2017 – 2018 Log1 Contest Round 1

Theta Counting and Probability

Name: ____________________

Units do not have to be included.

	4 points each	
1	If A is the set of all factors of 20 and B is the set of all factors of 36, find A∩B.	
2	A ball is randomly tossed and with 100% certainty it will land into one of two buckets. If the ball has a 0.24 chance of landing in one bucket, what is the probability that it might land in the 2nd bucket?	
3	Given 7 balls, each of a different color, how many combinations of balls in groups of 3 can be made?	
4	The 12 Lords of Kobol wish to be seated for dinner. How many permutations of the 12 lords are there when they are seated at a round table with exactly 12 chairs?	
5	Thirteen members of a math team are taking physics, music or both classes. They discover that 8 of them study physics and 7 of them study music. What is the probability of a single student studying both subjects?	

	5 points each	
6	What is the probability of flipping a fair coin 5 times and obtaining heads the first time followed by 3 additional heads from any of the remaining 4 flips?	
7	Seven athletes are to be divided into teams. Two of the athletes are captains. How many different teams of 4 can be selected from a squad of 7 if the teams must include only one captain?	
8	If $P(A) = \frac{1}{2}, P(B) = \frac{1}{3}, and\ P(A \cup B) = p$. Find p if A and B are mutually exclusive.	
9	Nine letters: A, H, M, T, S, E, L, U, R, are placed at random in a row. What is the probability that the word MATH is spelled out within this sequence of 9 letters? Assume the letters making up the word MATH must appear consecutively and in that order?	
10	There are 13 people on a math team. There are also 2 pairs of IDENTICAL TWINS that cannot be distinguished from each other. How many ways can the team DISTINCTLY reorganize themselves for a picture if the 3 officers must always be in the center within the president always in between the other two officers?	

6 points each		
11	How many different arrangements of four letters chosen from the letters of the word MONDAY are possible if at least one vowel (A or O) must be used?	
12	A box contains 2 red, 3 yellow and 3 green tickets. Five tickets are randomly selected from the box one by one without replacement. Find the probability that the 3rd ticket drawn is yellow.	
13	12 identical boxes are to be delivered to a house and then distributed among eight distinct people in the house. It is possible that everyone gets at least one box but some people could get no boxes. Boxes can be lost by the delivery person before arriving at the house so it is possible that less than 12 boxes are distributed to among the eight people. How many different ways can the boxes be distributed among the 8 people when boxes can be lost?	
14	Find the probability that a randomly chosen distinguishable permutation of the letters in PROBABILITY has either PR or RP, both letters adjacent to each other, in it?	
15	A coin with a diameter of 3 cm is thrown onto an infinite sheet of graph paper with grid lines forming boxes that are uniformly 5 cm square over the entire sheet. Assuming that the coin has a uniform probability of landing anywhere on the graph paper, calculate the probability that the coin touches any grid line when it lands.	

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4	A coin and a die are tossed simultaneously. Determine the probability of getting tails and a 5.	
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9	In a town called Nanville, 42% of the townsfolk speak Nanese. 24% of the townsfolk who speak Nanese also speak Villese. If 30% of the townsfolk speak Villese, what is the probability that a person who is randomly selected speaks Nanese if they speak Villese?	
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14	An archer has a 90% chance of hitting the target with each arrow. If 5 arrows are fired, determine the probability of hitting the target twice only.	
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2017 – 2018 Log1 Contest Round 1

Mu Counting and Probability

Name: ___________________

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2	A ball is randomly tossed and with 100% certainty it will land into one of two buckets. If the ball has a 0.24 chance of landing in one bucket, what is the probability that it might land in the 2^{nd} bucket?	
3	Given 7 balls, each of a different color, how many combinations of balls in groups of 3 can be made?	
4	A coin and a die are tossed simultaneously. Determine the probability of getting tails and a 5.	
5	A car accelerates, starting from rest, according to the equation $a = 2t + 1$, in units $\frac{m}{s^2}$, for a time of 5 seconds. If the speed of the car is measured at a random time between 0 and 5 seconds what is the probability that the car's velocity is no greater than 20 m/s?	

