2013 – 2014 Log1 Contest Round 2
Theta Areas & Volumes

Name: ________________

4 points each		
1	A circle encloses an area of $49\pi^2$ square feet. Find the length, in feet, of the radius of the circle.	
2	A triangle has sides of length 7, 8, and 13 feet. Find the area, in square feet, enclosed by the triangle.	
3	A sphere has surface area (in square feet) and volume (in cubic feet) that are numerically equivalent. Find the volume, in cubic feet, of a cube inscribed in the sphere.	
4	A rectangle has dimensions of 5 feet and 12 feet. The rectangle is cut along one of its diagonals to form two triangles. The two triangles are then joined back together along the cut to create a new quadrilateral that is not congruent to the original rectangle. Find the length, in feet, of the shorter diagonal of the new quadrilateral.	
5	An isosceles trapezoid has bases of length 24 feet and 40 feet and altitude of 15 feet. A second isosceles trapezoid is inscribed in the first trapezoid such that the longer base of the second trapezoid is the shorter base of the first trapezoid, while the shorter base of the second trapezoid lies on the longer base of the first trapezoid. Additionally, the acute base angles of both trapezoids are congruent. Find the area, in square feet, enclosed by the second trapezoid.	

5 points each		
6	A right circular frustum has bases with radii of lengths 4 feet and 2 feet. If the distance between these bases is 3 feet, find the volume, in cubic feet, of the frustum.	
7	A square is inscribed in a circle, which is in turn inscribed in another square. Find the ratio of the area enclosed by the smaller square to the area enclosed by the larger square.	
8	A regular hexagon has enclosed area (in square feet) numerically equivalent to its perimeter (in feet). Find the enclosed area, in square feet, of the hexagon.	
9	A pentagon is formed by connecting the following points, in order, with straight line segments: $(1,2)$, $(2,5)$, $(3,5)$, $(4,6)$, and $(5,0)$, then back to $(1,2)$. Find the area enclosed by this pentagon.	
10	A right circular cone has base diameter 30 feet and a height of 20 feet. Find the lateral surface area of the cone, in square feet.	

6 points each		
11	Find the area enclosed by the ellipse with equation $3x^2 + y^2 + 18x - 2y + 22 = 0$.	
12	A regular octahedron has an edge length of $6\sqrt{6}$ feet. Find the numerical value of the ratio of the volume (in cubic feet) of the octahedron to its surface area (in square feet).	
13	Find the surface area, in square feet, of an icosahedron with edge length of 2 feet.	
14	A cube with an edge length of 3 feet has the same surface area as that of a regular tetrahedron with what edge length, in feet?	
15	Four congruent circles with radius of length 1 foot are such that any one of the circles is externally tangent to exactly two of the other circles. Additionally, each of the four circles is internally tangent to a fifth, larger circle. Find the area, in square feet, enclosed by the fifth circle.	

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5	The area enclosed by the graphs of $y=2^x$ and the lines $y=0$, $x=0$, and $x=1$ can be written in the form $\log_2 a$. Find the value of a.	

	5 points each	
6	A square is inscribed in a circle, which is in turn inscribed in another square. Find the ratio of the area enclosed by the smaller square to the area enclosed by the larger square.	
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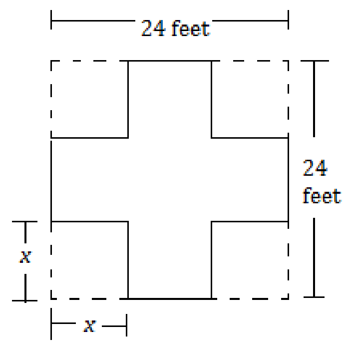
10	A right circular cone has base diameter 30 feet, a height that is an integral number of feet, and a lateral surface area whose number of square feet is an integral multiple of π. Find the sum of the lateral surface areas of all possible cones, in square feet.	

6 points each		
11	Find the area enclosed by the ellipse with equation $3x^2+y^2+18x-2y+22=0$.	
12	A regular octahedron has an edge length of $6\sqrt{6}$ feet. Find the numerical value of the ratio of the volume (in cubic feet) of the octahedron to its surface area (in square feet).	
13	Consider the complex numbers that are solutions to the equation $x^n=1$, where *n* is a positive integer satisfying $n\geq 3$. If the solutions are plotted in the Argand plane, and if those solutions are the vertices of a regular polygon whose enclosed area is A_n, find the value of $\lim\limits_{n\to\infty} A_n$.	
14	The region bounded by $y=x^2+2$ and the lines $y=0$, $x=0$, and $x=1$ is revolved about the *x*-axis. Find the volume of the resulting solid.	
15	An open box is to be made by cutting four equal squares with side length *x* from the corners of a 24 feet by 24 feet square piece of material , as shown in the diagram. The four remaining flaps are then folded up to form the sides of the box. What value of *x*, in feet, creates a box with maximum volume? 	

