1. A
2. E
3. B
4. A
5. C
6. A
7. B
8. B
9. D
10. C
11. A
12. C
13. C
14. E
15. A
16. B
17. D
18. A
19. C
20. B
21. E
22. B
23. E
24. A
25. A
26. B
27. B
28. A
29. E
30. C

1. A ObserAve $f''(t) = t^2 - 6t + 9 = (t-3)^2 \geq 0$. Thus the second derivative never changes signs, which means there are no inflection points.

2. E Observe that $\sqrt{4-(2+e)^2} = \sqrt{e^2 - 2e}$. For sufficiently small $e, e^2 - 2e < 0$. Thus, $\lim_{x \to 2} \sqrt{4 - x^2}$ does not exist.

3. B In a situation like this, solving a more general problem can be a better approach. In particular consider finding c_0 for the function:

$$g(x) = ax^2 + bx + c$$

on the interval (d, e).
Applying MVT yields:

$$2ac_0 + b = \frac{a(e^2 - d^2) + b(e-d)}{e-d} = a(e+d) + b \to c_0 = \frac{e+d}{2}$$

In the particular problem $d = 1331, e = 2022 \to c_0 = \frac{3353}{2}$

4. A This is a separable differential equation. Rearranging yields:

$$\int \frac{dy}{\sqrt{1+y^2}} = \int dx$$

For the first integral take $y = \sinh(z)\,, dy = \cosh\,(z)dz$ which gives:

$$\int dz = \int dx \to z = x + c \to y = \sinh(x+c).$$

With the initial condition it follows $c = 0$.
Now $\sinh(\ln(2)) = \frac{2-\frac{1}{2}}{2} = \frac{3}{4}$.
Note: In the integral above an alternate substitution could have been $y = \tan\,(z)$, which also would have led to the right answer.

5. C Writing $a_n = \cos\,(x)$ yields: $a_{n+1} = \sqrt{\frac{1+\cos(x)}{2}} = \cos\left(\frac{x}{2}\right)$.
From here it is a straightforward induction to show:

$$a_n = \cos\left(\frac{\pi}{3 \cdot 2^n}\right)$$

Since $\frac{\pi}{3 \cdot 2^n} \to 0, n \to \infty$, the requested limit is simply 1.

6. A Notice that if we shift the parabola such that the vertical line of symmetry is the y axis, the answer to #6 remains unchanged:
Since $g(x) = -(x-4)^2 + 1, g(x+4) = -x^2 + 1$.
Hence the answer is $\int_{-1}^{1} (1 - x^2)\, dx = \frac{4}{3}$

7. B Similar to #6 notice that shifting the parabola such that the vertical line of symmetry is the y axis does not change the answer.
So instead the maximum area of a rectangle that can be inscribed in the region bounded by $h(x) = 1 - x^2$ will be computed. Observe that the base of the rectangles are parallel to the x axis. To prove this, consider a slanted rectangle, tilt that

rectangle down until the lower base coincides with the x axis. Then expand out that rectangle until it is tangent to both sides of the curve.
With this in mind denote $(a, 1-a^2)$ to be the top right vertex of the rectangle where $a > 0$. Due to symmetry the top left vertex will be $(-a, 1-a^2)$.
Hence, we wish to maximize A = $2a(1-a^2)$. Taking the derivative gives $A' = 2 - 6a^2$. This gives critical points $a = \pm\frac{1}{\sqrt{3}}$, but since the area is clearly positive, discard the negative critical point. Plugging in $a = \frac{1}{\sqrt{3}}$, gives $\max(A) = \frac{4\sqrt{3}}{9}$

8. B Note: It is absolutely possible to solve this simply by a direct integral. However, the given solution will illustrate Pappus method for this. Again, note that it is easier to work with the shifted version $h(x) = 1 - x^2$ and the answer does not change. To use Pappus, first we will compute the centroid. Clearly the x coordinate is 0 of the centroid, as the parabola is symmetric about the y axis. To compute the y coordinate, we will first compute the volume of the solid formed when the region bound by $h(x)$ and the x axis is revolved about the x axis, and then use Pappus in reverse.
Notice that

$$V_{x=0} = \pi\int_{-1}^{1}(1-x^2)^2dx = 2\int_{0}^{1}(1-2x^2+x^4)\,dx - \frac{16\pi}{15}.$$

