1. B Using quotient and product rule: $\frac{p'qr-p(qr)'}{q^2r^2}=\frac{p'qr-p(q'r+qr')}{q^2r^2}=\frac{p'qr-pq'r-pqr'}{q^2r^2}$

2. B $\frac{x}{4-2x}=\frac{x}{4}\cdot\frac{1}{\left(1-\frac{x}{2}\right)}=\frac{x}{4}\sum_{n=0}^{\infty}\frac{x^n}{2^n}=\sum_{n=0}^{\infty}\frac{x^{n+1}}{2^{n+2}}$

3. B The slope of the line is $-\frac{1}{5}$ so the negative reciprocal is 5.

4. D $\frac{1}{2}\int_0^{2\pi} r^2\,d\theta=\frac{1}{2}\int_0^{2\pi} 3+\sin(\theta)d\theta=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 3+\sin(\theta)d\theta$ due to symmetry

5. A $g'(x)=\frac{-f^{-1}(x)'}{\left(f^{-1}(x)\right)^2}=-\frac{1}{\left(f^{-1}(x)\right)^2 f'\left(f^{-1}(x)\right)}\to g'(4)=-\frac{1}{\left(f^{-1}(4)\right)^2 f'\left(f^{-1}(4)\right)}=-\frac{1}{3^2\cdot\frac{4}{3}}=-\frac{1}{12}$

6. C $\lim_{x\to\infty}\ln\left(\frac{\frac{2}{x}}{tan^{-1}\left(\frac{5}{x}\right)}\right)\to L'Hospital\to\lim_{x\to\infty}\ln\left(\frac{\frac{-2}{x^2}}{-\frac{5}{x^2}\left(1+\left(\frac{5}{x}\right)^2\right)}\right)=\ln\left(\frac{2}{5}\right)$

7. D $\frac{dy}{dx}=2t^2$ and $t=1$ @ $(0,2)\to y-2=2(x-0)\to y=2x+2$

8. C $\frac{y+1}{y}dy=x\sin(x)\,dx\to\int\frac{y+1}{y}dy=\int x\sin(x)\,dx\to y+\ln|y|=-x\cos(x)+\sin(x)+C.\, y(0)=1\to c=1\to y+\ln|y|=-x\cos(x)+\sin(x)+1.$
However, since y is a function, and the initial condition has a positive value for y, the absolute value can be removed.

9. D $\ln(f(x))=\ln(3)+4\ln(x^3+1)+\ln(arctan(x))-\frac{1}{3}\ln(x)=\frac{1}{2}\ln(x^2+1)\to$
$f'(x)=\left(\frac{12x^2}{x^3+1}+\frac{1}{\arctan(x)(1+x^2)}-\frac{1}{3x}-\frac{x}{x^2+1}\right)f(x)\to f(1)=\left(\frac{12}{2}+\frac{1}{\frac{\pi}{4}\cdot 2}-\frac{1}{3}-\frac{1}{2}\right)\left(\frac{3\cdot 2^4\cdot\frac{\pi}{4}}{\sqrt{2}}\right)=12\sqrt{2}+31\sqrt{2}\pi$

10. A Let $f(x)=\sum_{n=0}^{\infty}\frac{x^n}{n+1}\to xf(x)=\sum_{n=0}^{\infty}\frac{x^{n+1}}{n+1}$ $so\ f'(x)=\sum_{n=0}^{\infty}x^n=\frac{1}{1-x}\to xf(x)=-\ln(1-x)\to f(x)=-\frac{\ln(1-x)}{x}$

11. E $\sin^2(x)=\frac{1}{2}-\frac{1}{2}\cos(2x)=\frac{1}{2}-\frac{1}{2}\left(1-\frac{2^2}{2!}x^2+\cdots\right)=\frac{2}{2!}x^2-\frac{2^3}{4!}x^4+\cdots=\sum_{n=1}^{\infty}(-1)^{n+1}\left(\frac{2^{2n-1}}{(2n)!}\right)x^{2n}$

