2017 – 2018 Log1 Contest Round 3
Theta Individual

Name: ___________________

Units do not have to be included.

4 points each		
1	What is measure of the exterior angle of a regular hexagon?	
2	How many positive distinct factors are there in the number 37800?	
3	Find the 21th term of the arithmetic sequence: 4, 7, 10...	
4	Evaluate: $\sqrt{12+\sqrt{12+\sqrt{12+\ldots}}}$	
5	The average of 50 numbers is 40. The average of 30 of these numbers is 30. Find the average of the other 20.	

5 points each		
6	Given the quadrilateral ABCD with side lengths of $\mathrm{AB} = 6, \mathrm{BC} = 5,$ and $\mathrm{CD} = 14$, how many possible integer values could there be for AD?	
7	Jeff owns a swimming pool. With his water hose, it takes 8 minutes to fill the pool to the top. With Mark's water hose, it also takes 8 minutes. Additionally, it takes 5 minutes to drain the swimming pool. If both water hoses were running and the drain was open, how long would it take to fill the pool?	
8	The letters of the word WATER are rearranged at random. What is the probability that W and A will be together in either order?	
9	One pulley has a radius of 3 feet and another pulley has a diameter of 18 feet. The center of the pulleys are 12 feet apart and there is a belt wrapped tightly around them. What is the length, in simplest radical form, of the part of the belt that is not touching either pulley?	
10	Find the determinant of the matrix. $$A = \begin{bmatrix} 2 & -2 & 0 \\ -1 & 5 & 1 \\ 3 & 4 & 5 \end{bmatrix}$$	

6 points each		
11	Given that $\log_9 a = \log_{12} b = \log_{16}(a+b)$ determine the value of $\frac{a}{b}$ and express your answer in proper radical form.	
12	A parallelogram has the vertices (0,2), (4,2), (6,5) and (2,5). If a 3-dimensional shape were to be generated by revolving the parallelogram about the y-axis, what would be its volume?	
13	A cone has a radius that is ½ its height. If the height of the cone is increased by a factor of 3/2, by what factor must the radius be changed to maintain a constant ratio of its lateral surface area to volume?	
14	Let $a = \sum_1^{20}(-1)^n(2n-1)$ and $b = \sum_1^{5}(2n(n^2+1)+1)$ What is the sum of the positive factors of (ab)	
15	Solve the following matrix equation for X. $$\begin{bmatrix}1 & -2\\0 & 3\end{bmatrix}X + \begin{bmatrix}2 & -1 & 1\\3 & 0 & 1\end{bmatrix}\begin{bmatrix}4 & 2\\1 & 0\\0 & -3\end{bmatrix} = \begin{bmatrix}1 & 2\\-3 & -2\end{bmatrix}$$	

2017 – 2018 Log1 Contest Round 3
Alpha Individual

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14	When the decimal number 123! is written in base 12, how many trailing zeroes will there be?	
15	Solve the following matrix equation for X. $$\begin{bmatrix} 1 & -2 \\ 0 & 3 \end{bmatrix} X + \begin{bmatrix} 2 & -1 & 1 \\ 3 & 0 & 1 \end{bmatrix} \begin{bmatrix} 4 & 2 \\ 1 & 0 \\ 0 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ -3 & -2 \end{bmatrix}$$	

2017 – 2018 Log1 Contest Round 3

Mu Individual

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5	Given the function $f(x) = \frac{3}{4}x^4 - 3x^2 + 2$, determine the x coordinate, in simplest radical form, of the inflection point where the slope of the function is negative.	

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10	Let $f(x) = -2\cos 2x$. If $z = f''\left(\frac{\pi}{24}\right)$, evaluate z^2 and express your answer in the form $a \pm b\sqrt{c}$ where a, b, and c are elements of the set of integers.	

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14	When the decimal number 123! is written in base 12, how many trailing zeroes will there be?	
15	Consider the curve defined by the following parametric equations. $$x = 3t^2$$ $$y = e^t - \frac{1}{t}$$ Evaluate the definite integral $\int_1^2 y\,dx$	

2017 – 2018 Log1 Contest Round 3

Theta Individual – Answer Key

Name: ____________________

Units do not have to be included.