	5 points each	
6	What is the probability of flipping a fair coin 5 times and obtaining heads the first time followed by 3 additional heads from any of the remaining 4 flips?	
7	Seven athletes are to be divided into teams. Two of the athletes are captains. How many different teams of 4 can be selected from a squad of 7 if the teams must include only one captain?	
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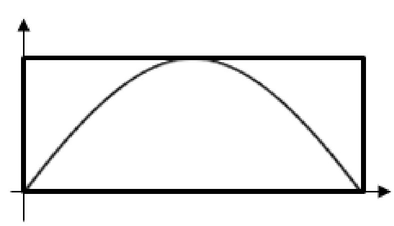
10	A sine function of the form $y = \sin(3x)$ is drawn on the domain $x = \left[0, \frac{\pi}{3}\right]$.A rectangle is drawn with vertices at $(0,1), \left(\frac{\pi}{3}, 1\right), (0,0)$, and $\left(\frac{\pi}{3}, 0\right)$. Determine the probability of randomly choosing a point that is within the rectangle but ABOVE the sine curve. 	

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14	An archer has a 90% chance of hitting the target with each arrow. If 5 arrows are fired, determine the probability of hitting the target twice only.	
15	A quantum particle has a 50% chance of being located inside an 8 m^3 cube centered at the origin of an X-Y coordinate system. The same particle has a 30% chance of being located inside a 64 m^3 cube centered at the origin of the same X-Y coordinate system but outside the 8 m^3 cube. A concave upward paraboloid, generated by revolving the equation, $y = x^2$, around the y-axix is inside the larger cube with its vertex at (0,0) and truncated at the plane where it intersects with the top surface of the larger cube. What is the probability that this particle is located inside the paraboloid?	

2017 – 2018 Log1 Contest Round 1

Theta Counting and Probability – Answer Key

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	4 points each	
1	If A is the set of all factors of 20 and B is the set of all factors of 36, find A∩B.	{1, 2, 4}
2	A ball is randomly tossed and with 100% certainty it will land into one of two buckets. If the ball has a 0.24 chance of landing in one bucket, what is the probability that it might land in the 2^{nd} bucket?	76% or 0.76
3	Given 7 balls, each of a different color, how many combinations of balls in groups of 3 can be made?	35
4	The 12 Lords of Kobol wish to be seated for dinner. How many permutations of the 12 lords are there when they are seated at a round table with exactly 12 chairs?	39,916,800
5	Thirteen members of a math team are taking physics, music or both classes. They discover that 8 of them study physics and 7 of them study music. What is the probability of a single student studying both subjects?	$\frac{2}{13}$

	5 points each	
6	What is the probability of flipping a fair coin 5 times and obtaining heads the first time followed by 3 additional heads from any of the remaining 4 flips?	$\frac{1}{8}$
7	Seven athletes are to be divided into teams. Two of the athletes are captains. How many different teams of 4 can be selected from a squad of 7 if the teams must include only one captain?	20
8	If $P(A) = \frac{1}{2}, P(B) = \frac{1}{3}, and\ P(A \cup B) = p$. Find p if A and B are mutually exclusive.	$\frac{5}{6}$
9	Nine letters: A, H, M, T, S, E, L, U, R, are placed at random in a row. What is the probability that the word MATH is spelled out within this sequence of 9 letters? Assume the letters making up the word MATH must appear consecutively and in that order?	$\frac{1}{504}$
10	There are 13 people on a math team. There are also 2 pairs of IDENTICAL TWINS that cannot be distinguished from each other. How many ways can the team DISTINCTLY reorganize themselves for a picture if the 3 officers must always be in the center within the president always in between the other two officers?	1814400