12. D $P(t)=P(0)e^{kt}\to P(10)=2P(0)=P(0)e^{10k}\to 2=e^{10k}\to k=\frac{\ln 2}{10}\to 100=P(0)e^{\frac{\ln 2}{10}*45}\to 100=P(0)2^{4.5}\to P(0)=\frac{100}{2^{4.5}}.\, P(75)=\frac{100}{2^{4.5}}*2^{7.5}=800$

13. B $\frac{dy}{dx}=-\tan(5x).\, L=\int_{\frac{\pi}{30}}^{\frac{\pi}{15}}\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx=\int_{\frac{\pi}{30}}^{\frac{\pi}{15}}\sec(5x)\,dx=\frac{\ln(sec(5x)+tan(5x))}{5}\Big|_{\frac{\pi}{30}}^{\frac{\pi}{15}}=\frac{1}{10}\ln\left[\frac{7}{3}+\frac{4}{\sqrt{3}}\right]$

14. D $\int\frac{(\ln(\ln x))^2}{x}dx=\int(\ln u)^2\,du=u(\ln u)^2-2u\ln u+2u+C=(\ln x)(\ln(ln\,x))^2-2(\ln x)(\ln(ln\,x))+2\ln x+C$

15. A $\int e^{\sqrt{x}}\,dx=\int 2ue^u\,du=2e^u(u-1)\Big]_{u=1}^{u=2}=2e^2$

16. D
$$\frac{dx}{dt}=1+\frac{1}{2\sqrt{t}},\frac{dy}{dx}=1-\frac{1}{2\sqrt{t}}\ so\ L=\int_0^1\sqrt{\left(1+\frac{1}{2\sqrt{t}}\right)^2+\left(1-\frac{1}{2\sqrt{t}}\right)^2}\,dt=\int_0^1\sqrt{2+\frac{1}{2t}}\,dt$$

17. E We can calculate the y-coordinate of the centroid through the equation,
$$\bar{y}=\frac{\int_0^1 y\cdot\left(2\sqrt{1-y^2}\right)dy}{\frac{\pi}{2}}$$
which can be evaluated by the u-substitution $u=1-y^2$ to give us,

$$\frac{2}{\pi}\int_0^1 \sqrt{u}\,du = \frac{2}{3}u^{3/2}\Big|_0^1 = \boxed{\frac{4}{3\pi}}.$$

18. D At time t, the light will be at a height of $y(x) = R(1-\cos(t))$ and at a horizontal position of $x(t) = R(t - \sin(t))$. The distance traveled by the light is equal to the arc-length of this curve drawn out by where the light has been. We can calculate this arc-length using the formula,

$$L = \int_0^{2\pi} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\ dt.$$

Differentiating and plugging in gives,

$$= R\int_0^{2\pi} \sqrt{(1-\cos t)^2 + \sin^2 t}\ dt.$$

Expanding and the Pythagorean identity make this equal to,

$$R\int_0^{2\pi} \sqrt{2-2\cos t}\ dt = R\int_0^{2\pi} 2\left|\sin\frac{t}{2}\right|\ dt = 8R$$

19. B The osculating circle will have the same curvature as $y = x\hat{}2$ at (0,0). We can calculate this curvature as

$$\kappa = \frac{|y''|}{(1+y'^2)^{3/2}} = 2$$

and since the curvature of the circle is the reciprocal of its radius, we have that,

$$r = \frac{1}{\kappa} = \boxed{\frac{1}{2}}.$$

20. C Consider the Taylor series for $\tan^{-1}(x)$,

$$x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots$$

Replacing this into the expression gives,

$$\lim_{x\to 0} \frac{\left(3x - x^3 + \frac{3x^5}{5} + O(x^7)\right) - 3x + x^3}{3x^5} = \lim_{x\to 0} \frac{1}{5} + O(x^2) = \frac{1}{5}.$$