	4 points each	
1	What is measure of the exterior angle of a regular hexagon?	60
2	How many positive distinct factors are there in the number 37800?	96
3	Find the 21th term of the arithmetic sequence: 4, 7, 10...	64
4	Evaluate: $\sqrt{12+\sqrt{12+\sqrt{12+\ldots}}}$	4
5	The average of 50 numbers is 40. The average of 30 of these numbers is 30. Find the average of the other 20.	55

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6	Given the quadrilateral ABCD with side lengths of $\mathrm{AB} = 6, \mathrm{BC} = 5$, and $\mathrm{CD} = 14$, how many possible integer values could there be for AD?	21
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8	The letters of the word WATER are rearranged at random. What is the probability that W and A will be together in either order?	$\frac{2}{5}$
9	One pulley has a radius of 3 feet and another pulley has a diameter of 18 feet. The center of the pulleys are 12 feet apart and there is a belt wrapped tightly around them. What is the length, in simplest radical form, of the part of the belt that is not touching either pulley?	$12\sqrt{3}$
10	Find the determinant of the matrix. $$A = \begin{bmatrix} 2 & -2 & 0 \\ -1 & 5 & 1 \\ 3 & 4 & 5 \end{bmatrix}$$	26

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11	Given that $\log_9 a = \log_{12} b = \log_{16}(a+b)$ determine the value of $\frac{a}{b}$ and express your answer in proper radical form.	$\dfrac{-1+\sqrt{5}}{2}$
12	A parallelogram has the vertices (0,2), (4,2), (6,5) and (2,5). If a 3-dimensional shape were to be generated by revolving the parallelogram about the y-axis, what would be its volume?	72π
13	A cone has a radius that is ½ its height. If the height of the cone is increased by a factor of 3/2, by what factor must the radius be changed to maintain a constant ratio of its lateral surface area to volume?	$\dfrac{6\sqrt{41}}{41}$
14	Let $a = \sum_1^{20}(-1)^n(2n-1)$ and $b = \sum_1^{5}(2n(n^2+1)+1)$ What is the sum of the positive factors of (ab)	21266
15	Solve the following matrix equation for X. $$\begin{bmatrix}1 & -2\\0 & 3\end{bmatrix} X + \begin{bmatrix}2 & -1 & 1\\3 & 0 & 1\end{bmatrix}\begin{bmatrix}4 & 2\\1 & 0\\0 & -3\end{bmatrix} = \begin{bmatrix}1 & 2\\-3 & -2\end{bmatrix}$$	$\begin{bmatrix}-16 & -\frac{7}{3}\\-5 & -\frac{5}{3}\end{bmatrix}$

2017 – 2018 Log1 Contest Round 3

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14	When the decimal number 123! is written in base 12, how many trailing zeroes will there be?	58
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5	Given the function $f(x) = \frac{3}{4}x^4 - 3x^2 + 2$, determine the x coordinate, in simplest radical form, of the inflection point where the slope of the function is negative.	$\frac{\sqrt{6}}{3}$

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10	Let $f(x) = -2\cos 2x$. If $z = f''\left(\frac{\pi}{24}\right)$, evaluate z^2 and express your answer in the form $a \pm b\sqrt{c}$ where a, b, and c are elements of the set of integers.	$32 + 16\sqrt{3}$

6 points each		
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15	Consider the curve defined by the following parametric equations. $$x = 3t^2$$ $$y = e^t - \frac{1}{t}$$ Evaluate the definite integral $\int_1^2 y\, dx$	$6e^2 - 6$ Or $6(e^2 - 1)$

2017 – 2018 Log1 Contest Round 3
Individual Solutions

<table>
<tr><th>Mu</th><th>Al</th><th>Th</th><th>Solution</th></tr>
<tr><td>1</td><td>1</td><td>1</td><td>Method 1: The sum of the exterior angles of any convex polygon must add up to 360 degrees. Since it is a regular hexagon, all interior angles are equal. Thus, all exterior angles are equal.

