### 2016 – 2017 Log1 Contest Round 1 Theta Applications

Name: \_\_\_\_\_

	4 points each	
1	On his mom's birthday, Izaya is 25 years old and his mom is 52 years old. Izaya notices that his age was the reverse of his mom's age. How many years have passed since their ages were also reversals of one another?	
2	On Monday, Kidney spends $\frac{1}{3}$ of her money on Froot Loops <sup>M</sup> . On Tuesday, she spends $\frac{1}{6}$ of her remaining money on off-brand cheese snacks. Now she has \$10 left. How much money did Kidney begin with, in dollars?	
3	A sunflower is one-foot-tall on Tuesday. By the following Tuesday (1 week later), the sunflower becomes one-third taller than it was the previous Tuesday. If this trend continues, how many Tuesdays <i>after the day it was one-foot-tall</i> will it take for the sunflower to grow so that it is over three feet tall?	
4	I have four charms. Two are identical. How many distinguishable ways can I arrange them on a charm bracelet?	
5	In the morning, Waltino Dawito drives to work with an average speed of 40 mph. At the end of his work day he drives back home with an average speed of 20 mph as traffic is much worse than it was in the morning. What was Waltino's overall average speed, in mph? Leave your answer as an improper fraction.	

	5 points each		
6	A cardboard party hat in the shape of a cone has a height of 6 inches. The diameter of its base is 4 inches. If a square inch of cardboard is equal to 1¢, how much did it cost to make the hat, in dollars? Do not estimate but rather provide an exact answer.		
7	Concert tickets cost \$5.34 for children and \$7.63 for adults. If 66 tickets were sold and \$501.29 was collected, how many adult tickets were sold?		
8	Banh Beo is standing on a building, holding a ball 645 feet above the ground. She drops the ball and discovers that after each bounce, it rebounds to only two-thirds of its previous height. How far has the ball traveled in feet when it comes to a rest?		
9	The National Bank offers a special compounding rate. Suppose Jack has \$2000 and invests in an account that pays 25% interest rate compounded annually. How much money does he have at the end of 2 years, in dollars?		
10	What is the reflection of the point (1,2) about the line $3y = 4x$ ?		

	6 points each		
11	Xavier's School for Gifted Youngsters is changing their student identification system by making everyone's student ID like so: digit-digit-letter-letter-letter-digit. Assuming that student IDs are assigned randomly, the first digit cannot be a zero and the letters cannot be an uppercase "I" or uppercase "O" and lowercase or uppercase letters are considered as being different from each other (i.e. A and a are different characters) what is the probability that the first and last initials of Jean Grey appear in the letters portion, from left-to-right OR right-to-left either, both or none capitalized? The initials do not have to be consecutive.		
12	Sahara Fig Newton drew four random cards from a standard 52 card deck. What are the chances that she drew a four card inside straight? An inside straight draw is when you have 4 of 5 cards needed for a straight but missing any number in the middle, such as 2,3,4,6. Aces count as high or low and "around-the-horn" straights are not allowed. (i.e. QKA23)		
13	Cap'n Geech of the Shrimp Shack Shooters is going to have a pool party, so he bought a doughnut-shaped pool tube with an inner diameter of 2 feet and an outer diameter of 4 feet when fully inflated. What is the volume of this tube when fully inflated, in feet cubed?		
14	Fry and Bender notice that in a football stadium, a section has 40 rows of seats. The odd numbered rows increase arithmetically by 2, starting with 60 seats in row 1. The even numbered rows increase arithmetically by 1, starting with 62 seats in row 2. How many total seats are there this section?		
15	Sarah Beluga plans to mop her floor. She has a bottle that contains 4 ounces of diluted solution at a 3:1 ratio of water-to-concentrated cleanser (25%). She has 2 bottles of concentrated solution, one is 90% and the other is 80%. She pours both concentrates simultaneously into the diluted solution until a 50% solution is obtained. How many total ounces COMBINED from both concentrated solutions will have been used once her diluted solution reaches 50% cleanser? Mixed number solutions are acceptable.		

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14	A property's boundary is to be fenced. Unfortunately, the boundary isn't a rectangle but rather a normal quadrilateral, ABCD, with A located at the origin of an X-Y coordinate system. AB is 40 feet in length at an angle of $30^{\circ}$ <i>East of North</i> . BC is 50 feet in length at an angle of $60^{\circ}$ <i>South of East</i> . CD is 70 feet in length at an angle of $30^{\circ}$ <i>West of South</i> . What is the length of DA, in feet?		
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4	Evaluate: $\lim_{x \to 0} \frac{\frac{1}{x+5} - \frac{1}{5}}{x}$		
5	The position of a particle moving along a line is given by the displacement equation $s(t) = \frac{1}{5}t^5 - 3t^3 + 2$ . Determine the acceleration of this particle at time $t = 4$ .		
	Units are arbitrary.		