6 points each		
11	How many different arrangements of four letters chosen from the letters of the word MONDAY are possible if at least one vowel (A or O) must be used?	336
12	A box contains 2 red, 3 yellow and 3 green tickets. Five tickets are randomly selected from the box one by one without replacement. Find the probability that the 3rd ticket drawn is yellow.	$\frac{3}{8}$
13	12 identical boxes are to be delivered to a house and then distributed among eight distinct people in the house. It is possible that everyone gets at least one box but some people could get no boxes. Boxes can be lost by the delivery person before arriving at the house so it is possible that less than 12 boxes are distributed to among the eight people. How many different ways can the boxes be distributed among the 8 people when boxes can be lost?	125,970
14	Find the probability that a randomly chosen distinguishable permutation of the letters in PROBABILITY has either PR or RP, both letters adjacent to each other, in it?	$\frac{2}{11}$
15	A coin with a diameter of 3 cm is thrown onto an infinite sheet of graph paper with grid lines forming boxes that are uniformly 5 cm square over the entire sheet. Assuming that the coin has a uniform probability of landing anywhere on the graph paper, calculate the probability that the coin touches any grid line when it lands.	$\frac{21}{25}$

2017 – 2018 Log1 Contest Round 1

Alpha Counting and Probability – Answer Key

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4 points each		
1	If A is the set of all factors of 20 and B is the set of all factors of 36, find A∩B.	{1, 2, 4}
2	A ball is randomly tossed and with 100% certainty it will land into one of two buckets. If the ball has a 0.24 chance of landing in one bucket, what is the probability that it might land in the 2nd bucket?	76% or 0.76
3	Given 7 balls, each of a different color, how many combinations of balls in groups of 3 can be made?	35
4	A coin and a die are tossed simultaneously. Determine the probability of getting tails and a 5.	$\frac{1}{12}$
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6	What is the probability of flipping a fair coin 5 times and obtaining heads the first time followed by 3 additional heads from any of the remaining 4 flips?	$\frac{1}{8}$
7	Seven athletes are to be divided into teams. Two of the athletes are captains. How many different teams of 4 can be selected from a squad of 7 if the teams must include only one captain?	20
8	If $P(A) = \frac{1}{2}, P(B) = \frac{1}{3}, and\ P(A \cup B) = p$. Find p if A and B are mutually exclusive.	$\frac{5}{6}$
9	In a town called Nanville, 42% of the townsfolk speak Nanese. 24% of the townsfolk who speak Nanese also speak Villese. If 30% of the townsfolk speak Villese, what is the probability that a person who is randomly selected speaks Nanese if they speak Villese?	0.336 or 33.6%
10	There are 13 people on a math team. There are also 2 pairs of IDENTICAL TWINS that cannot be distinguished from each other. How many ways can the team DISTINCTLY reorganize themselves for a picture if the 3 officers must always be in the center within the president always in between the other two officers?	1814400

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14	An archer has a 90% chance of hitting the target with each arrow. If 5 arrows are fired, determine the probability of hitting the target twice only.	0.0081 Or 0.81%
15	A coin with a diameter of 3 cm is thrown onto an infinite sheet of graph paper with grid lines forming boxes that are uniformly 5 cm square over the entire sheet. Assuming that the coin has a uniform probability of landing anywhere on the graph paper, calculate the probability that the coin touches any grid line when it lands.	$\frac{21}{25}$

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Name: ____________________

Units do not have to be included.

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1	If A is the set of all factors of 20 and B is the set of all factors of 36, find A∩B.	{1, 2, 4}
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4	A coin and a die are tossed simultaneously. Determine the probability of getting tails and a 5.	$\frac{1}{12}$
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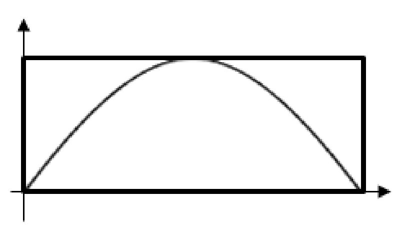
10	A sine function of the form $y = \sin(3x)$ is drawn on the domain $x = \left[0, \frac{\pi}{3}\right]$.A rectangle is drawn with vertices at (0,1), $\left(\frac{\pi}{3}, 1\right)$, (0,0), and $\left(\frac{\pi}{3}, 0\right)$. Determine the probability of randomly choosing a point that is within the rectangle but ABOVE the sine curve. 	$1 - \frac{2}{\pi}$