Now $2\pi rA = V_{x=0} \rightarrow 2\pi r\left(\frac{4}{3}\right) = \frac{16\pi}{15} \rightarrow r = \frac{2}{5}$
Now r is the distance from the centroid to the axis of revolution. In this case, this is simply the y coordinate. Thus, the centroid of $h(x)$ is $\left(0, \frac{2}{5}\right)$.
Finally, we can compute:

$$V_{x=4} = 2\pi\left(4 - \frac{2}{5}\right)\left(\frac{4}{3}\right) = \frac{48\pi}{5}$$

9. D We can use Pappus to solve this question. Notice in #8 the centroid when the parabola was shifted 4 units to the left was computed. Thus, shifting the centroid 4 units to the right will be the centroid of the original parabola. This means that the coordinates of the centroid is $\left(4, \frac{2}{5}\right)$. Now the line being revolved about can be written in standard form as $3y + 4x = 36$
Thus the distance from centroid to the axis of revolution is given by (using distance from point to a line formula):

$$d = \frac{\left|4(4) + 3\left(\frac{2}{5}\right) - 36\right|}{\sqrt{3^2+4^2}} = \frac{94}{25}$$

Now $V_l = 2\pi\left(\frac{94}{25}\right)\left(\frac{4}{3}\right) = \frac{752\pi}{75}$

10. C Denote a, b with $b > a$ to be the roots of the quadratic $g(x) - mx$.
We wish to solve the equation:

$$I = \int_a^b (g(x) - mx)\,dx = \int_a^b(-x^2 + (8-m)x - 15)dx = \frac{2}{3}$$

Now using power rule for integrals we see that:

$$I = -\frac{b^3 - a^3}{3} + \frac{(8-m)(b^2-a^2)}{2} - 15(b-a)$$

$$I = (b-a)\left(-\frac{(b+a)^2 - ab}{3} + \frac{(8-m)(a+b)}{2} - 15\right).$$

Now with Vieta's we find that $b + a = 8 - m, ab = 15$

Now using $(a-b)^2 + 4ab = (a+b)^2,$ we can compute:

$$b - a = \sqrt{(8-m)^2 - 4 \cdot 15} = \sqrt{m^2 - 16m + 4}.$$

Note: The positive square root is taken since by definition $b > a$.

Thus:

$$I = \sqrt{m^2 - 16m + 4}\left(-\frac{(8-m)^2 - 15}{3} + \frac{(8-m)^2}{2} - 15\right)$$

$$I = \sqrt{m^2 - 16m + 4}\left(\frac{(8-m)^2}{6} - 10\right)$$

$$I = \frac{(m^2 - 16m + 4)^{\frac{3}{2}}}{6}$$

Setting this equal to 2/3 yields:

$$4 = (m^2 - 16m + 4)^{\frac{3}{2}} \to * \; m^2 - 16m + 4 = 2\sqrt[3]{2}$$

$$(m-8)^2 = 60 + 2\sqrt[3]{2} \to m = 8 \pm \sqrt{60 + 2\sqrt[3]{2}}$$

Now notice $a + b = 8 - m = \pm\sqrt{60 + 2\sqrt[3]{2}}$. Since a, b are clearly positive, this means only the smaller value for m is valid. Thus, the answer is:

$$8 - 1 + 2 + 60 + 2 + 3 + 2 = 76$$

11. A For sufficiently negative x we can write:

$\sqrt{x^2 + 8x} \sim \sqrt{x^2 + 8x + 16} \sim -(x-4)$

Similarly, $\sqrt{x^2 + 4x} \sim -(x-2)$ for sufficiently negative x

Thus, we wish to compute:

$$\lim_{x \to -\infty} \left(-(x-4) - \left(-(x-2)\right)\right) = -2$$

12. C If we attempt to use the same trick as question #11 the limit evaluates to:

$$\lim_{x \to \infty} x\left((x+2) - (x+2)\right) = \infty \cdot 0,$$

which is an indeterminate form

Instead, note that:

$$\lim_{x \to \infty} x\left(\sqrt{x^2 + 4x + 5} - \sqrt{x^2 + 4x + 3}\right)$$

$$= \lim_{x \to \infty} x\left(\sqrt{x^2 + 4x + 5} - \sqrt{x^2 + 4x + 3}\right)\frac{\sqrt{x^2 + 4x + 5} + \sqrt{x^2 + 4x + 3}}{\sqrt{x^2 + 4x + 5} + \sqrt{x^2 + 4x + 3}}$$

$$= \lim_{x \to \infty} \frac{2x}{\sqrt{x^2 + 4x + 5} + \sqrt{x^2 + 4x + 3}}$$

Now we can use the approximation like Question #11 to get the denominator to be approximately $2x + 4$ for sufficiently large x. Thus the limit evaluates to 1

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13. C Observe:

$$S=\sum_{k=1}^{\infty}\frac{(k^2+k)}{k!}=\sum_{k=1}^{\infty}\frac{(k+1)}{(k-1)!}.$$

Now making the substitution $k=j+1$ yields:

$$S=\sum_{j=0}^{\infty}\frac{(j+2)}{j!}=\sum_{j=0}^{\infty}\frac{j}{j!}+2e=\sum_{j=1}^{\infty}\frac{j}{j!}+2e=\sum_{j=1}^{\infty}\frac{1}{(j-1)!}+2e=3e$$

14. E Notice that $\sqrt{3}>1$. Since $\sum_{n=0}^{\infty}(-1)^n t^{2n}$ only converges on $(-1,1)$, the integral diverges when $|x|\geq 1$, and thus $f(x)$ is not defined on this interval.

15. A Observe that the DE is separable. Rewriting gives:

$$\frac{dP}{P(50-P)}=dt\to\frac{dP}{50}\left(\frac{1}{P}+\frac{1}{50-P}\right)=dt$$

$$\to\frac{1}{50}\ln\left|\frac{P}{50-P}\right|=t+C\to\frac{P}{50-P}=Ce^{50t}$$

Now plugging in the initial condition gives:

$$C=24.$$

Finally, the question asks for the time at which the population is 24.
Hence:

$$\frac{24}{26}=24e^{50t}\to\frac{1}{26}=e^{50t}\to t=-\frac{\ln(26)}{50}.$$

16. B First observe that:

$$\sum_{j=0}^{5}x^{2j}y^{10-2j}\binom{5}{j}=(x^2+y^2)^5$$

So the relation reduces to:

$$(x^2+y^2)^{\frac{5}{2}}=2(x+y)(x^2-xy+y^2)\to(x^2+y^2)^{\frac{5}{2}}=2(x^3+y^3)$$

Now we will convert this to polar with the change of coordinates $x=rcos(\theta), y=rsin(\theta)$. Using the fact that $x^2+y^2=r^2$ the relation reduces to:

$$r^5=2(r^3\cos^3(\theta)+r^3\sin^3(\theta))$$

Notice that the graph $r(\theta)=0$ is simply the point at the origin and has no area. Thus we can divide out r^3 from the relation to get:

$$r^2(\theta)=2(\cos^3\theta+\sin^3\theta).$$

Thus the requested answer is:

$$I=\frac{1}{2}\int_0^{\frac{\pi}{2}}r^2(\theta)\,d\theta=\int_0^{\frac{\pi}{2}}(\cos^3\theta+\sin^3\theta).$$

Now using the fact that $\cos(\theta)$, $\sin(\theta)$ bound the same area on the interval $\left[0,\frac{\pi}{2}\right]$, we have that:

$$I = 2\int_0^{\frac{\pi}{2}} \cos^3\theta\, d\theta = 2\int_0^{\frac{\pi}{2}} \cos(\theta)(1-\sin^2(\theta)) \xRightarrow{u=\sin(\theta)} I = 2\int_0^1 (1-u^2)du = \frac{4}{3}$$

17. D Adding the system presented yields:

$$12\big(f'(x)+g'(x)\big) = -(f(x)+g(x))$$

Denote $u(x) = f(x)+g(x) \to u'(x) = f'(x)+g'(x)$ which gives:

$$12u'(x) = -u(x) \to \frac{12d\big(u(x)\big)}{dx} = -u(x) \to \frac{-12\left(d\big(u(x)\big)\right)}{u(x)} = dx$$

Integrating both sides yields:

$$-12\ln\big(u(x)\big) = x + C \to u(x) = Ce^{-\frac{x}{12}}.$$

We are additionally given the condition $f(0) = 5 - g(0) \to f(0)+g(0) = 5 \to u(0) = 5 \to C = 5$

$$u(x) = 5e^{-\frac{x}{12}} \to u\left(6\ln\left(\frac{16}{25}\right)\right) = 5e^{\ln\left(\frac{5}{4}\right)} = \frac{25}{4}$$

For the following questions denote $a = BC, b = AC, c = AB$.