Let x = the measure of the exterior angle of a regular hexagon
$$x = \frac{360}{6} = 60 \text{ degrees}$$Method 2: The exterior angle of a convex polygon is the supplement of its corresponding interior angle.
$$x = 180 - 120 = 60$$</td></tr>
<tr><td>2</td><td>2</td><td>2</td><td>Express the number as a product of prime factors.
$$37800 = 2^3 * 3^3 * 5^2 * 7^1$$The number of positive distinct factors is calculated as follows
$$(3+1)(3+1)(2+1)(1+1) = 96$$</td></tr>
<tr><td>3</td><td>3</td><td>3</td><td>For any arithmetic sequence, the formula for the nth term is given by
$$x_n = x_1 + (n-1)d; \text{ d is the common difference}$$$$x_{21} = 4 + (21-1)3 = 4 + 20(3) = 64$$</td></tr>
<tr><td>4</td><td>4</td><td></td><td>For a circle, arc length is given by the formula $s = r\theta$
The angle in question must be stated in radians so converting to radians, $\theta = 270° = \frac{3\pi}{2}$
$$s = 10\text{ft}\left(\frac{3\pi}{2}\right) = 15\pi\text{ft}$$</td></tr>
<tr><td></td><td></td><td>4</td><td>Set the expression equal to x and solve.
$$\sqrt{12 + \sqrt{12 + \sqrt{12 + \ldots}}} = x$$$$\sqrt{12 + x} = x \rightarrow 12 + x = x^2$$$$(x+3)(x-4) = 0 \rightarrow x = 4$$</td></tr>
</table>

5			The inflection points occur at the values of x for which the function's second derivative is equal to zero. $$f(x) = \frac{3}{4}x^4 - 3x^2 + 2$$ $$f'(x) = 3x^3 - 6x \to f''(x) = 9x^2 - 6$$ Setting the 2nd derivative equal to zero: $$9x^2 = 6 \to x = \pm\sqrt{\frac{2}{3}} = \pm\frac{\sqrt{6}}{3}$$ Analytically, you can verify that $f'\left(\frac{\sqrt{6}}{3}\right) = -\frac{4\sqrt{6}}{3}$ whereas $f'\left(-\frac{\sqrt{6}}{3}\right) = +\frac{4\sqrt{6}}{3}$ Alternatively, by inspection, when x approaches $+\infty$ or $-\infty$, the y value always diverges to $+\infty$. The first derivative has 3 zeroes so there are indeed 3 critical points and the only way this happens is with a concave downward portion in the middle of the graph. There are 2 infection points for this function so the one where the slope is negative would have to be the higher of the two x coordinate values.
	5	5	**Method 1: Weighted Averages:** $$\frac{20}{50}x + \frac{30}{50}y = 40 \to \frac{2}{5}x + \frac{3}{5}(30) = 40$$ $$2x + 90 = 200 \to 2x = 110$$ $$x = 55$$ **Method 2: Computing Sums** If the average of 50 numbers is 40, their sum is 2000. If the average of 30 numbers is 30, their sum is 900. The sum of the remaining 20 numbers must be 1100. Thus, the average of those 20 numbers is 55.
6	6	6	In a limiting sense, the greatest that the length of AD could be will occur when all four points of the quadrilateral are collinear forming a line segment labeled ABCD. In this scenario, AD must be less than 25 so the maximum integer length to form an actual quadrilateral is 24. To find the shortest length, the 4 points must be collinear with A and D as close to each other as possible, as would be the case with a line segment labeled DABC. Then AD would have to be greater than 3 so the minimum integer length to form an actual quadrilateral is 4. There are 21 integers between 3 and 24, inclusive.
7	7	7	Assume the pool has a volume of 1 in arbitrary units. Then both water hoses dispense water at a rate of $\frac{1}{8}\ \text{min}^{-1}$. With both water hoses running simultaneously, the rate is $\frac{1}{4}\text{min}^{-1}$. If the rate the pool drains is $\frac{1}{5}\text{min}^{-1}$, then we define the net fill rate as $$\frac{\Delta V}{\Delta t} = \frac{1}{4} - \frac{1}{5} = \frac{1}{20}\text{min}^{-1}$$ In arbitrary volume units, this means that $\frac{1}{20}$ of the total volume of the pool gets filled every minute so it takes 20 minutes to fill the pool.