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15	A quantum particle has a 50% chance of being located inside an 8 m^3 cube centered at the origin of an X-Y coordinate system. The same particle has a 30% chance of being located inside a 64 m^3 cube centered at the origin of the same X-Y coordinate system but outside the 8 m^3 cube. A concave upward paraboloid, generated by revolving the equation, $y = x^2$, around the y-axix is inside the larger cube with its vertex at (0,0) and truncated at the plane where it intersects with the top surface of the larger cube. What is the probability that this particle is located inside the paraboloid?	$\frac{11\pi}{280}$

2017 – 2018 Log1 Contest Round 1
Counting and Probability Solutions

Mu	Al	Th	Solution
1	1	1	A = {1, 2, 4, 5, 10, 20} B = {1, 2, 3, 4, 6, 9, 12, 18, 36} A∩B = the set of factors of both 20 and 36 = {1, 2, 4}
2	2	2	The probability of an event and its complement is 1. $$1 = P(A) + P(A^{-1}) = P(A) + 0.24$$ Thus, the probability of landing in a bin is 0.76 $$P(A) = 0.24 + P(2) = 0.76$$ $$P(2) = 0.76$$
3	3	3	Since order does not matter, use the formula $$C(7,3) = \frac{7!}{4!\,3!} = \frac{7*6*5}{3*2*1} = 35$$
4	4		Assuming that both events are independent of each other, $$P(T \cap 5) \;=\; P(T)\; x\; P(5)$$ $$= \frac{1}{2} x \frac{1}{6}$$ $$= \frac{1}{12}$$
		4	At a round table, a single permutation is counted 12 times due to rotation. Therefore, for cyclical permutations: $P_C(12,12) = (12-1)! = 11! =$ 39,916,800
5			Determine the car's velocity function by integrating the acceleration function. Since initial velocity is 0, the constant of integration is 0. $$v = \int a\,dt = \int 2t + 1 = t^2 + t$$ To determine the time that the car attains a velocity of $20\frac{m}{s}$, solve for t. $$20 = t^2 + t \rightarrow 0 = t^2 + t - 20$$ $$0 = (t+5)(t-4)$$ $t = 4\ seconds$ (the 2nd root is physically meaningless) $$P\left(\le 20\frac{m}{s}\right) = \frac{4}{5} = 0.8$$

	5	5	$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$ $$1 = \frac{8}{13} + \frac{7}{13} - P(A \cap B)$$ $$P(B) = \frac{2}{13}$$
6	6	6	The first coin has a ½ probability of coming up heads. For the remaining 4 flips, you want 3 heads. There are $\binom{4}{3}$ ways of doing this out of 2^4 possible combinations. Thus, the answer is $\frac{1}{2^1}\left(\frac{2^2}{2^4}\right) = 2^{-3} = \frac{1}{8}$
7	7	7	If one of the two captains must be include, then we only have 5 other athletes to choose from. Divide the teams into 4 slots. Slot 1 is for the captain and slots 2-4 or for non-captain athletes. Slot 1: $C(2,1) = 2$ Slots 2-4: $C(5,3) = \frac{5!}{2!3!} = \frac{5*4*3}{3*2} = 10$
8	8	8	If A and B are mutually exclusive, A∩B = 0 But P(A∪B) = P(A) + P(B) - P(A∩B) so $p = P(A \cup B) = \frac{1}{2} + \frac{1}{3} - 0 = \frac{5}{6}$
9	9		Use Bayes Rule: $P(N\|V) = \frac{P(V\|N)*P(N)}{P(V)}$ $$P(N\|V) = \frac{0.24 * 0.42}{0.30} =$$
		9	The number of permutations of the letters is 9! (9 factorial.) Treating the letters MATH as one unit, there are 6 places it can be placed in and around the other 5 letters: S, E, L, U, and R. The number of permutations that show MATH is $$6(5!) = 6!$$ $$P(MATH) = \frac{6!}{9!} = \frac{1}{9*8*7} = \frac{1}{504}$$
10			The area of the square is $\frac{\pi}{3}$. To find the area of the section bounded by the equation $y = \sin(3x)$ and the x-axis, you integrate from 0 to $\frac{\pi}{3}$. $\int_0^{\frac{\pi}{3}} \sin(3x)dx = -\frac{1}{3}\cos(3x)_0^{\pi/3} = \frac{2}{3}$. The probability of a point randomly chosen to be within the rectangle but below the sine curve is $\frac{2}{3}/\frac{\pi}{3} = \frac{2}{\pi}$. Since we wish to find the probability of the points within the rectangle but above the sine curve, the probability is $1 - \frac{2}{\pi}$.

