2018 – 2019 Log1 Contest Round 1

Theta Geometry

Name: _____

Units do not have to be included.

4 points each		
1	A garden is enclosed by a triangular fence with side lengths of 30, 30, and 48. Find the area of the garden.	
2	At Icy's Ice Cream shop, Icy sells ice cream cones, where the cone itself has a radius of 3 cm and a height of 15 cm. When a customer buys an ice cream cone with only one scoop, the cone is expected to be filled and has a hemispherical scoop on the top. What is the volume of all the ice cream?	
3	How many diagonals does a convex decagon have?	
4	A cylinder has a radius of 5 and a height of 2. A sphere has a radius of 3. A cube has a side length of 2. If you ignore π and multiply the volume of the cylinder by the volume of the sphere and divide the product by the volume of the cube, what value do you get?	
5	Determine the exterior angle of a regular nonagon.	

5 points each		
6	Two circles are given by the equations below. Circle A: $(x - 2)^2 + (y + 2)^2 = 90$ Circle B: $(x + 3)^2 + (y - 3)^2 = 91$ The circles intersect at the points A and B. Find the slope of the line segment $\overline{AB}$.	
7	Two roads start at point A and diverge from each other at an angle of 30 degrees. One road has a length of 8 km and the other has a length of 7 km. Find the area of the triangle formed by these two roads, in square kilometers	
8	Molly bought a new apartment. To decorate, she is going to buy a new lamp. However, she wants to buy the lamp which occupies the least amount of space. She is considering two options: a lamp in the shape of a frustum with a base radius of 2 in, a height of 14 in, and an upper radius of 1 in, OR a lamp in the shape of a cylinder with height of 4 in and radius of 5 in. What is the difference in size between these lamps, in cubic inches?	
9	Quadrilateral ABCD has coordinates $A(0,0)$, $B(5,1)$, $C(5,3)$, and $D(-2,4)$. Find the coordinates of a point P that minimizes the sum $AP + BP + CP + DP$.	
10	Cali has begun to form a triangle by choosing vertices at (6, 0) and (−2,6). She asks Valentina for a third ordered pair to complete her triangle. As it turns out, Valentina suggests a point (a, b) which doesn't complete a triangle at all. What is the value of $3a + 4b$?	

6 points each		
11	What is the area of a regular octagon if it has a side length of 2?	
12	Three consecutive vertices of a convex kite ABCD are $A(-2,4), B(1,3), \text{and } C(2,0)$. What are the coordinates of the fourth vertex, given that the area enclosed by the kite is 32 square units?	
13	Given that the bases of a trapezoid are 5 cm and 13 cm and legs that are equal to $2\sqrt{21}$ and 10, find the height of the trapezoid, in centimeters.	
14	A sphere with a surface of area of 12π is inscribed in a cube. Find the volume of the non-inscribed region between the sphere and cube.	
15	Rocco is building a linear fence on his ranch. The grid location of the first post he installs is at (−2, −7), and the fifth post he installs is found at (10,13). If he continues in the same direction, always maintaining equal spacing between posts, then what are the coordinates of the 63rd post Rocco installs?	

2018 – 2019 Log1 Contest Round 1

Alpha Geometry

Name: _____

Units do not have to be included.

4 points each		
1	A garden is enclosed by a triangular fence with side lengths of 30, 30, and 48. Find the area of the garden.	
2	At Icy's Ice Cream shop, Icy sells ice cream cones, where the cone itself has a radius of 3 cm and a height of 15 cm. When a customer buys an ice cream cone with only one scoop, the cone is expected to be filled and has a hemispherical scoop on the top. What is the volume of all the ice cream?	
3	How many diagonals does a convex decagon have?	
4	Consider an XYZ Cartesian coordinate system. Point A is at the origin and point B has the coordinates $(6, 13, 18)$. Determine the length of line segment $\overline{\mathrm{AB}}$.	
5	Determine the exterior angle of a regular nonagon.	

5 points each		
6	Two circles are given by the equations below. Circle A: $(\mathrm{x} - 2)^2 + (\mathrm{y} + 2)^2 = 90$ Circle B: $(\mathrm{x} + 3)^2 + (\mathrm{y} - 3)^2 = 91$ The circles intersect at the points A and B. Find the slope of the line segment $\overline{\mathrm{AB}}$.	
7	Two roads start at point A and diverge from each other at an angle of 30 degrees. One road has a length of 8 km and the other has a length of 7 km. Find the area of the triangle formed by these two roads, in square kilometers	
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9	Two circles intersect at two points. Both circles have a radius of 5. Their centers are separated by a distance of $5\sqrt{2}$. Find the area of the intersecting region of these two circles.	
10	Cali has begun to form a triangle by choosing vertices at (6, 0) and (−2,6). She asks Valentina for a third ordered pair to complete her triangle. As it turns out, Valentina suggests a point (a, b) which doesn't complete a triangle at all. What is the value of $3\mathrm{a} + 4\mathrm{b}$?	