8	8	8	There are 5P5 permutations of the letters in WATER. Treat the letters WA as 1 unit and there are 4P4 permutations of (WA), T, E, and R. For each of these permutations, the W and A can be reversed. Thus, there are 2*4P4 permutations with W and A adjacent to each other in any order. The probability is calculated as follows. $$\frac{2(4P4)}{5P5}=\frac{2*4!}{5!}=\frac{2*24}{120}=\frac{48}{120}=\frac{2}{5}$$ 0.40 or 40% are acceptable answers.
9	9		For a hyperbola, given by the standard form equation $$\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$$ the eccentricity, e, is defined as follows. $$e=\frac{\sqrt{a^2+b^2}}{a}$$ The equation stated in the problem must be rearranged into standard form. $$9x^2+36x-4y^2+8y=4$$ $$9(x^2+4x)-4(y^2-2y)=4$$ $$9(x^2+4x+4)-4(y^2-2y+1)=4+36-4$$ $$9(x+2)^2-4(y-1)^2=36$$ $$\frac{(x+2)^2}{4}-\frac{(y-1)^2}{9}=1$$ $$\frac{(x+2)^2}{2^2}-\frac{(y-1)^2}{3^2}=1$$ $$\frac{\sqrt{2^2+3^2}}{2}=\frac{\sqrt{4+9}}{2}=\frac{\sqrt{13}}{2}=\frac{\sqrt{13}}{2}$$

<table>
<tr>
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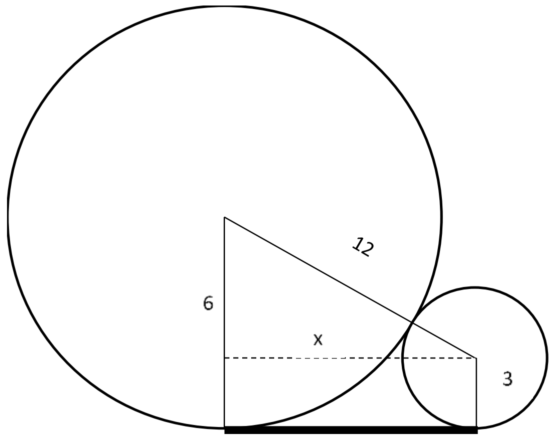
<td>9</td>
<td>
Consider the diagram below. The big pulley has a radius of 9 feet. The smaller pulley has a radius of 3 feet. The dotted line is the same length as the bold line, which represents the bottom part of the pulley belt that does not touch the pulleys. Since the dotted line is horizontal, it forms a triangle with the segment connecting the centers of the pulleys and a portion of the radius of the bigger pulley. Let the dotted line be labeled x. By the Pythagorean Theorem,



$$12^2 = x^2 + 6^2$$
$$x = 6\sqrt{3}$$

There is an upper part of the pulley belt not touching the pulleys. By symmetry, its length is the same as the bottom part.

The total length is $12\sqrt{3}$
</td>
</tr>
<tr>
<td>10</td>
<td></td>
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<td>
Evaluate the first and second derivatives and solve.

$$f'(x) = 4\sin 2x \rightarrow f''(x) = 8\cos 2x$$
$$f''\left(\frac{\pi}{24}\right) = 8\cos\left(\frac{\pi}{12}\right)$$
$$f''\left(\frac{\pi}{24}\right) = 8\cos\left(\frac{\pi/6}{2}\right)$$
$$f''\left(\frac{\pi}{24}\right) = 8\sqrt{\frac{1+\cos\frac{\pi}{6}}{2}}$$
$$f''\left(\frac{\pi}{24}\right) = \sqrt{32\left(1+\frac{\sqrt{3}}{2}\right)} = \sqrt{\frac{32\left(2+\sqrt{3}\right)}{2}} = \sqrt{16(2+\sqrt{3})} = z$$

Therefore; $z^2 = 32 + 16\sqrt{3}$
</td>
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</table>