	10	10	Just considering the 10 people that are not officers, the number distinct permutations is $\frac{10!}{2!\,2!} = 907200$ For each of the above distinct permutations, the officers can be ordered in only 2 ways if the president is to be at the true center. Total number of distinct permutations: 1814400
11	11	11	There are two ways to think of this problem. Method 1 is the easiest. Method 1: The number of 4 letter permutations of MONDAY using all the letters is ${}_{4}^{6}\mathrm{P} = 6 * 5 * 4 * 3 = 360$. The number of 4 letter permutations of MNDY is ${}_{4}^{4}\mathrm{P} = 4 * 3 * 2 * 1 = 24$. Therefore the number of permutations with either an A or an O is the difference of these two answers, $360 - 24 = 336$ Method 2: There are ${}_{3}^{4}\mathrm{P} = 4 * 3 * 2 = 24$ permutations of the letters M, N, D, and Y taken three at a time. To determine the number of ways to insert an A or an O but not both, consider a gap structure such as __X__X__X__. There are 8 ways to insert an A or an O into each gap. The number of permutations with just one A or one O is 192. In the situation where both A and O are present, there are ${}_{2}^{3}\mathrm{P} = 3 * 2 * 1 = 6$ ways to insert 2 gaps for the letters A and O within a 2 letter permutation of the letters M, N, D, and Y. Each possible gap arrangement has $4 * 3 * 2 * 1 = 24$ possible 4 letter permutations with both A and O present. Multiply by 6 to account for the various gap arrangements and the number of permutations with both A and O present is 144. Total possible 4 letter permutations is 336

12	12	12	There are 8 total tickets in the box and four unique scenarios where the 3rd ticket is yellow. The last two tickets drawn are irrelevant to the problem. Scenario 1: No yellow ticket is pulled either the 1st or 2nd draw but is pulled on the 3rd draw. $$\mathrm{P}(Y1) = \frac{5}{8} * \frac{4}{7} * \frac{3}{6} = \frac{5}{28}$$ OR Scenario 2: Yellow ticket is pulled on the 2nd and 3rd draws. $$P(Y1) = \frac{5}{8} * \frac{3}{7} * \frac{2}{6} = \frac{5}{56}$$ OR Scenario 3: Yellow ticket is pulled on the 1st and 3rd draws. By inspection, $P(Y3) = P(Y2) = \frac{5}{56}$ OR Scenario 4: Yellow ticket is pulled on the 1st, 2nd, and 3rd draws. $$P(Y4) = \frac{3}{8} * \frac{2}{7} * \frac{1}{6} = \frac{1}{56}$$ Therefore, $$P(Y\ 3rd\ draw) = P(Y1) + P(Y2) + P(Y3) + P(Y4)$$ $$P(Yellow\ on\ 3rd\ draw) = \frac{21}{56} = \frac{3}{8}$$

