1. C
2. B
3. C
4. D
5. C
6. E
7. B
8. C
9. A
10. C
11. B
12. C
13. E
14. A
15. E
16. C
17. C
18. A
19. C
20. C
21. C
22. C
23. A
24. C
25. B
26. C
27. B
28. D
29. C
30. C

1. C The system has infinitely many solutions when $r = -6, s = -10$.
2. B The third matrix is not invertible, the fourth needs more than two elementary matrices.
3. C $A^4 = -4\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$
4. D Matrix multiplication is not commutative.
5. C The square of each matrix is zero except for the second matrix.
6. E All statements are false.
7. B Multiply specific rows and columns of the matrices to identify individual entries one at a time. (for example, second row of the initial matrix with the second column of the inverse to identify the bottom middle entry.)
   It can be seen that the inverse is $\begin{bmatrix}4 & 3 & 3\\-1 & -1 & -1\\-3 & 0 & -1\end{bmatrix}$.
   The sum of the missing entries is -4
8. C $(M^T)^{-1} = (M^{-1})^T$
9. A $\det(A) = 48, \det\left(\frac{1}{2}A\right) = \frac{48}{2^4} = 3, \det\left(\left(\frac{1}{2}A\right)^3\right) = 3^3 = 27$
10. C With each step, the determinant changes to $5, -5, -20, -20, -8000, -50$.
11. B Based on the information, $A = \begin{bmatrix}0 & 0 & z\\0 & y & 0\\x & 0 & 0\end{bmatrix}$, which has determinant $-xyz$.
12. C $1{:}\,s = s{:}\,1$, so $s^2 = 1$, or $s = \pm 1$.
13. E The dot product is $s^4 - 5s^2 + 4 = (s+2)(s-2)(s+1)(s-1)$, so there are four possible values for s.
14. A $9 + 4 + 1 + s^2 = 25$, so $s = \pm\sqrt{11}$, the product of which is -11.
15. E $(s-5)^2 + 4 + 4 + s^2 = 25$, so $s = 1, 4$. The sum is 5.
16. C All but the second one are formed correctly.
17. C The sequence cycles $k, -j, -k, j$. $a_{2020} = a_0 = k$.
18. A Picking a point as the initial point, and find the three vectors, then the volume is $\frac{1}{6}$ of the absolute value of the scalar triple product. The simplest point to use is $(2, 2, 2)$, resulting in $\begin{vmatrix}-5 & 0 & -1\\3 & -3 & 0\\4 & 0 & -4\end{vmatrix} = -72$. So the volume is 12.
19. C The three points on the plane given by the equation $x + y + z = 2$
20. C It is sufficient to look at angle between the normal vectors, which we will call θ.
   Then $\cos\theta = \frac{1(2)-2(3)+1(-1)}{\sqrt{1^2+2^2+1^2}\sqrt{2^2+3^2+1^2}} = -\frac{5}{\sqrt{84}}$. The dihedral angle between the planes must be on $\left[0, \frac{\pi}{2}\right]$, so cosine of the dihedral angle is $\frac{5}{\sqrt{84}}$, and the tangent of the angle is $\frac{\sqrt{59}}{5}$
21. C Competing the square on the given equation to see that the sphere is centered at $(4, 5, 6)$ with a radius of 3. The magnitude of $\langle 1, 2, 2\rangle$ is also 3, so it's sufficient to subtract $\langle 1,2,2\rangle$ from $(4, 5, 6)$.

22. C Note that the second component is simply the sum of the other three components. For the vectors to be linearly dependent, it is sufficient for the remaining components to be linearly dependent. This results in

$$\begin{vmatrix} s & 1 & 0 \\ s & 0 & s \\ 0 & -s & 1 \end{vmatrix} = s^3 - s = 0$$

So there are 3 values in total.

23. A The displacement is simply velocity times time, which is $\langle 2,4,4 \rangle$

24. C The possible connections can be described by $A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$, with the rows/cols corresponding to MIA, JAX, PEN in that order. It is sufficient to find the $(3,1)$ entry of A^8, which is 13.

$$\begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}^8 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}^4 = \begin{bmatrix} 3 & 2 & 3 \\ 3 & 2 & 3 \\ 2 & 1 & 2 \end{bmatrix}^2 = \begin{bmatrix} 21 & 13 & 21 \\ 21 & 13 & 13 \\ 13 & 8 & 13 \end{bmatrix}$$

25. B $(6,0,0)$ is a point on the first plane. It is sufficient to find the distance from that point to the second plane: $\frac{|6+0+0-36|}{\sqrt{1^2+2^2+2^2}} = 10$

26. C The first clearly works, as it's the identity matrix. The third is rotation by $120°$. The fourth is a reflection that exchanges two vertices. The second does not work, as it is a reflection across the x-axis.

27. B Each of the 6 permutations of the three vertices corresponds to a unique transformation matrix.

28. D All four work.

29. C There are 8 possibilities for a given vertex, and for each, there are 6 possibilities to place its neighbors.

30. C The eigenvalues can be found by solving $\begin{vmatrix} 2-\lambda & 0 & 3 \\ 0 & 2-\lambda & 2 \\ 0 & 1 & 3-\lambda \end{vmatrix} = 0$

So $(2-\lambda)\big((2-\lambda)(3-\lambda)-2\big) = 0$, or $\lambda = 1, 2, 4$.

Each value can then be plugged into the equation below to find its corresponding eigenvector.

$$\begin{bmatrix} 2-\lambda & 0 & 3 \\ 0 & 2-\lambda & 2 \\ 0 & 1 & 3-\lambda \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

As a result, $\begin{bmatrix} 6 \\ 0 \\ -6 \end{bmatrix}$ is the only one that is not an eigenvector.