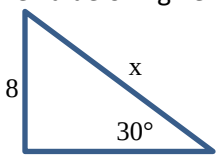
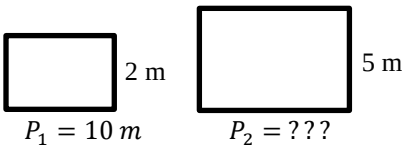
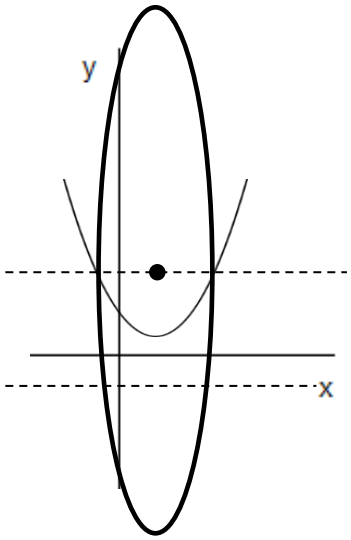
2016 – 2017 Log1 Contest Round 2
Theta Circles, Parabolas and Polygons

Name: _____

Units do not have to be included.

4 points each		
1	For the given right triangle, find the value of x given that 	
2	Find the area of a triangle given that it has side lengths 15, 20, 25.	
3	Given that the two shapes are similar, find the perimeter, P_2, of the larger rectangle given the perimeter, P_1 of the smaller rectangle. 	
4	How many sides does a regular polygon have if one measure of its interior angle is 179 degrees?	
5	How many sides does a heptadecagon have?	

5 points each		
6	Find the minimum distance from the point $(-2, 5)$ to the circle $(x - 10)^2 + (y + 11)^2 = 49$	
7	A rectangle is constructed such that its diagonals are 25 inches. Given that the side lengths are integers that are relatively prime, what is the perimeter of the rectangle.	
8	How many sides does a convex polygon have if the sum of its interior angles is equal to 10 straight angles?	
9	If a regular polygon has 819 diagonals, how many side does this regular polygon have?	
10	A circle circumscribes a triangle with side lengths 28,96, and 100. Determine the area of the circle.	

6 points each		
11	A wheel with a radius of $\frac{8}{\pi}$ meters rolls directly down a smooth $2^5\ meter$ path. How many revolutions does the wheel make on the path?	
12	Consider the parabola given by the equation $y = \frac{1}{12}x^2 - \frac{2}{3}x + \frac{7}{3}$. An ellipse is drawn with its center located at the focus of the parabola. The minor axis of the ellipse is the "latus rectum" of the parabola, which is the line segment formed by the points of intersection of the parabola and a horizontal line that passes through its focus. The semi-major axis is the vertical line segment joining the parabola's focus and its reflection point through its directrix. Calculate the area of this ellipse. 	
13	What is the surface area of a regular icosahedron with side length 10?	
14	What is the area, in square meters, of an isosceles right triangle that has a 14 meter hypotenuse?	
15	A cube has side length 8. Four distinct points are chosen from the centers of the cube's faces, not all coplanar. What is the volume of the tetrahedron whose vertices are these points?	

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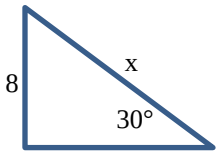
1	For the given right triangle, find the value of x given that 	
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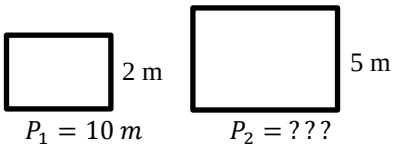
3	Given that the two shapes are similar, find the perimeter, P_2, of the larger rectangle given the perimeter, P_1, of the smaller rectangle. 	
4	How many diagonals does a regular nonagon have?	
5	How many sides does a heptadecagon have?	

