2018 – 2019 Log1 Contest Round 1

Theta Sequences and Series

Name: ___________________

Units do not have to be included.

	4 points each	
1	An arithmetic series has eight terms. The first term is 2 and the last term is 58. Find the sum of the sequence.	
2	What is the sum of the first eight pentagonal numbers?	
3	Find the sum of: $1^2 + 2^2 + 3^2 + 4^2 + \ldots + 20^2$	
4	Evaluate the following series. $$\sum_{n=0}^{\infty} 15^{-n}$$	
5	Evaluate: $$\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+\ldots}}}}$$	

	5 points each	
6	Find x given that $$\sum_{k=1}^{\infty}\left(\frac{5x}{2}\right)^{k-1} = 8$$	
7	Solve for x given that $1 + 4 + 7 + \cdots + x = 925$	
8	Group odd numbers as follows: {1}, {3,5}, {7,9,11}, {13,15,17,19} … Find the sum of the elements in the 38th set.	
9	Evaluate the double sum: $$\sum_{n=2}^{\infty}\sum_{k=2}^{\infty}\frac{1}{n^k}$$	
10	Find the sum of the first 41 terms of the Fibonacci sequence given that the 43nd term is 433,494,437.	

6 points each		
11	Consider the equation shown below $$S=\sqrt{12+\frac{12}{\sqrt{12+\frac{12}{\sqrt{12+\frac{12}{\cdots}}}}}}$$ Determine the largest integer less than or equal to S.	
12	Let $z_k = a_k + b_k$ where a_k is an arithmetic sequence and b_k is a geometric sequence. Suppose the first term is $z_1 = 2 + 3i$ and the 3rd term is $z_3 = 22 - 3i$. Calculate the sum of the first 50 terms, S_{50}.	
13	A ball bounces from a height of 4 meters and returns to 70% of its previous height on each bounce. Find the total distance travelled by the ball, in meters, until it is at rest. Express your answer in mixed number or improper fraction form.	
14	Evaluate: $$\sum_{n=10}^{\infty} \log_3(1-\frac{1}{n})$$	
15	The midpoints of the sides of a 512 by 512 square are joined to form another square, and then the midpoints of the side of the small square are joined to form an even smaller square. If this process continues, find the sum of the perimeters of the first 23 squares. Express your answer in the form $a + b\sqrt{c}$	

2018 – 2019 Log1 Contest Round 1

Alpha Sequences and Series

Name: ___________________

Units do not have to be included.

4 points each		
1	An arithmetic series has eight terms. The first term is 2 and the last term is 58. Find the sum of the sequence.	
2	What is the sum of the first eight pentagonal numbers?	
3	Find the sum of: $1^2 + 2^2 + 3^2 + 4^2 + \ldots + 20^2$	
4	Solve the following series. $$S = \sum_{n=1}^{23} i^n$$	
5	Evaluate: $$\sqrt{6 + \sqrt{6 + \sqrt{6 + \sqrt{6 + \ldots}}}}$$	

5 points each		
6	Find x given that $$\sum_{k=1}^{\infty} \left(\frac{5x}{2}\right)^{k-1} = 8$$	
7	Solve for x given that $1 + 4 + 7 + \cdots + x = 925$	
8	Group odd numbers as follows: {1}, {3,5}, {7,9,11}, {13,15,17,19} … Find the sum of the elements in the 38th set.	
9	x + 4 and x - 3 are the first two terms of a geometric **sequence,** respectively. Find the values of x for which the **sequence** converges.	
10	Find the sum of the first 41 terms of the Fibonacci sequence given that the 43nd term is 433,494,437.	

6 points each		
11	Consider the equation shown below $$S = \sqrt{12 + \frac{12}{\sqrt{12 + \frac{12}{\sqrt{12 + \frac{12}{\cdots}}}}}}$$ Determine the largest integer less than or equal to S.	
12	Let $z_k = a_k + b_k$ where a_k is an arithmetic sequence and b_k is a geometric sequence. Suppose the first term is $z_1 = 2 + 3i$ and the 3rd term is $z_3 = 22 - 3i$. Calculate the sum of the first 50 terms, S_{50}.	
13	A ball bounces from a height of 4 meters and returns to 70% of its previous height on each bounce. Find the total distance travelled by the ball, in meters, until it is at rest. Express your answer in mixed number or improper fraction form.	
14	What is the sum of all four-digit palindromes?	
15	The midpoints of the sides of a 512 by 512 square are joined to form another square, and then the midpoints of the side of the small square are joined to form an even smaller square. If this process continues, find the sum of the perimeters of the first 23 squares. Express your answer in the form $a + b\sqrt{c}$	

2018 – 2019 Log1 Contest Round 1

Mu Sequences and Series

Name: ___________________

Units do not have to be included.