6 points each		
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12	Three consecutive vertices of a convex kite ABCD are $A(-2,4), B(1,3), \text{and } C(2,0)$. What are the coordinates of the fourth vertex, given that the area enclosed by the kite is 32 square units?	
13	Given that the bases of a trapezoid are 5 cm and 13 cm and legs that are equal to $2\sqrt{21}$ and 10, find the height of the trapezoid, in centimeters.	
14	$\overrightarrow{AB}$, $\overrightarrow{BC}$, and $\overrightarrow{CD}$ are vectors. Expressed in polar form, $\overrightarrow{AB} = \left(6, \frac{\pi}{6}\right), \overrightarrow{BC} = \left(4\sqrt{3}, \frac{5\pi}{3}\right), \text{and } \overrightarrow{CD}\left(3\sqrt{2}, \frac{\pi}{4}\right)$. Let $\overrightarrow{AD} = \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD}$. Calculate the area of triangle ABD.	
15	Rocco is building a linear fence on his ranch. The grid location of the first post he installs is at (−2, −7), and the fifth post he installs is found at (10,13). If he continues in the same direction, always maintaining equal spacing between posts, then what are the coordinates of the 63rd post Rocco installs?	

2018 – 2019 Log1 Contest Round 1
Mu Geometry

Name: _____

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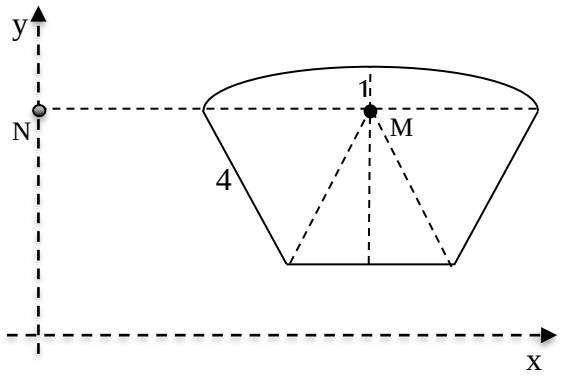
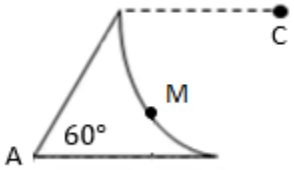
4 points each		
1	A garden is enclosed by a triangular fence with side lengths of 30, 30, and 48. Find the area of the garden.	
2	At Icy's Ice Cream shop, Icy sells ice cream cones, where the cone itself has a radius of 3 cm and a height of 15 cm. When a customer buys an ice cream cone with only one scoop, the cone is expected to be filled and has a hemispherical scoop on the top. What is the volume of all the ice cream?	
3	How many diagonals does a convex decagon have?	
4	Consider an XYZ Cartesian coordinate system. Point A is at the origin and point B has the coordinates $(6, 13, 18)$. Determine the length of line segment $\overline{\mathrm{AB}}$.	
5	Consider a right triangle. One leg has a length, x, and the second leg has a length, $\mathrm{x}^2 + 1$. If the initial length of the first leg is $\mathrm{x} = 1\ \mathrm{m}$ when t = 0 seconds, determine the rate of change of the area, $\frac{\mathrm{dA}}{\mathrm{dt}}$, at a time, t = 3 seconds, if the rate of change of the first leg is $\frac{\mathrm{dx}}{\mathrm{dt}} = 2\frac{\mathrm{m}}{\mathrm{s}}$.	