5 points each		
6	Find the minimum distance from the point $(-2, 5)$ to the circle $(x - 10)^2 + (y + 11)^2 = 49$	
7	A rectangle is constructed such that its diagonals are 25 inches. Given that the side lengths are integers that are relatively prime, what is the perimeter of the rectangle.	
8	How many sides does a convex polygon have if the sum of its interior angles is equal to 10 straight angles?	
9	What is the area of a regular hexagon that is inscribed in a circle if this circle also circumscribes a right triangle with legs of length 6 and 8?	
10	A circle circumscribes a triangle with side lengths 28,96, and 100. Determine the area of the circle.	

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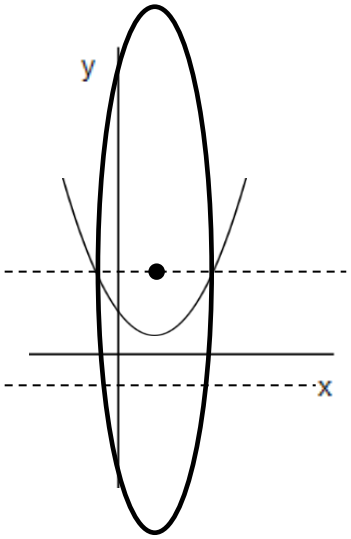
11	A wheel with a radius of $\frac{8}{\pi}$ meters rolls directly down a smooth $2^5\ meter$ path. How many revolutions does the wheel make on the path?	
12	Consider the parabola given by the equation $y = \frac{1}{12}x^2 - \frac{2}{3}x + \frac{7}{3}$. An ellipse is drawn with its center located at the focus of the parabola. The minor axis of the ellipse is the "latus rectum" of the parabola, which is the line segment formed by the points of intersection of the parabola and a horizontal line that passes through its focus. The semi-major axis is the vertical line segment joining the parabola's focus and its reflection point through its directrix. Calculate the area of this ellipse. 	
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14	Find the area of the circle that circumscribes an equilateral triangle with a side of 48. Leave your answer in terms of pi.	
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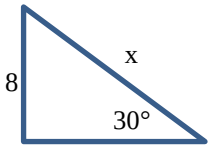
1	For the given right triangle, find the value of x given that 	
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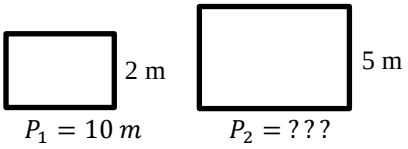
3	Given that the two shapes are similar, find the perimeter, P_2, of the larger rectangle given the perimeter, P_1, of the smaller rectangle 	
4	How many diagonals does a regular nonagon have?	
5	What is the area enclosed by the graph of $y = f(\|x\|)$ and the x-axis from $x = -3$ to $x = +3$ given that $f(x) = 4x^3$?	

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10	The region bounded by the coordinate axes and the portion of the line $y = -2x + 4$ in the first quadrant is rotated around the y-axis. What is the volume of the resulting solid?	

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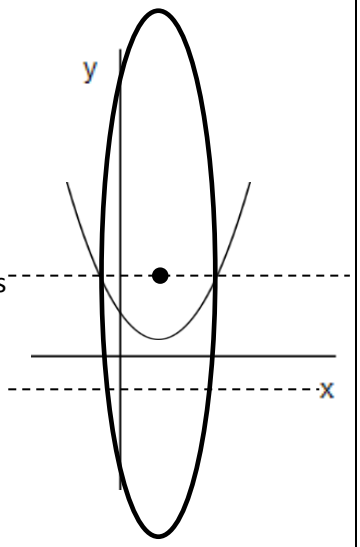
12	Consider the parabola given by the equation $y = \frac{1}{12}x^2 - \frac{2}{3}x + \frac{7}{3}$. An ellipse is drawn with its center located at the focus of the parabola. The minor axis of the ellipse is the "latus rectum" of the parabola, which is the line segment formed by the points of intersection of the parabola and a horizontal line that passes through its focus. The semi-major axis is the vertical line segment joining the parabola's focus and its reflection point through its directrix. Calculate the area of this ellipse. 	
13	What is the surface area of a regular icosahedron with side length 10?	
14	Find the area of the circle that circumscribes an equilateral triangle with a side of 48. Leave your answer in terms of pi.	
15	The definite integral $\pi \int_{-4}^{4} (\sqrt{16 - x^2})^2\, dx$ represents the volume of a sphere. What is the surface area of the sphere?	