	4 points each	
1	An arithmetic series has eight terms. The first term is 2 and the last term is 58. Find the sum of the sequence.	
2	What is the sum of the first eight pentagonal numbers?	
3	Find the sum of: $1^2 + 2^2 + 3^2 + 4^2 + \ldots + 20^2$	
4	Solve the following series. $$S = \sum_{n=1}^{23} i^n$$	
5	In the showing of Marvel's "Avengers: Infinity War", one person sits in the first row. In the second row, there are 4 people and in the third row there are 7 people. If this pattern continues and there are 28 rows, how many people are there in total?	

	5 points each	
6	Find x given that $$\sum_{k=1}^{\infty} \left(\frac{5x}{2}\right)^{k-1} = 8$$	
7	Solve for x given that $1 + 4 + 7 + \cdots + x = 925$	
8	Group odd numbers as follows: {1}, {3,5}, {7,9,11}, {13,15,17,19} … Find the sum of the elements in the 38th set.	
9	x + 4 and x - 3 are the first two terms of a geometric **sequence,** respectively. Find the values of x for which the **sequence** converges.	
10	If $f(x) = -x^3 + 3x^2 + 1$ and $g(x) = 5x$, evaluate $$\sum_{x=1}^{10} f''(g(x))$$	

6 points each		
11	Consider the equation shown below $$S = \sqrt{12 + \frac{12}{\sqrt{12 + \frac{12}{\sqrt{12 + \frac{12}{\cdots}}}}}}$$ Determine the largest integer less than or equal to S.	
12	Let $z_k = a_k + b_k$ where a_k is an arithmetic sequence and b_k is a geometric sequence. Suppose the first term is $Z_1 = 2 + 3i$ and the 3rd term is $Z_3 = 22 - 3i$. Calculate the sum of the first 50 terms, S_{50}.	
13	A ball bounces from a height of 4 meters and returns to 70% of its previous height on each bounce. Find the total distance travelled by the ball, in meters, until it is at rest. Express your answer in mixed number or improper fraction form.	
14	What is the sum of all four-digit palindromes?	
15	Given the function $f(x) = 3x^2 + 2x - 1$, determine the value of the series given below. $$S_{25} = \sum_{n=1}^{25} \sin\left(\frac{n\pi}{2}\right) f'(n)$$	

2018 – 2019 Log1 Contest Round 1

Theta Sequences and Series Answer Key

Name: ___________________

Units do not have to be included.

	4 points each	
1	An arithmetic series has eight terms. The first term is 2 and the last term is 58. Find the sum of the sequence.	240
2	What is the sum of the first eight pentagonal numbers?	288
3	Find the sum of: $1^2 + 2^2 + 3^2 + 4^2 + \ldots + 20^2$	2870
4	Evaluate the following series. $$\sum_{n=0}^{\infty} 15^{-n}$$	$\frac{15}{14}$
5	Evaluate: $$\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+\ldots}}}}$$	3

	5 points each	
6	Find x given that $$\sum_{k=1}^{\infty}\left(\frac{5x}{2}\right)^{k-1} = 8$$	$\frac{7}{20}$
7	Solve for x given that $1 + 4 + 7 + \cdots + x = 925$	73
8	Group odd numbers as follows: {1}, {3,5}, {7,9,11}, {13,15,17,19} … Find the sum of the elements in the 38th set.	54,872
9	Evaluate the double sum: $$\sum_{n=2}^{\infty}\sum_{k=2}^{\infty}\frac{1}{n^k}$$	1
10	Find the sum of the first 41 terms of the Fibonacci sequence given that the 43nd term is 433,494,437.	433,494,436

6 points each		
11	Consider the equation shown below $$S = \sqrt{12 + \frac{12}{\sqrt{12 + \frac{12}{\sqrt{12 + \frac{12}{\dots}}}}}}$$ Determine the largest integer less than or equal to S.	3
12	Let $z_k = a_k + b_k$ where a_k is an arithmetic sequence and b_k is a geometric sequence. Suppose the first term is $z_1 = 2 + 3i$ and the 3rd term is $z_3 = 22 - 3i$. Calculate the sum of the first 50 terms, S_{50}.	$12347 + 3i$
13	A ball bounces from a height of 4 meters and returns to 70% of its previous height on each bounce. Find the total distance travelled by the ball, in meters, until it is at rest. Express your answer in mixed number or improper fraction form.	$\frac{68}{3}$ or $22\frac{2}{3}$
14	Evaluate: $$\sum_{n=10}^{\infty} \log_3(1 - \frac{1}{n})$$	$-\infty$ Or diverges
15	The midpoints of the sides of a 512 by 512 square are joined to form another square, and then the midpoints of the side of the small square are joined to form an even smaller square. If this process continues, find the sum of the perimeters of the first 23 squares. Express your answer in the form $a + b\sqrt{c}$	$4095 + 2047\sqrt{2}$

2018 – 2019 Log1 Contest Round 1

Alpha Sequences and Series Answer Key

Name: ___________________

Units do not have to be included.