18. A

$$\frac{d(a+b+c)}{dt} = \frac{da}{dt} + \frac{db}{dt} + \frac{dc}{dt} = -1 + 2 - 3 = -2$$

19. C Using Law of Cosines we have:

$$b^2 + c^2 - 2bc(\cos(A)) = a^2$$

Taking the derivative yields:

$$2(bb' + cc' - (b'c + c'b)\cos(A) + bc(\sin(A))A') = 2aa'$$

$$2\big(13\cdot 2 + 14\cdot -3 - \big(2(14) - 13(3)\big)\cos(A) + 13\cdot 14(\sin(A))A'\big) = 2\cdot 15\cdot -1$$

$$1 = 11\cos(A) + 182\sin(A)\,A'$$

Now to calculate $\cos(A)$ we use LOC:

$$13^2 + 14^2 - 2\cdot 13\cdot 14\cos(A) = 15^2$$
$$140 = 2\cdot 13\cdot 14\cos(A)$$
$$\cos(A) = \frac{5}{13}, \sin(A) = \frac{12}{13}$$

So we have;

$$1 = 11\left(\frac{5}{13}\right) + 182\left(\frac{12}{13}\right)A' \to A' = -\frac{1}{52}$$

20. B Let $\angle AMB = \theta, \angle AMC - \pi - \theta$, Using LOC on triangles AMB ad AMC yields:

$$AM^2 + \frac{a^2}{4} - a \cdot AM \cdot \cos(\theta) = c^2$$
$$AM^2 + \frac{a^2}{4} + a \cdot AM \cdot \cos(\theta) = b^2$$
since $\cos(\pi - \theta) = -\cos(\theta)$.

Adding the two equations yields:

$$2(AM)^2 = b^2 + c^2 - \frac{a^2}{2}$$
and then taking the derivative yields:

$$4(AM)(AM)' = 2bb' + 2cc' - aa'$$

At time t_1 we have:
$$2(AM)^2 = 14^2 + 13^2 - \frac{15^2}{2} \to AM = \frac{\sqrt{505}}{2}$$

And so we have:
$$2\sqrt{505}(AM)' = 2(13)(2) + 2(14)(-3) - (15)(-1) \to (AM)'^2 = \frac{17^2}{2020}$$

21. E By definition:
$$\frac{F_{n+1}}{F_n} = \frac{F_n + F_{n-1}}{F_n} = 1 + \frac{F_{n-1}}{F_n}$$
Additionally, since $F_n \geq F_{n-1} \to \frac{F_{n-1}}{F_n} \leq 1$ it follows that $\frac{F_{n+1}}{F_n} \leq 2$ and converges.
Now let $L = \lim_{n\to\infty} \frac{F_{n+1}}{F_n}$. Since this limit was proven to converge above notice that:
$L = 1 + \frac{1}{L} \to L = \frac{1+\sqrt{5}}{2}$ since L clearly is positive.
Now notice the answer to this question is simply $\frac{1}{L} = \frac{-1+\sqrt{5}}{2}$

22. B Observe that:
$$h'(x) = 15x^2 + 24x + 14$$
Thus:
$$\sum h'(r_i) = 15\sum r_i^2 + 24\sum r_i + 42 =$$
$$15\left(\left(\sum r_i\right)^2 - 2\sum r_i r_j\right) + 24\sum r_i + 42 =$$
$$15\left(\left(-\frac{12}{5}\right)^2 - 2\left(\frac{14}{5}\right)\right) + 24\left(-\frac{12}{5}\right) + 42 = 15\left(\frac{4}{25}\right) + 24\left(-\frac{12}{5}\right) + 42 = -\frac{66}{5}$$