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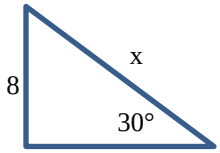
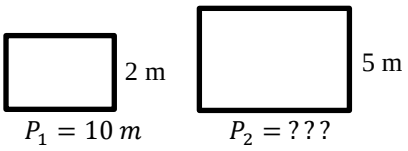
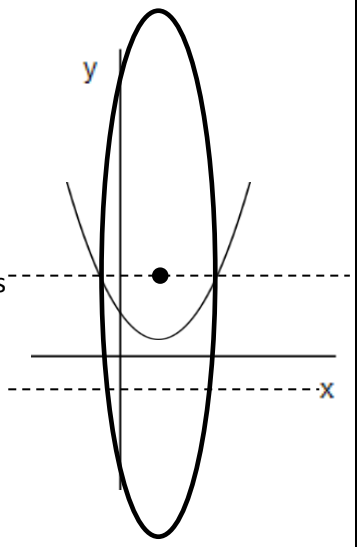
1	Find the value of x given that 	16
2	Find the area of a triangle given that it has side lengths 15, 20, 25.	150
3	Given that the two shapes are similar, find the perimeter of the larger rectangle. 	25
4	How many sides does a regular polygon have if one measure of its interior angle is 179 degrees?	360
5	How many sides does a heptadecagon have?	17

5 points each		
6	Find the minimum distance from the point $(-2, 5)$ to the circle $(x - 10)^2 + (y + 11)^2 = 49$	13
7	A rectangle is constructed such that its diagonals are 25 inches. Given that the side lengths are integers that are relatively prime, what is the perimeter of the rectangle.	62
8	How many sides does a convex polygon have if the sum of its interior angles is equal to 10 straight angles?	12
9	If a regular polygon has 819 diagonals, how many side does this regular polygon have?	42
10	A circle circumscribes a triangle with side lengths 28,96, and 100. Determine the area of the circle.	2500π

6 points each		
11	A wheel with a radius of $\frac{8}{\pi}$ meters rolls directly down a smooth $2^5\ meter$ path. How many revolutions does the wheel make on the path?	2
12	Consider the parabola given by the equation $y = \frac{1}{12}x^2 - \frac{2}{3}x + \frac{7}{3}$. An ellipse is drawn with its center located at the focus of the parabola. The minor axis of the ellipse is the "latus rectum" of the parabola, which is the line segment formed by the points of intersection of the parabola and a horizontal line that passes through its focus. The semi-major axis is the vertical line segment joining the parabola's focus and its reflection point through its directrix. Calculate the area of this ellipse. 	72π
13	What is the surface area of a regular icosahedron with side length 10?	$500\sqrt{3}$
14	What is the area, in square meters, of an isosceles right triangle that has a 14 meter hypotenuse?	49
15	A cube has side length 8. Four distinct points are chosen from the centers of the cube's faces, not all coplanar. What is the volume of the tetrahedron whose vertices are these points?	$\frac{64}{3}$

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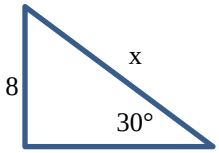
1	Find the value of x given that 	16
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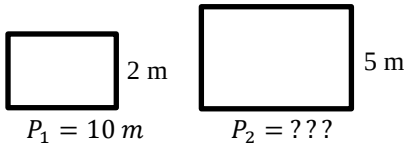
3	Given that the two shapes are similar, find the perimeter of the larger rectangle. 	25
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9	What is the area of a regular hexagon that is inscribed in a circle if this circle also circumscribes a right triangle with legs of length 6 and 8?	$\frac{75}{2}\sqrt{3}$
10	A circle circumscribes a triangle with side lengths 28,96, and 100. Determine the area of the circle.	2500π

