2016 – 2017 Log1 Contest Round 1
Theta Equations and Inequalities

Name: _____
Units do not have to be included.

	4 points each	
1	Simplify $\sqrt[4]{81a^{24}b^{8}}$	
2	Solve for x: $3x + \frac{2}{x} = -7$	
3	Find $(f(g(-5)) if\ f(x) = \sqrt{x+16}$ and $g(x) = 5x^2 - x + 23.$	
4	Find the vertex of: $f(x) = \frac{1}{2}x^2 - 4x - 2$	
5	Solve: $\left\|\frac{2x+1}{x+2}\right\| = 3$	

	5 points each	
6	Find the points where $y = x^2 - 6x + 3$ and $y = 2x^2 + 4x + 10$ meet.	
7	Evaluate: $x = 3 - \sqrt{3 - \sqrt{3 - \sqrt{3 - \cdots}}}$.	
8	What values of x satisfy the inequality $\frac{2^{3x+5}}{8^{x-2}} < \frac{(4^{3x})}{16^{-2x}}$?	
9	$If\ f(x) = \frac{3x+1}{x-1}$, solve for $f^{-1}\left(\frac{5}{2}\right)$	
10	Solve: $\frac{x(x+7)}{x+1} \leq 6$	

6 points each		
11	For what values of x is: $\frac{x-3}{x+4} < \frac{x+4}{x-3}$?	
12	If the polynomial, $2x^3 + x^2 + kx - 4$, has a root of -2, find k.	
13	The complex number z is a solution of the equation $\sqrt{z} = \frac{4}{1+i} + 7 - 2i$. If z can be expressed in the form $a + bi$, find $a + b$.	
14	Solve: $\frac{3x+1}{x+2} = \frac{6x+4}{2x+2}$	
15	What is the sum of the roots of the following equation? $2x^3 + 3x^2 - 8x + 3 = 0$	

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14	The cost of producing *x* candy hearts for Valentine's day is given by $C(x) = \frac{1}{125}x^2 + \frac{9}{25}x - \frac{47}{50}$ dollars and the selling price of each INDIVIDUAL candy heart is given by $R(x) = 44 - \frac{1}{5}x$, in cents. Find the maximum profit in cents.	
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5	Evaluate: $\lim_{x\to\infty} \frac{1-2x-x^2}{2x^2-4}$	

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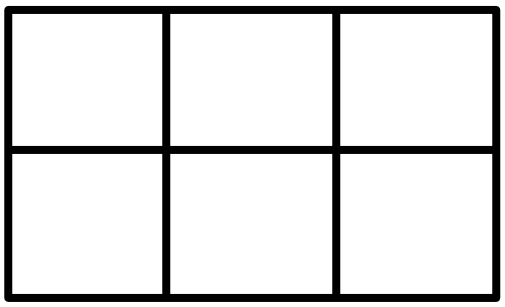
10	Farmers Nancy and Cindy want to construct 6 rectangular animal pens with 800m of fencing. The configuration of the pens will look like a rectangle divided into 6 sections each of equal area, as shown. Find the dimensions of each pen if each pen is to have a maximum area. The pens SHARE their INTERIOR walls. 	

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15	The equation $y = x^2 + ax + b$ function is tangent to the line $2x + y = 6$ at $x = 1$. Determine the values of **a** and **b**.	

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4	Find the vertex of: $f(x) = \frac{1}{2}x^2 - 4x - 2$	$(4, -10)$
5	Solve: $\left\|\frac{2x+1}{x+2}\right\| = 3$	$-5, -\frac{7}{5}$

	5 points each	
6	Find the points where $y = x^2 - 6x + 3$ and $y = 2x^2 + 4x + 10$ meet.	$x_1 = -5 - 3\sqrt{2}$ $y_1 = 76 + 48\sqrt{2}$ & $x_2 = -5 + 3\sqrt{2}$ $y_2 = 76 - 48\sqrt{2}$
7	Evaluate: $x = 3 - \sqrt{3 - \sqrt{3 - \sqrt{3 - \cdots}}}$.	$\frac{7 - \sqrt{13}}{2}$
8	What values of x satisfy the inequality $\frac{2^{3x+5}}{8^{x-2}} < \frac{(4^{3x})}{16^{-2x}}$?	$x > \frac{11}{14}$
9	$If\ f(x) = \frac{3x+1}{x-1}$, solve for $f^{-1}\left(\frac{5}{2}\right)$	-7
10	Solve: $\frac{x(x+7)}{x+1} \leq 6$	$x \leq -3$ $-1 < x \leq 2$