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10	What is the reflection of the point (1,2) about the line $3y = 4x$ ?		

	6 points each		
11	You are at an intersection and wish to get to the next intersection one block away to your right on the other side of the road. The road is 10 yards wide and the next intersection is 30 yards to your right. Strangely, you prefer to walk across the street at 1 yard/s but run at 3 yards/s when on the sidewalk. If you can choose any point on the other side of the street to run to before walking to your destination point, how far would that point be FROM THE DESTINATION POINT, in yards. Start		
12	Sahara Fig Newton drew four random cards from a standard 52 card deck. What are the chances that she drew a four card inside straight? An inside straight draw is when you have 4 of 5 cards needed for a straight but missing any number in the middle, such as 2,3,4,6. Aces count as high or low and "around-the-horn" straights are not allowed. (i.e. QKA23)		
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15	A person with binoculars is 3 horizontal kilometers away from an airport where he views a plane on a straight-line descent path into the airport with a speed of 2 km/min at an angle of 45 degrees above the horizontal, as depicted in the diagram below. How fast, in km/min, is the shortest distance between the plane and the person changing when the plane is at an altitude of 1 km? Person 3 km Airport		

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4	I have four charms. Two are identical. How many distinguishable ways can I arrange them on a charm bracelet?	2
5	In the morning, Waltino Dawito drives to work with an average speed of 40 mph. At the end of his work day he drives back home with an average speed of 20 mph as traffic is much worse than it was in the morning. What was Waltino's overall average speed, in mph? Leave your answer as an improper fraction.	$\frac{80}{3}$

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7	Concert tickets cost \$5.34 for children and \$7.63 for adults. If 66 tickets were sold and \$501.29 was collected, how many adult tickets were sold?	65	
8	Banh Beo is standing on a building, holding a ball 645 feet above the ground. She drops the ball and discovers that after each bounce, it rebounds to only two-thirds of its previous height. How far has the ball traveled in feet when it comes to a rest?	3225	
9	The National Bank offers a special compounding rate. Suppose Jack has \$2000 and invests in an account that pays 25% interest rate compounded annually. How much money does he have at the end of 2 years, in dollars?	3125	
10	What is the reflection of the point (1,2) about the line $3y = 4x$ ?	$\left(\frac{41}{25},\frac{38}{25}\right)$	

	6 points each		
11	Xavier's School for Gifted Youngsters is changing their student identification system by making everyone's student ID like so: digit-digit-letter-letter-letter-digit. Assuming that student IDs are assigned randomly, the first digit cannot be a zero and the letters cannot be an uppercase "I" or uppercase "O" and lowercase or uppercase letters are considered as being different from each other (i.e. A and a are different characters) what is the probability that the first and last initials of Jean Grey appear in the letters portion, from left-to-right OR right-to-left either, both or none capitalized? The initials do not have to be consecutive.	$\frac{3}{31250}$	
12	Sahara Fig Newton drew four random cards from a standard 52 card deck. What are the chances that she drew a four card inside straight? An inside straight draw is when you have 4 of 5 cards needed for a straight but missing any number in the middle, such as 2,3,4,6. Aces count as high or low and "around-the-horn" straights are not allowed. (i.e. QKA23)	$\frac{1536}{54145}$	
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14	Fry and Bender notice that in a football stadium, a section has 40 rows of seats. The odd numbered rows increase arithmetically by 2, starting with 60 seats in row 1. The even numbered rows increase arithmetically by 1, starting with 62 seats in row 2. How many total seats are there this section?	3010	
15	Sarah Beluga plans to mop her floor. She has a bottle that contains 4 ounces of diluted solution at a 3:1 ratio of water-to-concentrated cleanser (25%). She has 2 bottles of concentrated solution, one is 90% and the other is 80%. She pours both concentrates simultaneously into the diluted solution until a 50% solution is obtained. How many total ounces COMBINED from both concentrated solutions will have been used once her diluted solution reaches 50% cleanser? Mixed number solutions are acceptable.	$5\frac{5}{7}$	

