radius of the smaller circle, so the larger radius must be extended 3 units out to reach this line. Now, we have a right triangle with legs equal to the length of the internal tangent and 9, and a hypotenuse of the length between the centers (41). Therefore, the internal tangent is 40.

19. B By definition, the incenter does not lie on Euler's line.

20. D The formula for the radius of the inscribed circle is A=sr, where A is the area of the triangle and s is the semiperimeter of the triangle. Using heron's formula, the area of this triangle is 84, and the semiperimeter is 21, so the radius is 4. That means the circle has an area of 16π.

21. C Power of a point states that the length of a tangent to a circle from a point squared is equal to the length of the part of the secant outside the circle times the entire secant. In our problem, the tangent has a length of 12, and the part of the secant outside the circle is 9. If we call the entire secant x, then 9x = 144, and x = 16. We want the part of the secant inside the circle, so 16-9 = 7.

22. D If we draw a line from the center of the concentric circles to point Z, and a line from the center of the circles to the point of tangency of the inner circle, then we have created a right triangle because tangent lines are perpendicular to the radius that has an endpoint of the point of tangency. The hypotenuse of this triangle is 6, and one leg is 4, so the length of the other leg (half the external tangent) is $2\sqrt{5}$. We want the length of the whole tangent, so $4\sqrt{5}$.

23. A If we draw a line segment from point E to the midpoint of AB, then we have a right triangle with EA as the hypotenuse. The shorter leg is 4 (half the side length of the square) and the longer leg is $8 + 4\sqrt{3}$ (the height of the square plus the height of the triangle). Squaring these and adding them together, we get $128 + 64\sqrt{3}$.

24. A If we draw a line from the point of tangency on the smaller circle parallel to the line between the centers and draw the radius of the larger circle perpendicular to the

tangent, then we get a right triangle with one leg equal to the larger radius minus the smaller radius (5) and the hypotenuse equal to the distance between the centers (12). The other leg is the same length as the external tangent, which has a length of $\sqrt{119}$.

25. C If we call the length of AB 2x and the length of BC 2y, then $\sqrt{4x^2 + y^2} = 4\sqrt{2}$, and $\sqrt{x^2 + 4y^2} = 4\sqrt{3}$. Squaring these and adding them together, we get $5x^2 + 5y^2 = 80$. What we want is $\sqrt{4x^2 + 4y^2}$, so dividing by 5, multiplying by 4 and taking the square root gives us a hypotenuse of 8.

26. B The formula for radius of a circumscribed circle is $\frac{abc}{4A}$, where a, b, and c are side lengths and A is the area of the triangle. Using heron's formula, we get an area of 84 for this triangle. Plugging the numbers in the formula, we get a radius of $\frac{85}{8}$.

27. A Transform equation to: $(x-6)^2 + (y+4)^2 = 45$ so the center is (6,-4)

Find the slope between center and tangent point(-2). Tangent line has a slope that is the negative reciprocal of this. So equation is: $x - 2y = -1 \rightarrow A$

28. A If we call the side length of the square x, then the fraction of the side with length 3 that does not overlap with the square is 3-x, and similarly the fraction of the side with length 4 is 4-x. Because the square's sides are parallel to the legs of the triangle, then the two smaller triangles formed have the same angles and therefore are similar. Setting up a similarity ratio, we get $\frac{3-x}{x} = \frac{x}{4-x}$, and solving, we get x = $\frac{12}{7}$.

29. A Call the two diagonals r and s. Then, $r + s = 9$ and $r^2 + s^2 = 57$. If we square the first equation and subtract the second equation, we are left with $2rs = 24$. What we are looking for is $\frac{rs}{2}$, so dividing by 4 gives us an answer of 6.

30. D If we draw two circles externally tangent to the first circle and each other, and then connect the centers of these three circles, then we have formed an equilateral triangle with vertices at the 3 centers. An equilateral triangle has angles of 60 degrees, so we can only fit 6 of them in a circle (360 degrees).