13	13	13	It is best to consider the eight people as partitions and use separator bars to divide the boxes into all possible combinations of partitions. In general, if you have n items to distribute into r partitions, you need r-1 bars. However, instead of n items, you now have n+r-1 items to arrange. The total number of ways to distribute these items is given by the formula below which allows for any partitions to have either no items at a minimum up to a maximum of all items. Essentially, you want to count the number of ways you can count n+r-1 items into r-1 subgroups. $\binom{n+r-1}{r-1}$ For a situation where $n = 12$ and $r = 8$, the situation is straight forward. However, since boxes can get lost, add a "ninth" entity that will obtain the "lost boxes". The other eight people would never see the boxes delivered to the "ninth" entity so they would consider those boxes lost! Thus $n = 12$ and $r = 9$. $C = \binom{12+9-1}{9-1} = \binom{20}{8} = \frac{20!}{12!\,8!}$ $C = \frac{20*19*18*17*16*15*14*13}{8*7*6*5*4*3*2}$ $C = 19*17*15*13*2$ $C = 125{,}970$
14	14		Let H be the hitting of the target. $P(H) = 0.9$ and $P(H^C) = 0.1$ There are 10 ways this archer can hit the target twice: Hitting the first shot, the archer can hit the target a second time in 4 different ways: 2nd, 3rd, 4th and 5th shots. Hitting the second shot, the archer can hit the target a second time in 3 different ways: 3rd, 4th and 5th shots. Continuing this line of reasoning, the third time gives 2 different ways and the fourth time gives only 1 possibility. Each of the 10 ways to accomplish the task has a probability of: $P(2\ hits\ in\ 5\ shots) = 0.9^2 * 0.1^3 = 0.00081$ Therefore $P(2\ hits\ in\ 5\ shots) = 0.0081$

		14	Total permutations when accounting for the repeated letters B and I is given by $\frac{11!}{2!2!}$. Treating PR as one letter then the number of permutations reduces to $\frac{10!}{2!2!}$ Since RP is another distinct possibility, multiply by 2 to obtain $\frac{10!}{2!2!} * 2$. Probability: $P(RP\ or\ PR) = \left(2 * \frac{10!}{11!}\right) = \frac{2}{11}$
15			There are two scenarios to consider. One possibility is the particle is inside the smaller box AND the paraboloid. The second possibility is the particle is between the larger and smaller boxes AND inside the paraboloid. To find the volumes of the paraboloid sections under consideration, integrate using the disk method. $$\int_0^1 \pi x^2 dy = \int_0^1 \pi y dy = \frac{1}{2}\pi y^2 \begin{matrix} 1 \\ 0 \end{matrix} = \frac{\pi}{2}$$ $$\int_1^2 \pi x^2 dy = \int_1^2 \pi y dy = \frac{1}{2}\pi y^2 \begin{matrix} 2 \\ 1 \end{matrix} = \frac{3\pi}{2}$$ Event S: Inside smaller cube Event P: Inside the intersection of paraboloid and smaller cube. $$P(1) = P(S \cap P) = P(S) * P(P)$$ $$P(1) = 0.5 * \frac{\pi/2}{8} = \frac{\pi}{32}$$ Event B: In between cubes Event Z: Inside the intersection of the region between cubes and the paraboloid. $$\mathrm{P(2) = P(B \cap Z) = P(B) * P(Z)}$$ $$\mathrm{P(2)} = 0.3 * \frac{3\pi/2}{56} = \frac{3 * 3 * \pi}{2 * 10 * 56} = \frac{9\pi}{1120}$$ $$\mathrm{P(Total) = P(1) + P(2)} = \frac{\pi}{32} + \frac{9\pi}{1120} = \frac{44\pi}{1120}$$ $$\mathrm{P(Total)} = \frac{11\pi}{280}$$

	15	15	An infinite sheet is presented to disallow the possibility that the coin does not land on the graph paper. Geometric probabilities may then be calculated without considering that possibility. Since the boxes are 5 cm square and the coin is 3 cm in diameter, the center of the coin cannot be within $\frac{3}{2}$ cm from a grid line. The center of the square is located a distance of $\frac{5}{2}$ cm from a grid line. Draw a square that has sides $\frac{3}{2}$ cm from the edge of the larger square. The length of each side of this smaller square must be 2 cm. The coin would have to land with its center within this smaller square to avoid touching a grid line The area of the larger grid box 25 cm^2. The area of the smaller square is 4 cm^2 $$\text{P(Not Hitting a Grid Line)} = \frac{4}{25}$$ Thus the $\text{P(Hitting a Grid Line)} = \frac{21}{25}$