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12	If the polynomial, $2x^3 + x^2 + kx - 4$, has a root of -2, find k.	-8
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14	Solve: $\frac{3x+1}{x+2} = \frac{6x+4}{2x+2}$	$-\frac{3}{4}$
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Alpha Equations and Inequalities

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4	Given $f(x) = \frac{9-x}{3} + \frac{8+x}{2}$, evaluate $f^{-1}(10)$	18
5	Solve: $\left\|\frac{2x+1}{x+2}\right\| = 3$	$-5, -\frac{7}{5}$

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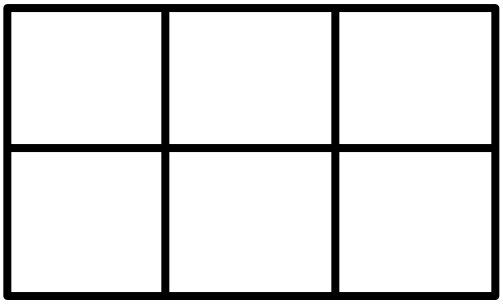
10	Farmers Nancy and Cindy want to construct 6 rectangular animal pens with 800m of fencing. The configuration of the pens will look like a rectangle divided into 6 sections each of equal area, as shown. Find the dimensions of each pen if each pen is to have a maximum area. The pens SHARE their INTERIOR walls. 	$\frac{400}{9}\ x\ 50$

6 points each		
11	For what values of x is: $\frac{x-3}{x+4} < \frac{x+4}{x-3}$?	$-4 < x < -\frac{1}{2}$ OR $x > 3$
12	If the polynomial, $2x^3 + x^2 + kx - 4$, has a root of -2, find k.	-8
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15	The equation $y = x^2 + ax + b$ function is tangent to the line $2x + y = 6$ at $x = 1$. Determine the values of **a** and **b**.	$a = -4$ $b = 7$

2016 – 2017 Log1 Contest Round 1
Equations and Inequalities Solutions

Mu	Al	Th	Solution
1	1	1	$(81a^{24}b^8)^{\frac{1}{4}} = 81^{\frac{1}{4}}a^{24*\frac{1}{4}}b^{8*\frac{1}{4}} = 3a^6b^2$
2	2	2	$3x + \frac{2}{x} = -7 \rightarrow 3x^2 + 2 = -7x$ $3x^2 - 7x + 2 = 0 \rightarrow (3x-1)(x-2) = 0$ $x = -\frac{1}{3}, -2$
3	3	3	$g(-5) = 5(-5)2 - (-5) + 23 = 153$ $f(g(-5)) = f(153)$ $f(153) = \sqrt{153+16} = 13$
4	4		Solve for y: $x = \frac{9-y}{3} + \frac{8+y}{2}$ $10 = \frac{9-y}{3} + \frac{8+y}{2} \rightarrow 60 = 18 - 2y + 24 + 3y = 42 + y$ $f^{-1}(10) = 6(10) - 42 = 18$
		4	Vertex is at (h,k). $h = -\frac{b}{2a}$, $k = f\left(-\frac{b}{2a}\right)$ $h = -\left(-\frac{4}{2*\frac{1}{2}}\right) = 4$ $k = \frac{1}{2}4^2 - 4(4) - 2 = -10$