	4 points each	
1	An arithmetic series has eight terms. The first term is 2 and the last term is 58. Find the sum of the sequence.	240
2	What is the sum of the first eight pentagonal numbers?	288
3	Find the sum of: $1^2 + 2^2 + 3^2 + 4^2 + \ldots + 20^2$	2870
4	Solve the following series. $$S = \sum_{n=1}^{23} i^n$$	-1
5	Evaluate: $$\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+\ldots}}}}$$	3

	5 points each	
6	Find x given that $$\sum_{k=1}^{\infty}\left(\frac{5x}{2}\right)^{k-1} = 8$$	$\frac{7}{20}$
7	Solve for x given that $1 + 4 + 7 + \cdots + x = 925$	73
8	Group odd numbers as follows: {1}, {3,5}, {7,9,11}, {13,15,17,19} … Find the sum of the elements in the 38th set.	54,872
9	x + 4 and x - 3 are the first two terms of a geometric **sequence,** respectively. Find the values of x for which the **sequence** converges.	$x > -\frac{1}{2}$
10	Find the sum of the first 41 terms of the Fibonacci sequence given that the 43nd term is 433,494,437.	433,494,436

6 points each		
11	Consider the equation shown below $$S = \sqrt{12 + \cfrac{12}{\sqrt{12 + \cfrac{12}{\sqrt{12 + \cfrac{12}{\dots}}}}}}$$ Determine the largest integer less than or equal to S.	3
12	Let $z_k = a_k + b_k$ where a_k is an arithmetic sequence and b_k is a geometric sequence. Suppose the first term is $Z_1 = 2 + 3i$ and the 3rd term is $Z_3 = 22 - 3i$. Calculate the sum of the first 50 terms, S_{50}.	$12347 + 3i$
13	A ball bounces from a height of 4 meters and returns to 70% of its previous height on each bounce. Find the total distance travelled by the ball, in meters, until it is at rest. Express your answer in mixed number or improper fraction form.	$\frac{68}{3}$ or $22\frac{2}{3}$
14	What is the sum of all four-digit palindromes?	495,000
15	The midpoints of the sides of a 512 by 512 square are joined to form another square, and then the midpoints of the side of the small square are joined to form an even smaller square. If this process continues, find the sum of the perimeters of the first 23 squares. Express your answer in the form $a + b\sqrt{c}$	$4095 + 2047\sqrt{2}$

2018 – 2019 Log1 Contest Round 1

Mu Sequences and Series Answer Key

Name: ___________________

Units do not have to be included.

	4 points each	
1	An arithmetic series has eight terms. The first term is 2 and the last term is 58. Find the sum of the sequence.	240
2	What is the sum of the first eight pentagonal numbers?	288
3	Find the sum of: $1^2 + 2^2 + 3^2 + 4^2 + \ldots + 20^2$	2870
4	Solve the following series. $$S = \sum_{n=1}^{23} i^n$$	-1
5	In the showing of Marvel's "Avengers: Infinity War", one person sits in the first row. In the second row, there are 4 people and in the third row there are 7 people. If this pattern continues and there are 28 rows, how many people are there in total?	1162

	5 points each	
6	Find x given that $$\sum_{k=1}^{\infty} \left(\frac{5x}{2}\right)^{k-1} = 8$$	$\frac{7}{20}$
7	Solve for x given that $1 + 4 + 7 + \cdots + x = 925$	73
8	Group odd numbers as follows: {1}, {3,5}, {7,9,11}, {13,15,17,19} … Find the sum of the elements in the 38th set.	54,872
9	x + 4 and x - 3 are the first two terms of a geometric **sequence,** respectively. Find the values of x for which the **sequence** converges.	$x > -\frac{1}{2}$
10	If $f(x) = -x^3 + 3x^2 + 1$ and $g(x) = 5x$, evaluate $$\sum_{x=1}^{10} f''(g(x))$$	-39750