2013 – 2014 Log1 Contest Round 2
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1	A circle encloses an area of $49\pi^2$ square feet. Find the length, in feet, of the radius of the circle.	$7\sqrt{\pi}$
2	A triangle has sides of length 7, 8, and 13 feet. Find the area, in square feet, enclosed by the triangle.	$14\sqrt{3}$
3	A sphere has surface area (in square feet) and volume (in cubic feet) that are numerically equivalent. Find the volume, in cubic feet, of a cube inscribed in the sphere.	$24\sqrt{3}$
4	A rectangle has dimensions of 5 feet and 12 feet. The rectangle is cut along one of its diagonals to form two triangles. The two triangles are then joined back together along the cut to create a new quadrilateral that is not congruent to the original rectangle. Find the length, in feet, of the shorter diagonal of the new quadrilateral.	$\frac{120}{13}$
5	An isosceles trapezoid has bases of length 24 feet and 40 feet and altitude of 15 feet. A second isosceles trapezoid is inscribed in the first trapezoid such that the longer base of the second trapezoid is the shorter base of the first trapezoid, while the shorter base of the second trapezoid lies on the longer base of the first trapezoid. Additionally, the acute base angles of both trapezoids are congruent. Find the area, in square feet, enclosed by the second trapezoid.	240

5 points each		
6	A right circular frustum has bases with radii of lengths 4 feet and 2 feet. If the distance between these bases is 3 feet, find the volume, in cubic feet, of the frustum.	28π
7	A square is inscribed in a circle, which is in turn inscribed in another square. Find the ratio of the area enclosed by the smaller square to the area enclosed by the larger square.	$1:2$ or $\frac{1}{2}$
8	A regular hexagon has enclosed area (in square feet) numerically equivalent to its perimeter (in feet). Find the enclosed area, in square feet, of the hexagon.	$8\sqrt{3}$
9	A pentagon is formed by connecting the following points, in order, with straight line segments: $(1,2)$, $(2,5)$, $(3,5)$, $(4,6)$, and $(5,0)$, then back to $(1,2)$. Find the area enclosed by this pentagon.	13
10	A right circular cone has base diameter 30 feet and a height of 20 feet. Find the lateral surface area of the cone, in square feet.	375π

6 points each		
11	Find the area enclosed by the ellipse with equation $3x^2 + y^2 + 18x - 2y + 22 = 0$.	$2\sqrt{3}\pi$
12	A regular octahedron has an edge length of $6\sqrt{6}$ feet. Find the numerical value of the ratio of the volume (in cubic feet) of the octahedron to its surface area (in square feet).	2 or 2:1 or $\frac{2}{1}$
13	Find the surface area, in square feet, of an icosahedron with edge length of 2 feet.	$20\sqrt{3}$
14	A cube with an edge length of 3 feet has the same surface area as that of a regular tetrahedron with what edge length, in feet?	$3\sqrt[4]{12}$
15	Four congruent circles with radius of length 1 foot are such that any one of the circles is externally tangent to exactly two of the other circles. Additionally, each of the four circles is internally tangent to a fifth, larger circle. Find the area, in square feet, enclosed by the fifth circle.	$\left(3+2\sqrt{2}\right)\pi$

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7	A regular hexagon has enclosed area (in square feet) numerically equivalent to its perimeter (in feet). Find the enclosed area, in square feet, of the hexagon.	$8\sqrt{3}$
8	Find the volume, in cubic feet, of the largest regular tetrahedron that can be inscribed in a cube with edge length 5 feet.	$\frac{125}{3}$
9	A pentagon is formed by connecting the following points, in order, with straight line segments: $(1,2)$, $(2,5)$, $(3,5)$, $(4,6)$, and $(5,0)$, then back to $(1,2)$. Find the area enclosed by this pentagon.	13
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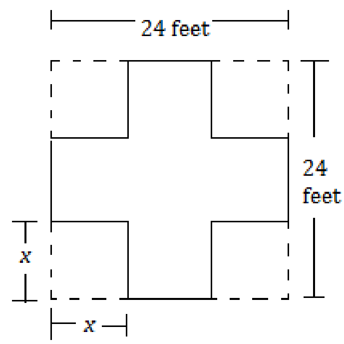
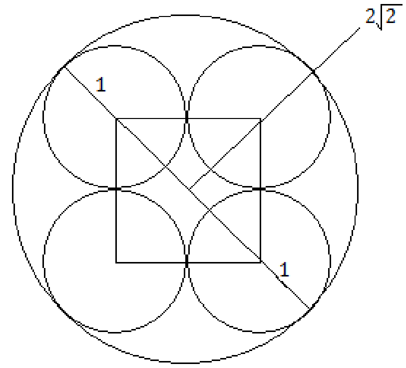
10	A right circular cone has base diameter 30 feet, a height that is an integral number of feet, and a lateral surface area whose number of square feet is an integral multiple of π. Find the sum of the lateral surface areas of all possible cones, in square feet.	2655π