5 points each		
6	Two circles are given by the equations below. Circle A: $(\mathrm{x} - 2)^2 + (\mathrm{y} + 2)^2 = 90$ and Circle B: $(\mathrm{x} + 3)^2 + (\mathrm{y} - 3)^2 = 91$. The circles intersect at the points A and B. Find the slope of the line segment $\overline{\mathrm{AB}}$.	
7	Two roads start at point A and diverge from each other at an angle of 30 degrees. One road has a length of 8 km and the other has a length of 7 km. Find the area of the triangle formed by these two roads, in square kilometers	
8	Molly bought a new apartment. To decorate, she is going to buy a new lamp. However, she wants to buy the lamp which occupies the least amount of space. She is considering two options: a lamp in the shape of a frustum with a base radius of 2 in, a height of 14 in, and an upper radius of 1 in, OR a lamp in the shape of a cylinder with height of $4\ \mathrm{in}$ and radius of 5 in. What is the difference in size between these lamps, in cubic inches?	
9	Two circles intersect at two points. Both circles have a radius of 5. Their centers are separated by a distance of $5\sqrt{2}$. Find the area of the intersecting region of these two circles.	

10	A two-dimensional shape is made such that it has a half-ellipse on the top and a half-regular hexagon on the bottom (i.e. it looks like a vertical cross-section of a cupcake.) The hexagonal sides have side lengths equal to 4. The elliptical side's semi-minor axis has a length equal to 1. When this cross-section is revolved around the y-axis, the center point, M, traces a circle parallel to the x-axis with a radius $\mathrm{NM} = 24$. The shape formed is toroidal, or "like a donut". Given this information, determine the volume of this special kind of toroid. 	

	6 points each	
11	What is the area of a regular octagon if it has a side length of 2?	
12	Three consecutive vertices of a convex kite ABCD are $\mathrm{A}(-2,4), \mathrm{B}(1,3), \text{and } \mathrm{C}(2,0)$. What are the coordinates of the fourth vertex, given that the area enclosed by the kite is 32 square units?	
13	Given that the bases of a trapezoid are 5 cm and 13 cm and legs that are equal to $2\sqrt{21}$ and 10, find the height of the trapezoid, in centimeters.	
14	$\overrightarrow{\mathrm{AB}}$, $\overrightarrow{BC}$, and $\overrightarrow{CD}$ are vectors. Expressed in polar form, $\overrightarrow{\mathrm{AB}} = \left(6, \frac{\pi}{6}\right), \overrightarrow{\mathrm{BC}} = \left(4\sqrt{3}, \frac{5\pi}{3}\right), \text{and } \overrightarrow{\mathrm{CD}}\left(3\sqrt{2}, \frac{\pi}{4}\right)$. Let $\overrightarrow{\mathrm{AD}} = \overrightarrow{\mathrm{AB}} + \overrightarrow{\mathrm{BC}} + \overrightarrow{\mathrm{CD}}$. Calculate the area of triangle ABD.	
15	Consider the shape shown to the right. The straight sides are each 2 meters long and form an angle of 60 degrees at the bottom-left vertex, A. The radius of curvature of the circular portion of the shape is also 2 meters. The center of curvature of the circular portion is located at the point C such that a radius drawn to the top vertex is horizontal. If we place the shape such that point A is at the origin of an X-Y coordinate system, determine the slope of the circular portion at point M, which has as its x-coordinate, $\mathrm{x} = \frac{3}{2}$. Express your answer in proper rationalized form. 	

2018 – 2019 Log1 Contest Round 1

Theta Geometry Answer Key

Name: _____

Units do not have to be included.

4 points each		
1	A garden is enclosed by a triangular fence with side lengths of 30, 30, and 48. Find the area of the garden.	432
2	At Icy's Ice Cream shop, Icy sells ice cream cones, where the cone itself has a radius of 3 cm and a height of 15 cm. When a customer buys an ice cream cone with only one scoop, the cone is expected to be filled and has a hemispherical scoop on the top. What is the volume of all the ice cream?	63π
3	How many diagonals does a convex decagon have?	35
4	A cylinder has a radius of 5 and a height of 2. A sphere has a radius of 3. A cube has a side length of 2. If you ignore π and multiply the volume of the cylinder by the volume of the sphere and divide the product by the volume of the cube, what value do you get?	225
5	Determine the exterior angle of a regular nonagon.	$40°$