5			$$\lim_{x\to\infty}\frac{1-2x-x^2}{2x^2-4}=\lim_{x\to\infty}\frac{(-x^2-2x+1)}{2x^2-4}$$ As x approaches infinity, the squared-terms dominate both functions in the limit. The limit may be re-written as follows: $$\lim_{x\to\infty}\left(\frac{-x^2}{2x^2}\right)=-\frac{1}{2}$$
	5	5	$$\left\lvert\frac{2x\ +1}{x\ +\ 2}\right\rvert=\ 3$$ $$\frac{2x+1}{x+2}=\ 3\rightarrow 2x+1=3(x+2)\rightarrow 2x+1=3x+6$$ $$-5=x$$ $$\frac{2x+1}{x+2}=\ -3\rightarrow 2x+1=-3(x+2)\rightarrow 2x+1=-3x-6$$ $$5x=-7$$ $$x=-\frac{7}{5}$$
6	6	6	$$y=x^2-6x+3,\ y=2x^2+4x+10$$ $$x^2-6x+3=2x^2+4x+10$$ $$0=x^2+10x+7$$ $$x=\frac{-10\pm\sqrt{100-4(1)(7)}}{2}=-5\pm\frac{1}{2}\sqrt{72}=-5\pm3\sqrt{2}$$ $$(x_1,y_1)=(-5-3\sqrt{2})\ ,(76+48\sqrt{2})$$ $$(x_2,y_2)=\left(-5+3\sqrt{2}\right),(76-48\sqrt{2})$$
7	7	7	$$x=3-\sqrt{x}\rightarrow x-3=-\sqrt{x}$$ $$(x-3)^2=x\rightarrow x^2-7x+9=0$$ $$x=\frac{7\pm\sqrt{49-4*1*9}}{2}=\frac{7\pm\sqrt{13}}{2}$$ The answer must be <3

8	8	8	$2^{3x+5}(16^{-2x}) < (4)^{3x}(8^{x-2}) \rightarrow 2^{3x+5}(2^4)^{-2x} < (2^2)^{3x}(2^3)^{x-2}$ $2^{3x+5}(2^{-8x}) < 2^{6x}2^{3x-6} \rightarrow 2^{3x+5-8x} < 2^{6x+3x-6}$ $5 - 5x < 9x - 6 \rightarrow 11 < 14x$ $x > \frac{11}{14}$
9	9		$(\sin x - 2)(\sin x + 1) = 0$ $\sin x = 2 \rightarrow Never$ $\sin x = -1; x = \frac{3\pi}{2}, \frac{7\pi}{2}, \ldots \rightarrow All\ other\ xolutions\ are > 4\pi$
		9	$f(x) = \frac{3x+1}{x-1} \rightarrow x = \frac{3y+1}{y-1}$ $x(y-1) = 3y + 1$ $xy - x - 3y = 1$ $y(x-3) = x + 1$ $y = \frac{x+1}{x-3}$ $When\ x = \frac{5}{2}: \rightarrow y = \frac{\frac{5}{2}+1}{\frac{5}{2}-3} = \frac{\frac{5}{2}+\frac{2}{2}}{\frac{5}{2}-\frac{6}{2}} = \frac{5+2}{5-6}$ $y = -7$

10			Let: $x = length\ of\ each\ pen$ $y = width\ of\ each\ pen$ $A_{total} = 6xy$ $P = 9x + 8y$ *(9 lengths, 8 widths)* $P = 800 \rightarrow 800 = 9x + 8y \rightarrow x = \frac{800 - 8y}{9}$ $A_{total} = 6\left(\frac{800 - 8y}{9}\right)y = \frac{4800y - 48y^2}{9}$ $\frac{dA}{dy} = \frac{4800}{9} - \frac{96}{9}y$ $0 = \frac{4800}{9} - \frac{96}{9}y \rightarrow y = 50 \rightarrow$ $x = \frac{800 - 8(50)}{9} \rightarrow x = \frac{400}{9}$
	10	10	$\frac{x(x+7)}{x+1} \leq 6 \rightarrow \frac{x(x+7)}{x+1} \leq \frac{6(x+1)}{x+1}$ $\frac{x^2 + x - 6}{x+1} \leq 0 \rightarrow \frac{(x+3)(x-2)}{x+1} \leq 0$ $x > -1\ AND\ x \geq -3\ \ AND\ \ x \leq 2;\ \ -1 < x \leq 2$ OR $x > -1\ AND\ x \leq -3\ \ AND\ \ x \geq 2;\ \ Never$ OR $x < -1\ AND\ x \geq -3\ \ AND\ \ x \geq 2;\ \ Never$ OR $x < -1\ AND\ x \leq -3\ \ AND\ \ x \leq 2;\ \ x \leq -3$