	6 points each	
11	Find the area enclosed by the ellipse with equation $3x^2+y^2+18x-2y+22=0$.	$2\sqrt{3}\pi$
12	A regular octahedron has an edge length of $6\sqrt{6}$ feet. Find the numerical value of the ratio of the volume (in cubic feet) of the octahedron to its surface area (in square feet).	2 or 2:1 or $\frac{2}{1}$
13	Consider the complex numbers that are solutions to the equation $x^n=1$, where n is a positive integer satisfying $n\geq 3$. If the solutions are plotted in the Argand plane, and if those solutions are the vertices of a regular polygon whose enclosed area is A_n, find the value of $\lim_{n\to\infty} A_n$.	π
14	The region bounded by $y=x^2+2$ and the lines $y=0$, $x=0$, and $x=1$ is revolved about the x-axis. Find the volume of the resulting solid.	$\frac{83\pi}{15}$
15	An open box is to be made by cutting four equal squares with side length *x* from the corners of a 24 feet by 24 feet square piece of material , as shown in the diagram. The four remaining flaps are then folded up to form the sides of the box. What value of *x*, in feet, creates a box with maximum volume? 	4

2013 – 2014 Log1 Contest Round 2
Areas & Volumes Solutions

Mu	Al	Th	Solution
1	1	1	$49\pi^2 = \pi r^2 \Rightarrow r^2 = 49\pi \Rightarrow r = 7\sqrt{\pi}$ (since $r > 0$)
2	2	2	Since the semiperimeter of the triangle is $s = \dfrac{7+8+13}{2} = 14$, the enclosed area is $A = \sqrt{14(14-7)(14-8)(14-13)} = \sqrt{14 \cdot 7 \cdot 6 \cdot 1} = 14\sqrt{3}$.
3	3	3	$\dfrac{4}{3}\pi r^3 = 4\pi r^2 \Rightarrow r = 3$. This makes the diameter $d = 6$ and the side length of the cube $s = \dfrac{6}{\sqrt{3}} = 2\sqrt{3}$. The volume of the cube, then, is $\left(2\sqrt{3}\right)^3 = 24\sqrt{3}$.
		4	The new quadrilateral is a kite whose longer diagonal is the length of the diagonal of the rectangle, which is 13. Therefore, since the two quadrilaterals have the same area, if *d* is the length of the shorter diagonal, $\dfrac{1}{2}(13)d = 5(12) \Rightarrow d = \dfrac{120}{13}$.
4	4	5	Since the longer base is 16 feet longer than the shorter base of the first trapezoid, and since the interior angles of the two trapezoids are congruent, this same relationship between the bases exists for the second trapezoid, making the shorter base of length 8 feet. Therefore, the enclosed area of the second trapezoid is $A = \dfrac{1}{2}(24+8)(15) = 240$.
	5	6	$V = \dfrac{1}{3}\pi(3)\left(4^2 + 2 \cdot 4 + 2^2\right) = 28\pi$
5			$\int_0^1 2^x\,dx = \left.\dfrac{2^x}{\ln 2}\right\|_0^1 = \dfrac{2-1}{\ln 2} = \dfrac{1}{\ln 2} = \dfrac{\ln e}{\ln 2} = \log_2 e$, so $a = e$.
6	6	7	If s is the side length of the smaller square, then the diagonal of that square, which equals the diameter of the circle, is $s\sqrt{2}$. The diameter of the circle is also the side length of the larger rectangle, so the desired ratio is $\dfrac{s^2}{\left(s\sqrt{2}\right)^2} = \dfrac{1}{2}$.
7	7	8	$\dfrac{3\sqrt{3}}{2}s^2 = 6s \Rightarrow s = \dfrac{4\sqrt{3}}{3}$, and since the area and the perimeter are numerically equivalent, the area is $6s = 6\left(\dfrac{4\sqrt{3}}{3}\right) = 8\sqrt{3}$.
8	8		This is a famous problem. The largest such tetrahedron has vertices at four corners of the cube, no two of which share an edge. The six edges are all face diagonals of the cube, thus having an edge of length $5\sqrt{2}$. Therefore, the volume of the tetrahedron is $\dfrac{\left(5\sqrt{2}\right)^3\sqrt{2}}{12} = \dfrac{500}{12} = \dfrac{125}{3}$.