5 points each		
6	Two circles are given by the equations below. Circle A: $(x-2)^2+(y+2)^2=90$ Circle B: $(x+3)^2+(y-3)^2=91$ The circles intersect at the points A and B. Find the slope of the line segment $\overline{AB}$.	1
7	Two roads start at point A and diverge from each other at an angle of 30 degrees. One road has a length of 8 km and the other has a length of 7 km. Find the area of the triangle formed by these two roads, in square kilometers	14
8	Molly bought a new apartment. To decorate, she is going to buy a new lamp. However, she wants to buy the lamp which occupies the least amount of space. She is considering two options: a lamp in the shape of a frustum with a base radius of 2 in, a height of 14 in, and an upper radius of 1 in, OR a lamp in the shape of a cylinder with height of 4 in and radius of 5 in. What is the difference in size between these lamps, in cubic inches?	$\frac{202}{3}\pi$
9	Quadrilateral ABCD has coordinates $A(0,0)$, $B(5,1)$, $C(5,3)$, and $D(-2,4)$. Find the coordinates of a point P that minimizes the sum $AP+BP+CP+DP$.	$\left(\frac{55}{18},\frac{11}{6}\right)$
10	Cali has begun to form a triangle by choosing vertices at (6, 0) and (−2,6). She asks Valentina for a third ordered pair to complete her triangle. As it turns out, Valentina suggests a point (a,b) which doesn't complete a triangle at all. What is the value of $3a+4b$?	18

6 points each		
11	What is the area of a regular octagon if it has a side length of 2?	$8+8\sqrt{2}$
12	Three consecutive vertices of a convex kite ABCD are $A(-2,4), B(1,3), \text{and } C(2,0)$. What are the coordinates of the fourth vertex, given that the area enclosed by the kite is 32 square units?	$(-7,-5)$
13	Given that the bases of a trapezoid are 5 cm and 13 cm and legs that are equal to $2\sqrt{21}$ and 10, find the height of the trapezoid, in centimeters.	$5\sqrt{3}$
14	A sphere with a surface of area of 12π is inscribed in a cube. Find the volume of the non-inscribed region between the sphere and cube.	$(24-4\pi)\sqrt{3}$
15	Rocco is building a linear fence on his ranch. The grid location of the first post he installs is at (−2, −7), and the fifth post he installs is found at (10,13). If he continues in the same direction, always maintaining equal spacing between posts, then what are the coordinates of the 63rd post Rocco installs?	(184,303)

2018 – 2019 Log1 Contest Round 1

Alpha Geometry Answer Key

Name: _____

Units do not have to be included.

4 points each		
1	A garden is enclosed by a triangular fence with side lengths of 30, 30, and 48. Find the area of the garden.	432
2	At Icy's Ice Cream shop, Icy sells ice cream cones, where the cone itself has a radius of 3 cm and a height of 15 cm. When a customer buys an ice cream cone with only one scoop, the cone is expected to be filled and has a hemispherical scoop on the top. What is the volume of all the ice cream?	63π
3	How many diagonals does a convex decagon have?	35
4	Consider an XYZ Cartesian coordinate system. Point A is at the origin and point B has the coordinates $(6, 13, 18)$. Determine the length of line segment $\overline{AB}$.	23
5	Determine the exterior angle of a regular nonagon.	$40°$

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9	Two circles intersect at two points. Both circles have a radius of 5. Their centers are separated by a distance of $5\sqrt{2}$. Find the area of the intersecting region of these two circles.	$25\left(\frac{\pi}{2} - 1\right)$
10	Cali has begun to form a triangle by choosing vertices at (6, 0) and (−2,6). She asks Valentina for a third ordered pair to complete her triangle. As it turns out, Valentina suggests a point (a, b) which doesn't complete a triangle at all. What is the value of $3a + 4b$?	18

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12	Three consecutive vertices of a convex kite ABCD are $\mathrm{A}(-2,4), \mathrm{B}(1,3), \text{and } \mathrm{C}(2,0)$. What are the coordinates of the fourth vertex, given that the area enclosed by the kite is 32 square units?	$(-7,-5)$
13	Given that the bases of a trapezoid are 5 cm and 13 cm and legs that are equal to $2\sqrt{21}$ and 10, find the height of the trapezoid, in centimeters.	$5\sqrt{3}$
14	$\overrightarrow{\mathrm{AB}}$, $\overrightarrow{\mathrm{BC}}$, and $\overrightarrow{\mathrm{CD}}$ are vectors. Expressed in polar form, $\overrightarrow{\mathrm{AB}}=\left(6,\frac{\pi}{6}\right), \overrightarrow{\mathrm{BC}}=\left(4\sqrt{3},\frac{5\pi}{3}\right), \text{and } \overrightarrow{\mathrm{CD}}\left(3\sqrt{2},\frac{\pi}{4}\right)$. Let $\overrightarrow{\mathrm{AD}}=\overrightarrow{\mathrm{AB}}+\overrightarrow{\mathrm{BC}}+\overrightarrow{\mathrm{CD}}$. Calculate the area of triangle ABD.	$\frac{15\sqrt{3}+9}{2}$
15	Rocco is building a linear fence on his ranch. The grid location of the first post he installs is at (−2, −7), and the fifth post he installs is found at (10,13). If he continues in the same direction, always maintaining equal spacing between posts, then what are the coordinates of the 63rd post Rocco installs?	(184,303)