11	11	11	$$\frac{x-3}{x+4}-\frac{x+4}{x-3}<0$$ $$\frac{(x-3)^2}{(x-3)(x+4)}-\frac{(x+4)^2}{(x-3)(x+4)}<0$$ $$\frac{(x-3)^2-(x+4)^2}{(x-3)(x+4)}<0$$ $$\frac{(x^2-6x+9)-(x^2+8x+16)}{(x-3)(x+4)}<0$$ $$\frac{-14x-7}{(x-3)(x+4)}<0 \rightarrow \frac{-7(2x+1)}{(x-3)(x+4)}<0$$ $$\frac{7(2x+1)}{(x-3)(x+4)}>0$$ Possibilities: $$x>-\frac{1}{2}\ AND\ x<-4\ \therefore\ Not\ possible$$ $$x>-\frac{1}{2}\ AND\ x>3\ \therefore x>3$$ $$x<-\frac{1}{2}\ AND\ x<-4\ AND\ x>3\ \therefore Not\ possible$$ $$x<-\frac{1}{2}\ AND\ x<3\ AND\ x>-4\ \therefore\ -4<x<-\frac{1}{2}$$
12	12	12	$$y=2x^3+x^2+kx-4$$ $$y=2(x+2)(x^2+bx-1)$$ $$y=(2x+4)(x^2+bx-1)$$ $$y=2x^3+4x^2+2bx^2+4bx-2x-4$$ $$y=2x^3+(4+2b)x^2+(4b-2)x-4$$ $$4+2b=1$$ $$b=-\frac{3}{2}$$ $$k=4b-2=4\left(-\frac{3}{2}\right)-2$$ $$k=-8$$

13	13	13	$\sqrt{z} = \frac{4}{1+i} + 7 - 2i \rightarrow \sqrt{z} = \frac{4(1-i)}{(1+i)(1-i)} + 7 - 2i$ $\sqrt{z} = \frac{4-4\mathrm{i}}{2} = 7 - 2\mathrm{i} = 2 - 2\mathrm{i} + 7 - 2\mathrm{i} = 9 - 4\mathrm{i}$ $Z = (9-4i)^2 = 81 - 72i + 16i^2 = 65 - 72\mathrm{i} = 65 + (-72)\mathrm{i}$ $a = 65, b = -72 \ \therefore a + b = -7$
14	14		$C(x) = \frac{4}{5}x^2 + 36x - 94 \ (in\ cents)$ $R(x) = 44 - \frac{1}{5}x \rightarrow P(x) = xR(x) - C(x)$ $P(x) = 44x - \frac{1}{5}x^2 - \frac{4}{5}x^2 - 36x + 94$ $P(x) = -x^2 + 8x + 94$ $Vertex: \ -\frac{b}{2a} \rightarrow Maximum\ profit: \ -\frac{8}{2(-1)}$ $x = 4$ $P(4) = -(4)^2 + (32) + 94 = 110$
		14	$\frac{3x\ +\ 1}{x\ +\ 2} = \frac{6x\ +\ 4}{2x\ +\ 2}$ $(3x+1)(2x+2) = (6x+4)(x+2)$ $6x^2 + 8x + 2 = 6x^2 + 16x + 8$ $0 = 8x + 6$ $x = -\frac{3}{4}$
15			$y = -2x + 6; \ \ y = x^2 + ax + b \rightarrow Both\ intersect\ when\ x = 1$ Therefore; $6 - 2x = x^2 + ax + b \rightarrow When\ x = 1, a + b = 3$ $The\ slope\ of\ 2x + y = 6\ is\ equal\ to - 2.$ $\frac{dy}{dx} = 2x + a = -2$ At $x = 1, a = -4$. Thus $b = 7$

	15	15	Possible roots may be one of the factors of 3: $\pm 1, \pm 3$. Test these. $f(1) = 2 + 3 - 8 + 3 = 0$ $f(-1) = -2 + 3 + 8 + 3 = 12$ $f(3) = 54 + 27 - 24 + 3 = 60$ $f(-3) = -54 + 27 + 24 + 3 = 0$ Once you know at least one of the above roots, you can factor the cubic equation. $(x - 1)(2x^2 + 5x - 3) = 0$ The sum of the roots of the quadratic factor are: $-\frac{b}{a} = -\left(\frac{5}{2}\right) = -\frac{5}{2}$ The sum of all roots is $1 - \frac{5}{2} = -\frac{3}{2}$ Alternatively: $(x + 3)(2x^2 - 3x + 1)$ The sum of the roots of the quadratic factor are: $-\frac{b}{a} = -\left(-\frac{3}{2}\right) = \frac{3}{2}$ The sum of all roots is $-3 + \frac{3}{2} = -\frac{3}{2}$