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9	A boat is driven North at a speed of 15 knots per hour relative to the ocean. Suddenly, the boat meets rough waters, encountering an ocean current of 9 knots at 30° South of West, relative to the Earth. What is the resultant speed, in knots, of the boat, relative to the Earth?	3√19
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4	Evaluate: $\lim_{x \to 0} \frac{\frac{1}{x+5} - \frac{1}{5}}{x}$	$-\frac{1}{25}$
5	The position of a particle moving along a line is given by the displacement equation $s(t) = \frac{1}{5}t^5 - 3t^3 + 2$ . Determine the acceleration of this particle at time $t = 4$ . Units are arbitrary.	184

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12	Sahara Fig Newton drew four random cards from a standard 52 card deck. What are the chances that she drew a four card inside straight? An inside straight draw is when you have 4 of 5 cards needed for a straight but missing any number in the middle, such as 2,3,4,6. Aces count as high or low and "around-the-horn" straights are not allowed. (i.e. QKA23)	$\frac{1536}{54145}$
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15	A person with binoculars is 3 horizontal kilometers away from an airport where he views a plane on a straight-line descent path into the airport with a speed of 2 km/min at an angle of 45 degrees above the horizontal, as depicted in the diagram below. How fast, in km/min, is the shortest distance between the plane and the person changing when the plane is at an altitude of 1 km? Person 3 km Airport	$-\frac{1}{5}\sqrt{10}$

## 2016 – 2017 Log1 Contest Round 1 Applications Solutions

Mu	Al	Th	Solution
1	1	1	11 years ago Izaya was 14 and his mom was 41.
2	2	2	$\frac{1}{3}x + \frac{1}{6}(\frac{2}{3}x) + 10 = x$ $\frac{1}{3}x + \frac{2}{18}x + 10 = x$ $\frac{4}{9}x + 10 = x$ x = 18
3	3	3	One Tuesday: $1 \times \frac{4}{3} = \frac{4}{3}$ , $3 > \frac{4}{3}$ Two Tuesdays: $\frac{4}{3} \times \frac{4}{3} = \frac{16}{9}$ , $3 > \frac{16}{9}$ Three Tuesdays: $\frac{16}{9} \times \frac{4}{3} = \frac{64}{27}$ , $3 > \frac{64}{27}$ Four Tuesdays: $\frac{64}{27} \times \frac{4}{3} = \frac{256}{81}$ , $3 < \frac{256}{81}$ Thus, the answer is four Tuesdays. Alternatively, let n represent the number of Tuesday's following the initial Tuesday, and set up the equation $(\frac{4}{3})^n > 3$ where n has to be a whole number, and solve for n.
4			$\lim_{x \to 0} \frac{\frac{1}{x+5} - \frac{1}{5}}{x} \times (\frac{x+5}{x+5})(\frac{5}{5})$ $\lim_{x \to 0} \frac{5 - (x+5)}{x(x+5)(5)} = \lim_{x \to 0} \frac{-x}{x(x+5)(5)}$ $\lim_{x \to 0} \frac{-1}{(x+5)(5)} = \lim_{x \to 0} \frac{-1}{5x+25} = -\frac{1}{25}$
	4	4	Let the charms be represented by AABC, where AA represents the two identical charms. If the two identical charms are next to each other, then there is only one unique way to arrange the charms, AABC. AACB is achieved through rotations and flipping. The other possibility is ABAC. ACAB is achieved through rotations and flipping. Therefore, there are only 2 distinguishable charm arrangements.
5			The acceleration is the 2 <sup>nd</sup> derivative of displacement. $a(t) = \frac{d^2s}{dt^2} = \frac{d}{dt}(t^4 - 9t^2) = 4t^3 - 18t$ $a(4) = 4^4 - 18(4) = 184$