21. A We can find the centroid of a polar region by

$$\bar{x} = \frac{\frac{1}{3}\int_\alpha^\beta r^3\cos\theta\ d\theta}{\frac{1}{2}\int_\alpha^\beta r^2\ d\theta} \qquad \bar{y} = \frac{\frac{1}{3}\int_\alpha^\beta r^3\sin\theta\ d\theta}{\frac{1}{2}\int_\alpha^\beta r^2\ d\theta}$$

Here, since we have symmetry about the line $y = x$, we need only find one of these coordinates in order to have uniquely identified the centroid. We can calculate the numerator as:

$$\int_\alpha^\beta r^3\cos\theta\ d\theta = \int_0^{\pi/2} \sin^3(2\theta)\cos\theta\ d\theta = \frac{16}{35}.$$

Then, we can get the denominator as,

$$\int_\alpha^\beta r^2\ d\theta = \int_0^{\pi/2} \sin^2(2\theta)\ d\theta = \frac{\pi}{4}.$$

This means that

$$\bar{x} = \frac{\frac{1}{3}\int_\alpha^\beta r^3 \cos\theta \ d\theta}{\frac{1}{2}\int_\alpha^\beta r^2 \, d\theta} = \frac{1}{3}\cdot\frac{16}{35}\cdot 2\cdot\frac{4}{\pi} = \frac{128}{105\pi}.$$

And therefore, the value of the radius in polar coordinates must be,

$$\frac{128\sqrt{2}}{105\pi}.$$

22. E $\lim_{n\to\infty}\left|\frac{\frac{(n+1)(2x-1)^{n+1}}{4^{n+1}}}{\frac{n(2x-1)^n}{4^n}}\right| = \frac{|2x-1|}{4} < 1 \to \left(-\frac{3}{2},\frac{5}{2}\right)$ *since at the end points we diverge*

23. A I. Converges by the integral test. $\int_3^\infty \frac{1}{n(\ln(n))^2}dn = \frac{1}{\ln 3}$. II. Diverges by the limit comparison test with $\sum\frac{1}{\sqrt{n}}$. III. Converges with the comparison test with $\sum\frac{1}{n^2}$. IV. Diverges using the root test: $\lim_{n\to\infty}\left(\frac{n+1}{n}\right)^n = e$. Thus, I, III, IV converges.

24. B $\frac{dy}{dx} = -\frac{x}{\sqrt{r^2-x^2}}dx. SA = \int_{r^2}^{r} 2\pi\sqrt{r^2-x^2}\sqrt{1+\left(\frac{x}{\sqrt{r^2-x^2}}\right)^2}\,dx = \int_{r^2}^{r} 2\pi r\,dx = 2\pi(r^2-r^3)$. To find the r that maximizes area: $\frac{d}{dr} = 0 \to 2\pi(2r-3r^2) = 0 \to r = \frac{2}{3}$. Plug r back in to get: $\frac{8\pi}{27}$.

25. A We can recognize the integrand is,

$$\int_0^1 \frac{-x^2}{1+x^2}\,dx = -x + \tan^{-1}(x)\Big|_0^1 = \frac{\pi}{4} - 1.$$

Alternatively, we can integrate term by term and see the solution is,

$$\sum_{k=1}^{\infty}\frac{(-1)^k}{2k+1}$$

and knowing that,

$$tan^{-1}(x) = x - \frac{x^3}{3} + \frac{x^5}{5} + \ldots$$

we see that this is

$$\tan^{-1}(1) - 1 = \boxed{\frac{\pi}{4} - 1}.$$

26. A The particles speed squared is equal to,

$$(3t^2+2t)^2 + (6t)^2$$

which is non-negative and equal to 0 only when $t = 0$.