2018 – 2019 Log1 Contest Round 1
Mu Geometry Answer Key

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5	Consider a right triangle. One leg has a length, x, and the second leg has a length, $x^2 + 1$. If the initial length of the first leg is $x = 1$ m when t = 0 seconds, determine the rate of change of the area, $\frac{dA}{dt}$, at a time, t = 3 seconds, if the rate of change of the first leg is $\frac{dx}{dt} = 2\frac{m}{s}$.	148

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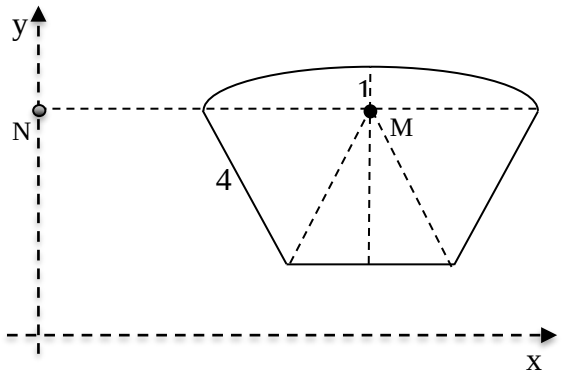
9	Two circles intersect at two points. Both circles have a radius of 5. Their centers are separated by a distance of $5\sqrt{2}$. Find the area of the intersecting region of these two circles.	$25\left(\frac{\pi}{2} - 1\right)$

10	A two-dimensional shape is made such that it has a half-ellipse on the top and a half-regular hexagon on the bottom (i.e. it looks like a vertical cross-section of a cupcake.) The hexagonal sides have side lengths equal to 4. The elliptical side's semi-minor axis has a length equal to 1. When this cross-section is revolved around the y-axis, the center point, M, traces a circle parallel to the x-axis with a radius $\mathrm{NM} = 24$. The shape formed is toroidal, or "like a donut". Given this information, determine the volume of this special kind of toroid. 	$96\pi^2 + 576\pi\sqrt{3}$

6 points each		
11	What is the area of a regular octagon if it has a side length of 2?	$8 + 8\sqrt{2}$
12	Three consecutive vertices of a convex kite ABCD are $\mathrm{A}(-2,4), \mathrm{B}(1,3), \text{and } \mathrm{C}(2,0)$. What are the coordinates of the fourth vertex, given that the area enclosed by the kite is 32 square units?	$(-7,-5)$
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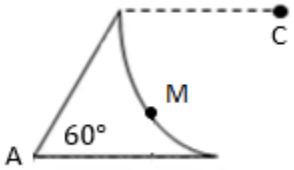
15	Consider the shape shown to the right. The straight sides are each 2 meters long and form an angle of 60 degrees at the bottom-left vertex, A. The radius of curvature of the circular portion of the shape is also 2 meters. The center of curvature of the circular portion is located at the point C such that a radius drawn to the top vertex is horizontal. If we place the shape such that point A is at the origin of an X-Y coordinate system, determine the slope of the circular portion at point M, which has as its x-coordinate3, $\mathrm{x} = \frac{3}{2}$. Express your answer in proper rationalized form. 	$-\frac{3\sqrt{7}}{7}$

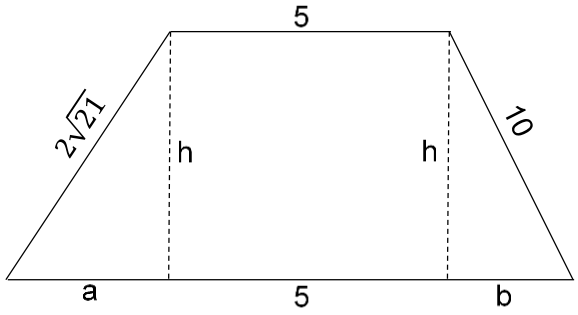
2017 – 2018 Log1 Contest Round 1
Geometry Solutions