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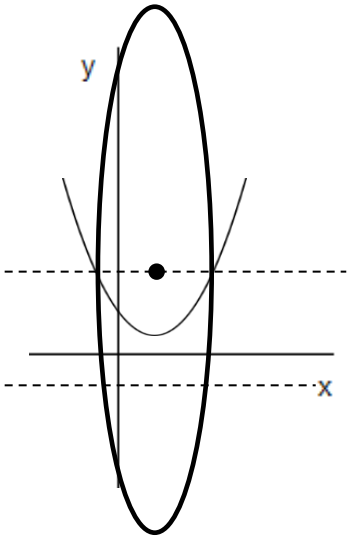
12	Consider the parabola given by the equation $y = \frac{1}{12}x^2 - \frac{2}{3}x + \frac{7}{3}$. An ellipse is drawn with its center located at the focus of the parabola. The minor axis of the ellipse is the "latus rectum" of the parabola, which is the line segment formed by the points of intersection of the parabola and a horizontal line that passes through its focus. The semi-major axis is the vertical line segment joining the parabola's focus and its reflection point through its directrix. Calculate the area of this ellipse. 	72π
13	What is the surface area of a regular icosahedron with side length 10?	$500\sqrt{3}$
14	Find the area of the circle that circumscribes an equilateral triangle with a side of 48. Leave your answer in terms of pi.	768π
15	A cube has side length 8. Four distinct points are chosen from the centers of the cube's faces, not all coplanar. What is the volume of the tetrahedron whose vertices are these points?	$\frac{64}{3}$

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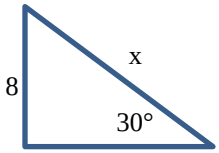
1	Find the value of x given that 	16
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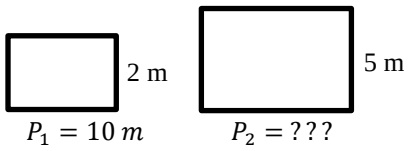
3	Given that the two shapes are similar, find the perimeter of the larger rectangle. 	25
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8	How many sides does a convex polygon have if the sum of its interior angles is equal to 10 straight angles?	12
9	What is the area of the region that is outside a regular hexagon AND inside the circle that circumscribes it if the circle has a circumference of $16\sqrt{\sqrt{3}\pi}$?	$\frac{75}{2}\sqrt{3}$
10	The region bounded by the coordinate axes and the portion of the line $y = -2x + 4$ in the first quadrant is rotated around the y-axis. What is the volume of the resulting solid?	$\frac{16\pi}{3}$

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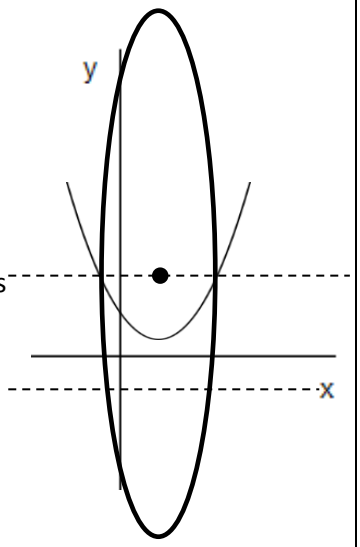
12	Consider the parabola given by the equation $y = \frac{1}{12}x^2 - \frac{2}{3}x + \frac{7}{3}$. An ellipse is drawn with its center located at the focus of the parabola. The minor axis of the ellipse is the "latus rectum" of the parabola, which is the line segment formed by the points of intersection of the parabola and a horizontal line that passes through its focus. The semi-major axis is the vertical line segment joining the parabola's focus and its reflection point through its directrix. Calculate the area of this ellipse. 	72π
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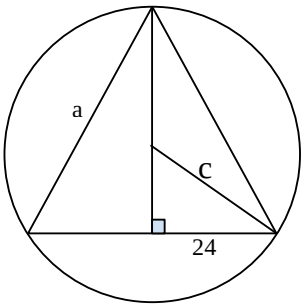
2016 – 2017 Log1 Contest Round 2
Circles, Parabolas and Polygons Solutions