27. C $u = (\ln x)^4, dv = 1 \to du = \frac{4(\ln x)^3}{x}dx, v = x \ \int_0^1(\ln(x))^4dx = x(\ln x)^4\Big|_0^1 - \int_0^1 4(\ln x)^3dx.$

Repeat again and again to eventually get

$24x - 24x\ln x + 12x(\ln x)^2 + 4x(\ln x)^3 + x(\ln x)^4|_0^1$

$= 24 - \lim_{x\to 0} 24x - 24x\ln x + 12x(\ln x)^2 + 4x(\ln x)^3 + x(\ln x)^4$

$= 24 - \lim_{x\to\infty} \frac{24\ln x}{x} + \frac{12(\ln x)^2}{x} - \frac{4(\ln x)^3}{x} + \frac{(\ln x)^4}{x}.$

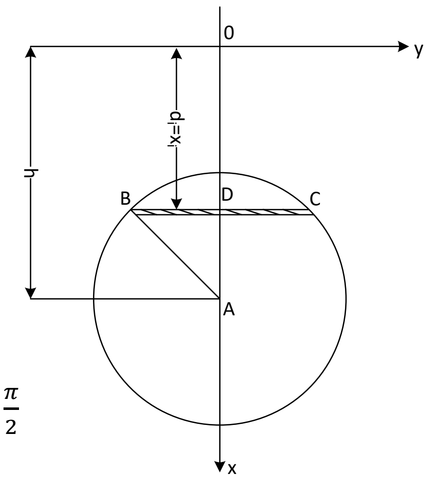
With continuous use of L'Hospital it is apparent that $\lim_{x\to 0} x\,(\ln x)^n$ *where n is a finite number is* 0.

$Final\ anser\ is\ thus\ 24.$

28. C The arc-length formula in polar is, $\int \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta = \int_0^{2\pi} \sqrt{(2+2\cos(\theta))^2 + (2\sin\theta)^2} d\theta =$ $2\sqrt{2}\int_0^{2\pi} \sqrt{1+\cos\theta}\, d\theta = 2\sqrt{2}\int_0^{2\pi} \sqrt{2\cos^2\frac{\theta}{2}}\, d\theta = 4\int_0^{2\pi} \left|\cos\frac{\theta}{2}\right| d\theta = 16$

29. D Telescoping series: $\sum_{n=1}^{\infty} \ln\left(\frac{tan^{-1}(n)}{tan^{-1}(n+1)}\right) = \ln\frac{tan^{-1}(1)}{tan^{-1}(2)} + \ln\frac{tan^{-1}(2)}{tan^{-1}(3)} + \cdots + \ln\ \frac{tan^{-1}(n)}{tan^{-1}(n+1)} = \ln\frac{\pi}{4} -$ $\ln\tan^{-1}(\infty) = \ln\frac{\pi}{4} - \ln\frac{\pi}{2} = -\ln 2$

30. B Choose x-axis and y-axis as shown:



$F_i = \rho g d_i A_i$. Now $d_i = x_i, A_i = 2\overline{BD}\cdot\Delta x$.

$\overline{BD} = \sqrt{\overline{AB} - \overline{AD}} = \sqrt{1-(h-x_i)^2}$ so

$A_i = 2\sqrt{1-(h-x_i)^2}\Delta x \rightarrow$

$F_i = 10000 x_i \cdot 2\sqrt{1-(h-x_i)^2}\Delta x$

So $F = \int_{h-1}^{h+1} 20000x\sqrt{1-(h-x)^2}dx$.

$Let\ h - x = \sin(\theta), \frac{\pi}{2} \le \theta \le \frac{\pi}{2}, dx = -\cos\theta\ d\theta$

When $x = h-1, \sin\theta = 1, \theta = \frac{\pi}{2}$, when $x = h+1, \theta = -\frac{\pi}{2}$

$So\ F = \int_{\frac{\pi}{2}}^{-\frac{\pi}{2}} 20000(h - \sin\theta)\cos^2\theta\, d\theta = 10000\pi h$