Mu	Al	Th	Solution
1	1	1	$$\text{Garden Area} = \frac{1}{2}(\text{Base})(\text{Height})$$ $$\text{Area} = \frac{1}{2}(48)(\text{Height})$$ The height of the triangular fence forms two right triangles. Since the triangle is equilateral, the height bisects the base. Therefore, $$30^2 = \text{Height}^2 + 24^2$$ $$\text{Height} = 18$$ $$\text{Area} = 432$$
2	2	2	The equation for the volume of a cone is given by $$V_{cone} = \frac{1}{3}\pi r^2 \text{ h.}$$ $$V_{cone} = \frac{1}{3}\pi(3)^2\ 15$$ $$V_{cone} = 45\pi \text{ cm}^2$$ The equation for the volume of a sphere is given by $$V_{sphere} = \frac{4}{3}\pi r^3$$ $$V_{sphere} = \frac{4}{3}\pi(3)^3$$ $$V_{sphere} = 36\pi \text{ cm}^2$$ $$V_{cone} + \frac{1}{2}V_{sphere} = 63\pi$$
3	3	3	The general formula for solving for diagonals is $\frac{n(n-3)}{2}$, where n is the number of sides. $$\frac{10(10-3)}{2} = 35$$
4	4		The distance formula may be used in this case. $$d = \sqrt{6^2 + 13^2 + 18^2} = \sqrt{36 + 169 + 324}$$ $$d = \sqrt{529} = 23$$
		4	$$V_{cylinder} = \pi r^2 h = \pi(5)^2 2 = 50\pi$$ $$V_{sphere} = \frac{4}{3}\pi r^3 + \frac{4}{3}\pi(3)^3 = 36\pi$$ $$V_{cube} = x^3 = 2^3 = 8$$ Ignoring π: $$\frac{50 \times 36}{8} = 225$$

5			The area function of this triangle is expressed as $$A = \frac{1}{2}(x)(x^2+1) = \frac{1}{2}x^3 + \frac{1}{2}x$$ $$\frac{dA}{dt} = \left(\frac{3}{2}x^2 + \frac{1}{2}\right)\frac{dx}{dt}$$ $$\frac{dA}{dt} = \left(\frac{3}{2} * 7^2 + \frac{1}{2}\right)(2) = \frac{3*49+1}{2} * 2 = 148\frac{m^2}{s}$$ Since units are not required for the Log 1 tests, 148 is sufficient as a final answer.
	5	5	The exterior angle will be 360 degrees divided by 9, or 40 degrees.
6	6	6	The centers of the two circles are (2, -2) and (-3, 3) and the slope between the centers is equal to -1. Since the points A and B lie on the same circle, then the center of the circle (or of both circles) must lie on the perpendicular bisector of $\overline{AB}$. Then the slope of $\overline{AB}$ is perpendicular to the slope of the segment between the centers, and $\overline{AB}$ has a slope of 1.
7	7	7	Know the area of a triangle can be found by ½*a*b*sin c Using basic right triangle trigonometry, the height of the triangle is $h = 7\sin 30$. With a base of 8, the area is given by the formula, $\text{area} = \frac{1}{2} * \text{base} * \text{height} * \sin\theta$ Thus, $\text{area} = \frac{1}{2} * 8\text{ km} * 7\text{ km} * \sin 30° = 14\text{ km}^2$
8	8	8	Without knowing the formula for the volume of a frustum, one can derive it by knowing the volume of cones and the rules for similar triangles. Consider the frustum. This frustum is missing a small cone on its top. Since the upper radius is 1 in and the bottom radius is 2 in, the missing cone must also have a height of 14 in. This is because the smaller cone, is mathematically similar to the larger cone, which is the frustum with its missing smaller cone. These are similar triangles in the ratio of 1:2, respectively. The calculation is as follows. $V_{frustum} = \frac{1}{3}\pi r_b^2 h_b - \frac{1}{3}\pi r_s^2 h_s = \frac{1}{3}\pi(2^2 * 28 - 1^2 * 14)$ The volume of the frustum is $\frac{98}{3}\pi\text{ in}^3$, The volume of the cylindrical lamp is $V_{cylinder} = \pi r^2 h = \pi(5^2)(4) = 100\pi\text{ in}^3$ The difference between these values is $\frac{202}{3}\pi\text{ in}^3$