6 points each		
11	Consider the equation shown below $$S=\sqrt{12+\frac{12}{\sqrt{12+\frac{12}{\sqrt{12+\frac{12}{\cdots}}}}}}$$ Determine the largest integer less than or equal to S.	3
12	Let $z_k = a_k + b_k$ where a_k is an arithmetic sequence and b_k is a geometric sequence. Suppose the first term is $z_1 = 2 + 3i$ and the 3rd term is $z_3 = 22 - 3i$. Calculate the sum of the first 50 terms, S_{50}.	$12347 + 3i$
13	A ball bounces from a height of 4 meters and returns to 70% of its previous height on each bounce. Find the total distance travelled by the ball, in meters, until it is at rest. Express your answer in mixed number or improper fraction form.	$\frac{68}{3}$ or $22\frac{2}{3}$
14	What is the sum of all four-digit palindromes?	495,000
15	Given the function $f(x) = 3x^2 + 2x - 1$, determine the value of the series given below. $$S_{25}=\sum_{n=1}^{25}\sin\left(\frac{n\pi}{2}\right)f'(n)$$	80

2018 – 2019 Log1 Contest Round 1
Sequences and Series Solutions

Mu	Al	Th	Solution
1	1	1	$$\text{In general, } S_n = n\frac{a_1 + a_n}{2}$$ $$S_8 = \frac{8(2+58)}{2} = \frac{480}{2} = 240$$
2	2	2	The nth pentagonal number is determined by the formula $$P_n = \frac{n(3n-1)}{2}$$ Solving by brute force, 1 + 5 + 12 + 22 + 35 + 51 + 70 + 92 = 288
3	3	3	$$\sum_{n=1}^{20} n^2 = \frac{(n)(n+1)(2n+1)}{6} = \frac{20*21*41}{6} = 2870$$ Can also be solved through brute force methods.
4	4		The first four terms are $i = \sqrt{-1}, i^2 = -1, i^3 = -i, i^4 = 1$. These add to zero and this pattern continues for every set of 4 terms in the sequence. For $n = 23$, this is -1. $a_{21} + a_{22} = i + (-i) = 0$. Thus $S_{23} = \sum_{n=1}^{23} i^n = -1$
		4	This is a geometric series, therefore $$\sum_{n=0}^{\infty} 15^{-n} = \frac{a}{1-r}, \text{where } r = \frac{1}{15}$$ $$\sum_{n=0}^{\infty} 15^{-n} = \frac{1}{1-\frac{1}{15}} = 1\left(\frac{15}{14}\right) = \frac{15}{14}$$

5			Since $a_2 = 4$ and $a_3 = 7$, the common difference is 3. Thus $a_1 = 1$. The 28th term in this sequence is $$a_{28} = a_1 + (28-1)3 = 1 + 27(3) = 82$$ Therefore, $S_{28} = 28\frac{1+82}{2} = 14(83) = 1162$
	5	5	$$\text{Let } x = \sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+\ldots}}}}$$ $$\text{Then, } x = \sqrt{6+x}$$ $$\text{Thus, } x^2 = 6 + x$$ $$\text{Rearranging, } x^2 - x - 6 = 0$$ $$\text{Factoring, } (x-3)(x+2) = 0$$ $$\text{Taking the positive root, } x = 3$$
6	6	6	This may be equivalently expressed as $$\sum_{k=0}^{\infty}\left(\frac{5x}{2}\right)^k = 8$$ If this series is to converge, then it must be true that $$\frac{5x}{2} < 1$$ $$S = \frac{a_1}{1-r} = 8, \text{where } a_1 = 1 \text{ and } r = \frac{5x}{2}$$ $$\text{Thus, } \frac{1}{1-r} = 8, \text{so } 1 - r = \frac{1}{8} \rightarrow r = \frac{7}{8}$$ $$\text{Therefore, } \frac{5x}{2} = \frac{7}{8} \rightarrow \frac{14}{40} = x \rightarrow x = \frac{7}{20}$$