Mu	Al	Th	Solution
1	1	1	$\sin 30 = \frac{1}{2}$ Therefore, $\sin 30 = \frac{8}{x}$ So, $\frac{1}{2} = \frac{8}{x}$ $x = 8(2)$ $x = 16$
2	2	2	Using Heron's formula $\sqrt{s(s-a)(s-b)(s-c)}$ Where $s = \frac{a+b+c}{2}$ the side perimeters. $$s = \frac{15+20+25}{2} = 30$$ $A = \sqrt{30(30-15)(30-20)(30-25)} \rightarrow A = \sqrt{22500} = 150$
3	3	3	$P_1 = 10$, so $\frac{10-(2+2)}{2} = 3$, 3 is the missing side length With that information we can set up a relationship $\frac{2}{5} = \frac{3}{x}$ $$2x = 15, x = 7.5,$$ $$P_2 = 2(5+7.5) = 25$$
4	4		The general formula for solving for diagonals is $\frac{n(n-3)}{2}$, where n is the number of sides. $$\frac{9(9-3)}{2} = 27$$
		4	The adjacent exterior angle is 1° since the polygon is regular the number of sides is found by $$\frac{360°}{1°} = 360.$$

5			Evaluate the integral of the function f(x) from x = 0 to x = +3. $$\int_0^3 4x^3 dx = 3^4 - 0^4 = 81$$ For $x < 0$, $f(\|x\|)$ generates a graph that is the reflection over the y-axis of the function $f(x)$ for $x > 0$. Therefore, the area is $2(81) = 162$
	5	5	A heptadecagon has 17 sides
6	6	6	The center of the circle is (10, -11) $D = \sqrt{(x_2 - x_1)^2 + (y_2 + y_1)^2}$ Using the point give $(-2, 5)$ We can plug it into the equation. $D = \sqrt{(10 - (-2))^2 + (-11 - 5)^2}$ $D = \sqrt{144 + 256}$ $D = \sqrt{400}$ $D = 20$ Then we subtract 7 for the radius and the answer is 13.
7	7	7	$D = \sqrt{l^2 + w^2}, 25 = \sqrt{l^2 + w^2}, 625 = l^2 + w^2$, There is only one possible pair that could work which is the side lengths being 7 and 24. So the Perimeter is $P = 2(7 + 24) = 62$
8	8	8	Sum of interior angles = $(n - 2)180$; 1800 = $(n - 2)180$; 10 = $n - 2$; $n = 12$

9	9		The right triangle inscribed in the circle has a hypotenuse of 10. This implies that the hypotenuse is also the diameter of the circumscribing circle. For a regular hexagon inscribed in a circle, the line segments that connect directly opposing vertices are also diameters of this circle. Each line segment connecting opposing vertices bisects each $120°$ angle of the hexagon, thus forming 6 identical, equilateral triangles of side length 5, which corresponds to ½ the diameter of the circle. Therefore, $A_H = 6\left(\frac{5^2\sqrt{3}}{4}\right) = \frac{75}{2}\sqrt{3}$
		9	$\text{Number of diagonals } = \frac{n(n-3)}{2}$ $819 = \frac{n(n-3)}{2}$ $1638 = n(n-3)$ $n = 42$
10			The x-intercept is at $(2,0)$, and the y-intercept is at $(0,4)$. The figure is a cone of radius 2 and height 4, which has volume $\frac{1}{3}\pi r^2 h = \frac{16\pi}{3}$.
	10	10	Note that this triangle is right (if you divide the sides by 4, you get 7-24-25, a well-known right triangle with integer side lengths). The right angle subtends an arc of 180 degrees, so the hypotenuse goes through the center of the circle. So the radius of the circle is 100/2 = 50, so the area of the circle is 2500π