9	9		If we call the intersection points A and B, and the centers C and D, then ΔCAD and ΔCBD are both right triangles as the lengths form a Pythagorean Triple. Let $CA = CB = BD = AD = 5$. Since ABCD is a square, then $AB = CD = 5\sqrt{2}$, and $\angle A = \angle D = 90°$. Thus, the area of sectors $ACB = ADB = \frac{1}{4}\pi(5^2) = \frac{25}{4}\pi$ $$A_{\Delta ACB} = \frac{1}{2}\text{Base} * \text{Height}$$ $$A_{\Delta ACB} = \frac{1}{2}(5\sqrt{2})\frac{(5\sqrt{2})}{2} = \frac{1}{4}50 = \frac{25}{2}$$ Subtracting yields the area of the chord. $$A_{\text{chord}AB} = \frac{25}{4}\pi - \frac{25}{2}$$ Multiply by 2 to get the area of the intersecting region of both circles. $A = 25(\frac{\pi}{2} - 1)$ or any equivalent form.
		9	If we think about AP + CP, we know that if P is between A and C, then AP + PC = AC. We also know from the triangle inequality theorem that if P is not between A and C, then AP + PC > AC. We are trying to minimize the sum, so we know P must be between A and C. By similar reasoning, we determine that P must be between B and D. To minimize the sum of those 4 segments, point P must be the intersection of the diagonals of the quadrilateral. Determine the equations of the diagonals and equate them to find the coordinates of the intersection point. Using the definition of slope and the slope-intercept form, the equations are BD: $3x + 7y = 22$ & AC: $y = \frac{3}{5}x$ Substituting y, $x = \frac{55}{18}$. Evaluating for y, $y = \frac{11}{6}$

10			Calculate the area of each region separately and add them. Hexagonal region: Find the area of the triangular sector and multiply by 3. Each triangular sector of a hexagon is an equilateral triangle. Given the length of the hexagon side as 4, the area is easily calculated. $$A_{sector} = \frac{1}{2}bh = \frac{1}{2}(4)(2\sqrt{3}) = 4\sqrt{3}$$ $$A_{hexagon} = 12\sqrt{3}$$ Elliptical region: Find the area of the half-ellipse, noting that the semi-major axis is equal to 4 since it is part of the equilateral triangle of a sector of the half-hexagon on the bottom. $$A_{ellipse} = \frac{1}{2}\pi ab = \frac{1}{2}\pi(1)(4) = 2\pi$$ The total area is found by addition. $A = 2\pi + 12\sqrt{3}$ To find the toroidal volume, multiply the area above by the circumference of the circle of revolution. $$V = A * 2\pi r = (2\pi + 12\sqrt{3})(2\pi * 24) = 96\pi^2 + 576\pi\sqrt{3}$$ Any equivalent form of the answer for this volume is acceptable.
	10	10	To fail to complete a triangle, the point Valentina chose must be on the line containing the first two points. This line is 3x + 4y = 18, so for any point on that line, specifically the one chosen in the form (a, b), it must be true that 3a + 4b = 18
11	11	11	An octagon is a square with its four corners cutoff. If the side length of an octagon is 2, then each corner of the square to be cut off is an isosceles right triangle with hypotenuse equal to 2. The length of the legs of these triangles, must be $\sqrt{2}$. This is shown below. $$2x^2 = 4 \rightarrow x^2 = 2 \rightarrow x = \sqrt{2}$$ The area of these triangles is $$A_{triangle} = \frac{1}{2}bh = \frac{1}{2}\sqrt{2}\sqrt{2} = \frac{1}{2} * 2 = 1$$ Since there are 4 corner triangles to be cutoff, we need to subtract 4 from the area of the square. Each side of the square has a length, $L = 2 + 2\sqrt{2}$ The area of the square is $$A_{square} = (2 + 2\sqrt{2})^2 = 4 + 8\sqrt{2} + 8 = 12 + 8\sqrt{2}$$ Therefore, $$A = A_{square} - 4A_{triangle} = 8 + 8\sqrt{2}$$