23. E We have:

$$h(x) = 3x^3 \int_0^x dt = 3x^4 \to h'(1) = 12$$

For the following two solutions the ellipse:

$$\frac{x^2}{4} + \frac{y^2}{9} = 1$$

will be used which satisfies the conditions and is easy to work with for computations

24. A Since the semi-circular cross sections are perpendicular to the minor axis that means for a given x. the diameter of the semi-circle is 2|y|. This also means that the area of the semi-circle is $\frac{\pi}{2}y^2$. Thus the answer is simply equivalent to :

$$I = \frac{\pi}{2}\int_{-2}^{2} y^2\,dx = \frac{\pi}{2}\int_{-2}^{2} \frac{(36 - 9x^2)}{4}\,dx = \frac{\pi}{8}\int_{-2}^{2}(36 - 9x^2)\,dx = 12\pi$$

25. A Using washer method this is simply:

$$\pi \int_{-2}^{2} y^2 dx$$

Which is double the answer computed in Question 24. Thus, the answer is 24π.

26. B Observe that:

$$\ln\left(f(x)\right) = \ln\left(e^{x^2} + 5x + 6\right) + \frac{\ln(1 - 2x^2) + \ln(1 - \sin(x))}{4} - 5\left(\ln(1 + x) + \frac{\ln(1 - x)}{2}\right)$$

Note: The terms $2x^2 - 1, \sin(x) - 1$ were negated when the terms were broken up to account for the fact that at x = 0 the terms should be positive in order for the derivative to be defined. This has no impact on the final answer but is included for the sake of rigor.

Taking the derivative yields:

$$\frac{f'(x)}{f(x)} = \frac{2xe^{x^2} + 5}{e^{x^2} + 5x + 6} + \frac{1}{4}\left(-\frac{4x}{1 - 2x^2} - \frac{\cos(x)}{1 - \sin(x)}\right) - 5\left(\frac{1}{1 + x} - \frac{1}{2(1 - x)}\right)$$

Plugging in x = 0 yields:

$$\frac{f'(0)}{f(0)} = \frac{5}{7} + 0 - \frac{1}{4} - 5 + \frac{5}{2} \to f'(0) = f'(0) = -\frac{57}{28}f(0)$$

Trivially note that $f(0) = 7$.

Thus $f'(0) = -\frac{57}{4}$

27. B We will first work on computing the indefinite integral.
The simplest solutions to search for of those where:

$$\left(\frac{f(x)}{5\cos(x) + \sin(x)}\right)' = \frac{26}{(5\cos(x) + \sin(x))^2}$$

Using quotient rule on the LHS gives the DE:

$$(5\cos(x)+\sin(x))p'(x)-p(x)(-5\sin(x)+\cos(x))=26$$

Rather than directly attempt to solve this DE think about the RHS. The RHS is simply a constant with no trigonometric function:
Recall the identity $\sin^2(x)+\cos^2(x)=1$
With that in mind the simplest form that p(x) could take on is:

$$\mathrm{a}\sin(x)+b cos(x)$$

Plugging this into the above DE and simplifying magically gives:

$$5a-b=26$$

Now observe that the answer to this question would be:

$$\frac{\mathrm{a}\sin(x)+b cos(x)}{5\cos(x)+\sin(x)}\Big|_0^{\frac{\pi}{2}}=a-\frac{b}{5}=\frac{5a-b}{5}=\frac{26}{5}$$

28. A We will first evaluate the more general sum:

$$S=\sum_{n=0}^{\infty}\binom{2n}{n}x^n$$

where the sum converges.
Notice for $n\geq 1$:

$$\binom{2n}{n}x^n=\frac{2n!}{(n!)^2}x^n=(2n-1)!!\frac{2^n x^n}{n!}=\frac{(2n-1)!!}{2^n n!}4^n x^n=$$
$$\frac{(-1)^n(2n-1)!!}{2^n n!}(-4x)^n=\binom{-\frac{1}{2}}{n}(-4x)^n=1^{-\frac{1}{2}-n}(-4x)^n\binom{-\frac{1}{2}}{n}$$

Now applying binomial theorem in reverse it is clear that:

$$S=(1-4x)^{-\frac{1}{2}}\to S^2=\frac{1}{1-4x}$$

With the particular value $x_0=\frac{1}{6}$ the sum evaluates to 3.