	5	5	The average velocity is the harmonic mean of the two speeds.
			$y_{ave} = \frac{1}{1 + 1} = \frac{1}$
			$\frac{1}{2}\left(\frac{1}{40} + \frac{1}{20}\right) = \frac{1}{2}\left(\frac{60}{800}\right) = \frac{30}{800} = 3$
			Alternatively; from physics principles
			$v_{ave} = \frac{\text{distance}}{1} = \frac{40t_1 + 20t_2}{1}$
			time $t_1 + t_2$
			$v_{ave} = \frac{40t_1 + 20(2t_1)}{t_1 + 2t_1} = \frac{80t_1}{3t_1} = \frac{80}{3}mph$
6	6	6	The lateral surface area of the party hat is given by the formula A = $\pi rL$ where L =
			Slant Height of the cone and r = the base radius. Since it is a hat, you may ignore the
			base area and use the lateral area to calculate the cost. The slant height is related to
			the radius and height, h, of the cone. L = $\sqrt{r^2 + h^2} = \sqrt{2^2 + 6^2} = 2\sqrt{10}$
			Therefore, A = $\pi(2)(2)\sqrt{10} = 4\pi\sqrt{10}$ . Divide by 100 to convert to dollars. Cost = $\frac{\pi\sqrt{10}}{25}$
7	7	7	5.34c + 7.63a = 501.29 and $c + a = 66$
			5.34c + 7.63a = 501.29 and $5.34a + 5.34c = 352.44$
			7.63a - 5.34a = 2.29a and $501.29 - 352.44 = 148.85$
			$148.85 \div 2.29 = a$
			a = 65
8	8	8	The bounce heights can be represented as an infinite geometric series with common
			ratio of 2/3. It has the form 645, 645 $\left(\frac{2}{3}\right)$ , 645 $\left(\frac{2}{3}\right)^2$ ,
			$S_{\infty} = \frac{a_1}{1-r} = \frac{645}{1-\left(\frac{2}{3}\right)} = \frac{645}{\frac{1}{3}} = 1935 \text{ feet.}$
			Each bounce counts for twice the distance, so $d = 3870$ <i>feet</i> . You have to subtract
			645 feet since the first term assumes that the ball started from the ground instead of
			begin dropped.
			d = 3870 - 645 = 3225 feet
9	9		The speeds, when directed as vectors, of the boat and the current add like vectors. The
			angle between the two vectors is 60°. According to the Law of Cosines, $c^2 = a^2 + b^2 - b^2$
			2abCosC
			$c^2 = 15^2 + 9^2 - 2(15)(9)\cos(60^\circ)$
			$c^{2} = 225 + 81 - 270(\frac{1}{2}) = 306 - 135 = 171.$
			Therefore, $c = \sqrt{171} = \sqrt{9 \times 19} = 3\sqrt{19}$

		9	After 1 year $P = $2000(1.25)$
			After 2 years $P = $2000(1.25^2) = $3125$
10	10	10	The equation of the line perpendicular to $3y = 4x$ going through (1,2) is $4y + 3x = 11$ ,
			which intersects the line $3y = 4x$ at the point $(\frac{33}{25}, \frac{44}{25})$ . The reflection is then $(2(\frac{33}{25}) - $
			1), $(2(\frac{44}{35}) - 2)$
			or $\left(\frac{41}{25}, \frac{38}{25}\right)$ .
11			Label your starting point S. Pick a point across the street to initially run to and label it
			B. The point directly across from you at S is A and your destination point is C. Let
			AB=x and BC=30-x. The distance from S to B is $\sqrt{10^2 + x^2}$ .
			The total time to reach your destination is $t = \frac{\sqrt{10^2 + x^2}}{1} + \frac{30 - x}{3}$
			To maximize this function, find $\frac{dt}{dx}$ and set it equal to zero.
			$\frac{dt}{dx} = \frac{1}{2}(10^2 + x^2)^{\left(-\frac{1}{2}\right)}(2x) - \left(\frac{1}{3}\right) = 0$
			$x(10^{2} + x^{2})^{-\frac{1}{2}} = \frac{1}{3} \rightarrow x^{2}(10^{2} + x^{2})^{-1} = \frac{1}{9}$
			$x^{2} = \left(\frac{1}{9}\right)(10^{2} + x^{2}) \rightarrow x^{2} = \frac{100}{9} + \frac{1}{9}x^{2}$
			$\frac{8}{9}x^2 = \frac{100}{9} \to x^2 = \frac{100}{8} \to x = \frac{5}{\sqrt{2}} = \frac{5}{2}\sqrt{2} \therefore BC = 30 - \frac{5}{2}\sqrt{2} \text{ yards}$
	11	11	The number of distinct student IDs is
			$9 * 10^2 * 50^3$
			The number of 2 letter combinations in a 3 letter sequence is ${}_{2}^{2}C = 3$
			If upper and lower case letters are distinct and order does matter, then each
			combination has 4 different permutations. Thus the number of ways that Jean Grey's initials appear is 12. Total passible IDs with Ioan Crew's initials are $(0 + 10^2)(12)$
			initials appear is 12. Total possible for with Jean Grey S initials are $(9 \times 10^{-})(12)$
			$p = (9 * 10^2)(12) = 12 = 3$
			$P = \frac{1}{9 * 10^2 * 50^3} = \frac{1}{125000} = \frac{1}{31250}$