9	9	9	Using the shoelace method, the enclosed area is: $\begin{array}{r\|rr\|l} & 1 & 2 & \\ 4 & 2 & 5 & 5 \\ 15 & 3 & 5 & 10 \\ 20 & 4 & 6 & 18 \\ 30 & 5 & 0 & 0 \\ 0 & 1 & 2 & 10 \\ 69 & & & 43 \end{array} = \frac{1}{2}\|69-43\| = \frac{1}{2}(26) = 13$.
	10	10	Since the radius of the cone is 15, the slant height of the cone is $\sqrt{15^2+20^2}=25$. Therefore, the lateral surface area of the cone is $\pi(15)(25)=375\pi$.
10			If the height of the cone is *h* and its slant height is *l*, then $225=15^2=l^2-h^2$ $=(l-h)(l+h)$, and the five factorizations of 225 are $1\cdot 225$, $3\cdot 75$, $5\cdot 45$, $9\cdot 25$, and $15\cdot 15$. Setting $(l-h)(l+h)$ equal to each of these, the first three yield positive integral values of *l* and *h*, with l equaling 113, 39, and 25, respectively. Therefore, the sum of the three possible lateral surface areas is $\pi\cdot 15\cdot 113+\pi\cdot 15\cdot 39+\pi\cdot 15\cdot 25$ $=2655\pi$.
11	11	11	Completing the square on both variables yields $3(x+3)^2+(y-1)^2=6$. Dividing both sides by 6 yields $\frac{(x+3)^2}{2}+\frac{(y-1)^2}{6}=1$, so the enclosed area is $\pi\sqrt{2}\cdot\sqrt{6}=2\sqrt{3}\pi$.
12	12	12	The volume of the octahedron is $\frac{\sqrt{2}\left(6\sqrt{6}\right)^3}{3}=864\sqrt{3}$, and the surface area of the octahedron is $2\sqrt{3}\left(6\sqrt{6}\right)^2=432\sqrt{3}$, so the desired ratio is $\frac{864\sqrt{3}}{432\sqrt{3}}=2$.
	13	13	Each of the 20 faces of the icosahedron is an equilateral triangle, so the area of each face is $\frac{2^2\sqrt{3}}{4}=\sqrt{3}$. Therefore, the total surface area is $20\sqrt{3}$.
13			The vertices are equally spread on the unit circle, so as $n\to\infty$, the polygon approaches the unit circle, the enclosed area of which is $\pi(1)^2=\pi$.
	14	14	A regular tetrahedron's surface is four equilateral triangles, so the total surface area of a regular tetrahedron is $4\cdot\frac{s^2\sqrt{3}}{4}=s^2\sqrt{3}$, where *s* is the edge length of the tetrahedron. The total surface area of the cube is $6\cdot 3^2=54$, so $s^2\sqrt{3}=54\Rightarrow s^2=18\sqrt{3}=\sqrt{972}$ $\Rightarrow s=\sqrt[4]{972}=3\sqrt[4]{12}$.
14			$V=\pi\int_0^1\left(x^2+2\right)^2dx=\pi\int_0^1\left(x^4+4x^2+4\right)dx=\pi\left(\frac{x^5}{5}+\frac{4x^3}{3}+4x\right)\Big\|_0^1=\pi\left(\frac{1}{5}+\frac{4}{3}+4\right)=\frac{83\pi}{15}$

	15	15	A diagram of the circles appears to the right. The four centers of the smaller circles are the vertices of a square whose side length equals a diameter of any one of the smaller circles, which is 2. Therefore, the diagonal of the square, which also goes through the center of the larger circle, has length $2\sqrt{2}$. Add the two radii of the smaller circles as shown to get a diameter of the larger circle equal to $2+2\sqrt{2}$. Thus, the radius of the larger circle has length $1+\sqrt{2}$, making the enclosed area $\pi\left(1+\sqrt{2}\right)^2$ $=\left(3+2\sqrt{2}\right)\pi$. 
15			The volume is $V=x\left(24-2x\right)^2=576x-96x^2+4x^3$, with $0\le x\le 12$. $\frac{dV}{dx}=576-192x+12x^2=12\left(4-x\right)\left(12-x\right)$, so $\frac{dV}{dx}=0$ if $x=4$ or $x=12$. Using the closed interval method, since $V\left(0\right)=V\left(12\right)=0$ and $V\left(4\right)=1024$, V is a maximum when $x=4$.