	10	10	For a 3x3 matrix: $$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$ $$\det A = a \det \begin{bmatrix} e & f \\ h & i \end{bmatrix} - b \det \begin{bmatrix} d & f \\ g & i \end{bmatrix} + c \det \begin{bmatrix} d & e \\ g & h \end{bmatrix}$$ $$\det A = 2 \det \begin{bmatrix} 5 & 1 \\ 4 & 5 \end{bmatrix} - (-2) \det \begin{bmatrix} -1 & 1 \\ 3 & 5 \end{bmatrix} + 0$$ $$\det A = 2(25 - 4) - (-2)(-5 - 3)$$ $$\det A = 2(21) - (-2)(-8)$$ $$\det A = 42 - 16$$ $$\det A = 26$$
11	11	11	Rewrite the log expressions in exponential form. $a = 9^x$, $b = 12^x$, and $a + b = 16^x$ $$\frac{a}{b} = \frac{9^x}{12^x} = \left(\frac{3}{4}\right)^x$$ $$\frac{b}{a+b} = \frac{12^x}{16^x} = \left(\frac{3}{4}\right)^x$$ $$\frac{a}{b} = \frac{b}{a+b} \rightarrow a(a+b) = b^2$$ $$a^2 + ab = b^2 \rightarrow \frac{a^2}{b^2} + \frac{ab}{b^2} = 1$$ $$\frac{a^2}{b^2} + \frac{a}{b} - 1 = 0$$ $$\frac{a}{b} = \frac{-1 \pm \sqrt{1 - 4(1)(-1)}}{2} = \frac{-1 \pm \sqrt{5}}{2}$$ Both a and b must be positive numbers so the root that is positive must be the correct answer. $$\frac{a}{b} = \frac{-1 + \sqrt{5}}{2}$$

12	12	12	**Method 1: Volume of a Frustum** The revolved parallelogram forms an upside-down frustum with an upside-down cone removed from its top-center region. The frustum's height is 3 with top and bottom radii equal to 6 and 4 respectively. The cone is centered on the y-axis and has a radius of 2 and a height of 3. $$V_f = \frac{1}{3}\pi h(R^2 + r^2 + R * r)$$ $$V_f = \frac{1}{3}\pi 3(6^2 + 4^2 + 6 * 4)$$ $$V_f = 76\pi$$ $$V_c = \frac{1}{3}\pi r^2 h$$ $$V_c = \frac{1}{3}\pi 2^2(3) = 4\pi$$ $$V_t = V_f - V_c = 76\pi - 4\pi = 72\pi$$ **Method 2: Volumes of Cones** If one does not know the general formula for a frustum, the problem can be solved using cones only. The shape can be expanded to a full cone. The line formed by the points (6,5) and (4,2) has a slope of 3/2. Extend this line and it intersects the y-axis at (0,-4). The height of the full cone is 9 and its radius is 6. The volume of the revolved parallelogram is the volume of the full cone with the volumes of the smaller cone on top and the lower cone below subtracted. The lower cone has a radius of 4 and a height of 6. $$V_{sc} = 4\pi$$ $$V_{lc} = \frac{1}{3}\pi 4^2(6) = 32\pi$$ $$V_{fc} = \frac{1}{3}\pi 6^2(9) = 108\pi$$ $$V_t = (108 - 32 - 4)\pi = 72\pi$$

13	13	13	The volume of a cone is $V = \frac{1}{3}\pi r^2 h$ The lateral surface area of a cone is $SA = \pi r L$; where the slant height is $L = \sqrt{h^2 + r^2}$ The ratio of lateral surface area to volume is $$\frac{SA}{V} = \frac{\pi r\sqrt{h^2 + r^2}}{\frac{1}{3}\pi r^2 h} = \frac{3\sqrt{h^2 + r^2}}{rh} = 3\sqrt{\frac{1}{r^2} + \frac{1}{h^2}}$$ The ratio $\frac{SA}{V}$ will remain constant when $\frac{1}{r^2} + \frac{1}{h^2}$ remains constant. Thus, $\frac{1}{r_1^2} + \frac{1}{h_1^2} = \frac{1}{r_2^2} + \frac{1}{h_2^2}$. Given that $h_1 = 2r_1$ and $h_2 = \frac{3}{2}h_1$, solve for $\frac{r_2}{r_1}$ $$\frac{1}{r_1^2} + \frac{1}{(2r_1)^2} = \frac{1}{r_2^2} + \frac{1}{\left(\frac{3}{2}h_1\right)^2}$$ $$\frac{1}{r_1^2} + \frac{1}{4r_1^2} = \frac{1}{r_2^2} + \frac{4}{9h_1^2}$$ $$\frac{5}{4r_1^2} = \frac{1}{r_2^2} + \frac{4}{9(2\mathrm{r}_1)^2} \rightarrow \frac{5}{4r_1^2} - \frac{4}{36r_1^2} = \frac{1}{r_2^2}$$ $$\frac{45}{36r_1^2} - \frac{4}{36r_1^2} = \frac{1}{r_2^2} \rightarrow \frac{41}{36r_1^2} = \frac{1}{r_2^2}$$ $$\frac{r_2^2}{r_1^2} = \frac{36}{41}$$ $$\frac{r_2}{r_1} = \frac{6}{\sqrt{41}} = \frac{6\sqrt{41}}{41}$$
14	14		One needs to count the factors of 2 and 3 in 123! as it will take two 2's and one 3 to make a factor of 12. The number of factors of 3 can be counted by repeated division by 3 as 1/3 of the numbers have a factor of 3 but 1/3 of those have a second factor of 3 (multiples of 9), etc. $$\frac{123}{3} = 41\, r.0 \rightarrow \frac{41}{3} = 13\, r.2$$ $$\frac{13}{3} = 4\, r.1 \rightarrow \frac{4}{3} = 1\, r.1$$ Add the quotients to get the number of factors of 3. $$\#Factors\ of\ 3 = 41 + 13 + 4 + 1 = 59$$ In like manner there are 117 factors of 2 so only 58 factors of $2^2 = 4$. We need to choose the smaller one or 58.