12	12	12	There are $\binom{52}{4}$ ways to draw four cards from a standard 52 card deck. For each card A-
			10, there are $3 * 4^4$ ways to draw an inside straight. E.g. A,2,3,5 or A,3,4,5 or A,2,4,5 -
			10,J,K,A or 10,Q,K,A or 10,J,Q,A. Thus there are $10 * 3 * 4^4$ possible inside straights.
			$P = \frac{30 * 4^4}{\frac{52!}{4! \cdot 48!}} = \frac{30 * 4! * 48! * 4^4}{52!} = \frac{1536}{54145}$
13	13	13	The shape of this tube is the same as a circular cylinder bent so that its ends are
			attached. The volume of the tube is equal to the volume of this cylinder. With an outer
			RADIUS of 2 feet and an inner RADIUS of 1 foot, the diameter of the cylinder is 1 foot.
			Therefore its radius is $\frac{1}{2}$ foot. The height of the cylinder is the average of the outer and
			inner circumferences of the pool tube, or $\frac{2\pi(1)+2\pi(2)}{2} = 3\pi$ feet
			$V = \pi \left(\frac{1}{2}\right)^2 3\pi = \frac{3\pi^2}{4} f t^3$
14	14		Resolve AB, BC and CD into right triangles. The sum of the X components of AB and BC
			= the sum of the X components of CD and DA. The sum of the Y components of AB and
			DA = the sum of the components of BC and CD.
			$AB_x = 40Sin30 = 20 \rightarrow AB_y = 40Cos30 = 20\sqrt{3}$
			$BC_x = 50Cos60 = 25 \rightarrow BC_y = 50Sin60 = 25\sqrt{3}$
			$CD_x = 70Sin30 = 35 \rightarrow CD_y = 70Cos30 = 35\sqrt{3}$
			$\therefore DA_x = AB_x + BC_x - CD_x = 10 \rightarrow DA_y = BC_y + CD_y - AB_y = 40\sqrt{3}$
			$DA = \sqrt{100 + 4800} = \sqrt{4900} = 70;$
		14	$u_{n-odd} = 60 + (n-1)2 \rightarrow u_{20} = 60 + (19 * 2) = 98$
			$u_{n-even} = 62 + (n-1)1 \rightarrow u_{20} = 62 + (19 * 1) = 81$
			$S_{20-odd} = \frac{20}{2}(60+98) = 1580$
			20
			$S_{20-even} = \frac{1}{2}(62+81) = 1430$
			<i>S</i> = 3010

15			Draw a triangle with the person located at vertex A and the airport at B. The plane is
			at vertex C. Line segment BC forms an angle of 45 degrees with respect to the
			horizontal. Label the segment AC with the variable s to denote the distance between
			the person and the plane. Also label the segment BC with the variable x to denote the
			distance between the plane and the airport. According to the Law of Cosines,
			$s^2 = 3^2 + x^2 - 2(3)x\cos 45 = 9 + x^2 - 3\sqrt{2}x \rightarrow$
			$s^2 = x^2 - 3\sqrt{2}x + 9$
			$2s\frac{ds}{dt} = (2x - 3\sqrt{2})\frac{dx}{dt} \rightarrow \frac{ds}{dt} = \frac{2x - 3\sqrt{2}}{2s}\left(\frac{dx}{dt}\right)$
			By the definition of the Sine, $x = \frac{h}{Sin45} = \sqrt{2}$
			You must also find the distance, s, at this altitude, h. The segment, AB, can be divided
			into 2 smaller segments, AH and HB, where H is the point where the altitude of the
			triangle intersects the segment AB. Since the angle of descent is 45 degrees, If CH=1,
			then HB=1 and AH=2
			By the Pythagorean Theorem, $s^2 = 1^2 + 2^2 = 1 + 4 = 5 \rightarrow s = \sqrt{5}$
			$ds = 2\sqrt{2} - 3\sqrt{2} = \sqrt{5}(-\sqrt{2}) = -\sqrt{10} = \frac{1}{\sqrt{10}}$
			$\frac{1}{dt} = \frac{1}{2\sqrt{5}} = \frac{1}{5} =$
	15	15	If she starts off with a solution that has a water to concentrated cleanser ratio of 3:1
			and adds x ounces of concentrated cleanser at a ratio of 1:9 simultaneously with x
			ounces of concentrated cleanser solution at a ratio of 1:8 then the new solution would
			have a water to cleanser ratio of 1:1 only if the following proportion holds true:
			$3 + \frac{1}{10}x + \frac{2}{10}x$
			$\frac{10}{1+\frac{9}{1+\frac{8}{10}}x+\frac{8}{10}x} = 1$
			$-10^{10} \cdot 10^{10}$
			$\frac{56}{14} = 20 = 26 = 10 + 172$
			$14x = 20 \rightarrow x = 2\frac{-}{7}$ ounces; This is the number of ounces from each bottle.
			Multiply by 2 to obtain the final answer. $5\frac{5}{7}ounces$