12	12	12	We know the area of a kite is $$\text{Area} = (½)d_1d_2$$ Using the distance formula, we know $AC = 4\sqrt{2}$. $A = (½)d_1d_2 = (½)(4\sqrt{2})d_2 = 32 \Rightarrow d_2 = 8\sqrt{2}$ Now since $\overline{AC}$ has slope -1, and the diagonals of a kite are perpendicular, then the slope of $\overline{BD}$ is 1, and we must travel $8\sqrt{2}$ units, which specifically has us moving down 8 units and left 8 units from point B to $(-7,-5)$.
13	13	13	Consider the diagram below.  $h^2 + a^2 = 84$ and $h^2 + b^2 = 100$ Thus, $a^2 - b^2 = 16$, or $(a-b)(a+b) = 16$ Assume that all lengths must be positive. Since $a + b = 8$, then $a - b = 2$. The only possible values for a and b for this to be true are $a = 3$ and $b = 5$. Solve for h. $$h^2 = 3^2 + 84 \rightarrow h^2 = 75$$ $$h = 5\sqrt{3}\text{ cm}$$
14	14		Solve by resolving into components. <table><tr><th></th><th>X</th><th>Y</th></tr><tr><td>AB</td><td>$6\cos\frac{\pi}{6} = +3\sqrt{3}$</td><td>$6\sin\frac{\pi}{6} = +3$</td></tr><tr><td>BC</td><td>$4\sqrt{3}\cos\frac{5\pi}{3} = +2\sqrt{3}$</td><td>$4\sqrt{3}\sin\frac{5\pi}{3} = -6$</td></tr><tr><td>CD</td><td>$3\sqrt{2}\cos\frac{\pi}{4} = +3$</td><td>$3\sqrt{2}\sin\frac{\pi}{4} = +3$</td></tr><tr><td>AD</td><td>$5\sqrt{3}+3$</td><td>0</td></tr></table>Assume that point A is located at (0,0). Since point D has a y-coordinate of 0, this means that line segment AD is parallel to the x-axis and has a length of $5\sqrt{3}+3$. Since point B has a y-coordinate of +3, this means that triangle ABD has a height of +3. Thus, the area of triangle ABD is $$\text{Area} = \frac{1}{2}bh = \frac{1}{2}(5\sqrt{3}+3)(3) = \frac{15\sqrt{3}+9}{2}$$

		14	One must determine the volume of the cube and the inscribed sphere. The radius of the sphere is easily determined using $$A_{sphere} = 12\pi = 4\pi r^2 \rightarrow r = \sqrt{3}$$ Since the diameter of the sphere, when inscribed in the cube, is equal to the length of the side of the cube, $x = 2\sqrt{3}$. $$V_{sphere} = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi\left(\sqrt{3}\right)^3 = \frac{4}{3}\pi 3\sqrt{3} = 4\pi\sqrt{3}$$ $$V_{cube} = x^3 = 24\sqrt{3}$$ $$V = V_{cube} - V_{sphere} = (24 - 4\pi)\sqrt{3}$$ Any equivalent form is acceptable.
15			Imagine the complete circle formed by the circular portion. A line drawn from the top vertex to the center of this circle is parallel to the x-axis and 2 meters long, as stated in the problem. This forms the third side of a rhombus. The fourth side may be drawn in for clarity. The length of the longer diagonal of the rhombus is twice the height of the equilateral triangle, which is $2\ (2\cos 30) = 2\sqrt{3}$. Assuming the bottom-left vertex is at the origin, the center of the circle may be found as follows. $$h = 2\sqrt{3}\cos 30 = 3$$ $$k = 2\sqrt{3}\sin 30 = \sqrt{3}$$ The equation for this circle will be $$(x-3)^2 + \left(y - \sqrt{3}\right)^2 = 4$$ When $x = \frac{3}{2}, y = \pm\frac{\sqrt{7}}{2} + \sqrt{3}$ Implicitly differentiating, $$2(x-3) + 2\left(y - \sqrt{3}\right)\frac{dy}{dx} = 0$$ $$\frac{dy}{dx} = \frac{2(3-x)}{2\left(y-\sqrt{3}\right)} = \frac{3-x}{y-\sqrt{3}}$$ Substituting, $$\frac{dy}{dx} = \frac{3-\frac{3}{2}}{-\frac{\sqrt{7}}{2} + \sqrt{3} - \sqrt{3}} = -\frac{\frac{3}{2}}{\frac{\sqrt{7}}{2}} = -\frac{3}{\sqrt{7}} = -\frac{3\sqrt{7}}{7}$$ We take the value of y that yields a negative slope since this must be the case by inspection. If students have not learned how to implicitly differentiate, then they can solve for y and use the chain rule to arrive at the same form for $\frac{dy}{dx}$ $$\frac{dy}{dx} = \pm\frac{1}{2}(4 - (x-3)^2)^{-\frac{1}{2}}(2(x-3))$$ The value for the slope will be the same.

	15	15	Rocco begins at (-2, -7) and the fifth post is at (10, 13). So, 4 moves take him right 12 units and up 20 units. This means each move will take him right 3 and up 5. To get to the 63rd post he must make 62 moves, which will take him right 186 and up 310 from (-2, -7) to arrive at (184, 303).