29. E Denote:

$$S(m,n)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{m^2n}{n^4m^3+m^4n^3}$$

Observe that:

$$S(m,n)+S(n,m)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{m^2n+n^2m}{n^4m^3+m^4n^3}=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{mn(m+n)}{m^3n^3(m+n)}.$$

Thus:

$$S(m,n)+S(n,m)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{1}{m^2n^2}=\left(\sum_{m=1}^{\infty}\frac{1}{m^2}\right)^2=\frac{\pi^4}{36}$$

Also notice that:

$$S(n,m)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{n^2m}{n^4m^3+m^4n^3}=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{m^2n}{n^4m^3+m^4n^3}=$$

$$\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{m^2n}{n^4m^3+m^4n^3}=S(m,n).$$

Thus:

$$2S(m,n)=\frac{\pi^4}{36}\to S(m,n)=\frac{\pi^4}{72}$$

30. C Denote (where $0<x<1$:

$$f(x)=\sum_{n=1}^{\infty}\frac{x^n}{n^2}\to f'(x)=\sum_{n=1}^{\infty}\frac{x^{n-1}}{n}\to xf'(x)=\sum_{n=1}^{\infty}\frac{x^n}{n}$$

$$\left(xf'(x)\right)'=\sum_{n=1}^{\infty}x^{n-1}=\frac{1}{1-x}\to xf'(x)=-\ln(1-x)$$

So:

$$f(x)=\int_0^x\frac{-\ln(1-t)}{t}dt$$

$$F=f(x)+f(1-x)=\int_0^x\frac{-\ln(1-t)}{t}dt+\int_0^{1-x}\frac{-\ln(1-t)}{t}dt$$

Now observe:

$$\int_0^{1-x}\frac{-\ln(1-t)}{t}dt=\int_x^1-\frac{\ln(t)}{1-t}dt$$

And:

$$\int_0^x\frac{-\ln(1-t)}{t}dt=\int_0^1-\frac{\ln(1-t)}{t}dt+\int_x^1\frac{\ln(1-t)}{t}dt$$

Thus:

$$F=\int_0^1-\frac{\ln(1-t)}{t}dt+\int_x^1\frac{\ln(1-t)}{t}-\frac{\ln(t)}{1-t}dt$$

Now observe:

$$\int_x^1\frac{\ln(1-t)}{t}-\frac{\ln(t)}{1-t}dt=\int_x^1(\ln(1-t)\ln(t))'dt=$$

$$\lim_{h\to1^-}\ln(h)\ln(1-h)-\ln(1-x)\ln(x)$$

Denote: $L=\lim_{h\to1^-}\ln(h)\ln(1-h)$

Making the substitution $u=1-h$ yields the equivalent limit:

$$L=\lim_{u\to0^+}\ln(u)\ln(1-u)=\frac{\ln(1-u)}{\frac{1}{\ln(u)}}$$

Using L'hopital yields:

$$L = \lim_{u\to 0^+} \frac{uln^2(u)}{1-u}$$

Now apply the substitution $u = e^{-t}$ which yields:

$$L = \lim_{t\to\infty} \frac{e^{-t}t^2}{1-e^{-t}}$$

Now observe that the exponential term e^{-t} is decreasing much faster than t^2 is growing.
Thus, this limit evaluates to 0

Additionally note:

$$\int_0^1 -\frac{\ln(1-t)}{t}dt = -\int_0^1 \frac{-\sum_{n=1}^{\infty}\frac{t^n}{n}}{t}dt = \int_0^1 \sum_{n=1}^{\infty}\frac{t^{n-1}}{n}dt = \sum_{n=1}^{\infty}\frac{1}{n^2} = \frac{\pi^2}{6}$$

Combining all the previous results yields:

$$F(x) = \frac{\pi^2}{6} - \ln(x)\ln(1-x).$$

The question asks for$F\left(\frac{1}{4}\right)$ which is simply:

$$\frac{\pi^2}{6} - \ln\left(\frac{1}{4}\right)\ln\left(\frac{3}{4}\right) = \frac{\pi^2 + 6\ln(4)\ln(3) - 6\ln^2 4}{6}$$