		14	$a_{odd} = -1 - 5 + \cdots - 37 = 10\frac{(-1-37)}{2} = -190$ $a_{even} = 3 + 7 + \cdots + 39 = 10\left(\frac{3+39}{2}\right) = 210$ $a = 20$ $b = \sum_{1}^{5} (2n^3 + 2n + 2) = 2\left(\frac{5*6}{2}\right)^2 + 2\left(\frac{5*6}{2}\right) + 5$ $b = \frac{30^2}{2} + 30 + 5 = 485$ $ab = 9700 = 2^2 5^2 97$ Let s = sum of positive factors $s = (1+2+4)(1+5+25)(1+97)$ $s = (7)(31)(98) = 465(57) = 21266$
15			Use substitution to setup the definite integral. $dx = 6tdt$ $\int_1^2 (e^t - \frac{1}{t})(6t)dt \to \int_1^2 (6te^2 - 6)dt$ $\left(6\int_1^2 te^t dt\right) - (12 - 6)$ Use integration by parts to evaluate the integral $\int_1^2 te^t dt$ $u = t; du = dt$ $v = e^t; dv = e^t dt$ $(te^t) - \int_1^2 e^t dt$ $(2e^2 - e) - (e^2 - e) = e^2$ $6e^2 - 6$

	15	15	Simplify $\begin{bmatrix}1 & -2\\0 & 3\end{bmatrix}X+\begin{bmatrix}8-1 & 4-3\\12 & 6-3\end{bmatrix}=\begin{bmatrix}1 & 2\\-3 & -2\end{bmatrix}$ $\begin{bmatrix}1 & -2\\0 & 3\end{bmatrix}X+\begin{bmatrix}7 & 1\\12 & 3\end{bmatrix}=\begin{bmatrix}1 & 2\\-3 & -2\end{bmatrix}$ $\begin{bmatrix}1 & -2\\0 & 3\end{bmatrix}X=\begin{bmatrix}1 & 2\\-3 & -2\end{bmatrix}-\begin{bmatrix}7 & 1\\12 & 3\end{bmatrix}$ $\begin{bmatrix}1 & -2\\0 & 3\end{bmatrix}X=\begin{bmatrix}1-7 & 2-1\\-3-12 & -2-3\end{bmatrix}$ $\begin{bmatrix}1 & -2\\0 & 3\end{bmatrix}X=\begin{bmatrix}-6 & 1\\-15 & -5\end{bmatrix}$ $X=\begin{bmatrix}1 & -2\\0 & 3\end{bmatrix}^{-1}\begin{bmatrix}-6 & 1\\-15 & -5\end{bmatrix}$ $X=\left(\frac{1}{3-0}\begin{bmatrix}3 & 2\\0 & 1\end{bmatrix}\right)\begin{bmatrix}-6 & 1\\-15 & -5\end{bmatrix}$ $X=\begin{bmatrix}1 & \frac{2}{3}\\0 & \frac{1}{3}\end{bmatrix}\begin{bmatrix}-6 & 1\\-15 & -5\end{bmatrix}$ $X=\begin{bmatrix}-6-10 & 1-\frac{10}{3}\\0-5 & -\frac{5}{3}\end{bmatrix}$ $X=\begin{bmatrix}-16 & -7/3\\-5 & -5/3\end{bmatrix}$