11	11	11	The wheel's circumference is $2\pi r = 2\pi\frac{8}{\pi} = 16 = 2^4;\ \frac{2^5}{2^4} = 2^1 = 2$ Thus, the wheel makes 2 revolutions.
12	12	12	Rewrite the equation for the parabola in standard form, $(x-h)^2 = 4p(y-k)$ $12y = x^2 - 8x + 28 \rightarrow 12y - 12 = x^2 - 8x + 16$ $12(y-1) = (x-4)^2 \rightarrow 4(3)(y-1) = (x-4)^2$ Focus: $F = (h, k+p) = (4,1+3) = (4,4)$ Vertex: $V = (h,k) = (4,1)$ Directrix: $y = k - p = 1 - 3 = -2;\ y = -2$ Latus Rectum: $LR = 4p = 4(3) = 12$ Semi $-$ minor axis: $a = \frac{1}{2}LR = \frac{1}{2}(12) = 6$ Semi $-$ major axis: $b = 2\big(4-(-2)\big) = 2(6) = 12$ Area of an Ellipse: $A = \pi ab = \pi(6)(12) = 72\pi$
13	13	13	$SA = 5(a^2)(\sqrt{3})$ Where a = side length. So $5(10^2)(\sqrt{3}) = 500\sqrt{3}$

14	14		We are given that the sides are 48 units. Draw the perpendicular bisector from one of the vertices of the triangle. Also, draw a line from another vertex of the triangle to its circumcenter. This forms a right triangle with a base equal to 24 and a hypotenuse that is the radius of the circumscribed circle. Since the triangle is equilateral, its circumcenter and incenter are concurrent. Thus, the radius bisects the vertex angle. Consequently, the right triangle is a 30-60-90 right triangle with a base angle formed at the vertex of the inscribed triangle equal to 30 degrees. The ratio of the base to the hypotenuse must be $\frac{\sqrt{3}}{2}$. Therefore,  $\frac{\sqrt{3}}{2} = \frac{24}{c}$ $c = \frac{48}{\sqrt{3}}$ The area of the circumscribed circle is $A = \pi\left(\frac{24(2)}{\sqrt{3}}\right)^2 = 768\pi$
		14	Given that the triangle is an isosceles right triangle we know that its two legs must be equal in length. Set these lengths equal to x and use the Pythagorean Theorem. $14^2 = x^2 + x^2$ $196 = 2x^2$ $x^2 = 98$ For this triangle, the legs correspond to its base and height. Thus $x^2 = \text{Base x Height}$. Since the area of a triangle is $A = \frac{1}{2}Bh$, $x^2 = 2 \text{ x Area}$. Therefore, $\text{Area} = 49$
15			The quantity squared comes from equation $y = \sqrt{16 - x^2}$ which is the top half of a circle of radius 4. The sphere would have surface area $4\pi(4^2)$

	15	15	Note that any tetrahedron satisfying these properties is the same. To see this, since we have 6 faces and 4 points to choose from, by the Pigeonhole Principle, two points must be on opposite faces. Choosing a third point always yields the same resulting triangle, and the fourth point cannot be opposite the third point. This leaves two possible points, and both yield identical tetrahedrons. Now orient the tetrahedron so that there is a right triangle in the middle layer and a point on the top face. The right triangle is isosceles and has legs of length $4\sqrt{2}$, yielding a base of area 16. The height is half the length of the cube, or 4. The volume of a pyramid is $\frac{1}{3}\mathrm{Ah}$, A = area of base, h = height, so the volume of the tetrahedron is $\frac{1}{3}(16)(4) = \frac{64}{3}$