7	7	7	The nth term of this series is given by the formula $$a_n = a_1 + (n-1)d. \text{ Let } x = a_n$$ $$x = 1 + 3(n-1) = 3n - 2. \text{ We must solve for n to determine x.}$$ $$S_n = n\frac{a_1 + a_n}{2} \to 925 = n\frac{1+x}{2} \to 1850 = n(1+x)$$ $$1850 = n\big(1 + (3n-2)\big) = n(3n-1) = 3n^2 - n$$ $$3n^2 - n - 1850 = 0$$ Using the quadratic formula, $$n = \frac{1 \pm \sqrt{1 - 4(3)(1850)}}{2(3)} = \frac{1 \pm \sqrt{22201}}{6} = \frac{1 \pm 149}{6}$$ $$n = 25 \text{ or } -24\frac{2}{3}$$ The positive root must be taken. Thus, $x = 3(25) - 2 = 73$
8	8	8	The sequence of element sums is equivalent to the sequence of the first n cubes. For set n = 1, the sum is 1^3, for n = 2, the sum is 2^3. The 38^{th} set would have as its sum of elements, $$38^3 = 54872$$
9	9		$$u_1 = x + 4, u_2 = ru_1 \; r = \frac{x-3}{x+4}$$ $$\text{Therefore } r = \frac{u_2}{u_1} = \frac{x-3}{x+4}$$ The series will converge if $\lvert r \rvert < 1$ $$\left\lvert \frac{x-3}{x+4} \right\rvert < 1$$ $$\lvert x-3 \rvert < \lvert x+4 \rvert$$ $$(x-3)^2 < (x+4)^2$$ $$x^2 - 6x + 9 < x^2 + 8x + 16$$ $$-14x < 7$$ $$x > \frac{-1}{2}$$

		9	$$\text{Let } x_1 = \sum_{k=2}^{\infty} \frac{1}{n^k} = \sum_{k=0}^{\infty} \frac{1}{n^2}\left(\frac{1}{n}\right)^k$$ $$\text{Then } x_1 = \frac{\frac{1}{n^2}}{1-\left(\frac{1}{n}\right)}$$ $$x_1 = \frac{1}{n(n-1)}$$ $$\text{Let } x_2 = \sum_{n=2}^{\infty} x_1 = \sum_{n=2}^{\infty} \frac{1}{n(n-1)}$$ $$x_2 = \frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \cdots$$ $$x_2 = \left(\frac{1}{1}-\frac{1}{2}\right) + \left(\frac{1}{2}-\frac{1}{3}\right) + \left(\frac{1}{3}-\frac{1}{4}\right) + \left(\frac{1}{4}-\frac{1}{5}\right) + \cdots$$ $$\text{By inspection, } x_2 = 1$$
10			Evaluate the composite function f(g(x)) and then determine its 2nd derivative. $$f(g(x)) = -125x^3 + 75x^2 + 1$$ $$f'(g(x)) = -375x^2 + 150x$$ $$f''(g(x)) = -750x + 150$$ This can be used to define an arithmetic sequence. $$a_n = -600 - 750(n-1)$$ $$S_{10} = \frac{10}{2}(-600 - 7350) = 5(-7950) = -39750$$

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<td>10</td>
<td>10</td>
<td>
Consider the following sequence of numbers.
<table>
<tr><td>1</td><td>1</td><td>2</td><td>3</td><td>5</td><td>8</td><td>13</td><td>21</td><td>34</td></tr>
<tr><td>1</td><td>2</td><td>4</td><td>7</td><td>12</td><td>20</td><td>33</td><td>54</td><td>88</td></tr>
</table>

The top row is the Fibonacci sequence
The bottom row is the sum of the first n Fibonacci numbers.

A pattern appears where the sum of the first n Fibonacci numbers seems to be one less than the $(n^{th} + 2)$ Fibonacci number.

The general formula, which may be proven easily, is
$$\sum_{k=1}^{n} F_k = F_{n+2} - 1$$
Therefore,
$$\sum_{k=1}^{41} F_k = F_{43} - 1 = 433{,}494{,}437 - 1$$
$$\sum_{k=1}^{41} F_k = 433{,}494{,}436$$
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<td>11</td>
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<td>
$$S = \sqrt{12 + \frac{12}{S}}$$
$$S^2 = 12 + \frac{12}{S}$$
$$S^3 = 12S + 12$$
$$S^3 - 12S - 12 = 0$$
Because there is only 1 sign change in this equation, by Descartes' Rule of Signs, this polynomial has only one positive root.
$$S^3 - 12S = 12$$
$$S(S^2 - 12) = 12$$
It is simple to check that S is a number between 3 and 4. Therefore, the largest integer less than or equal to the answer is 3.
</td>
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</table>

12	12	12	If $z_1 = 2 + 3i$ and $z_3 = 22 - 3i$, it follows that $\mathrm{Re}(z(k)) = 10k - 8$ & $\mathrm{Im}(z(k)) = 3i^k$ Consider the real parts. $$\mathrm{Re}(S_{50}) = \frac{50}{2}(2 + 492) = 12350$$ Consider the imaginary parts. Every four terms add up to $0 + 0i$. Thus, for 48 terms, all the imaginary parts add to $0 + 0i$. Let $S' = \mathrm{Im}(S_{50})$ $$S' = \mathrm{Im}(z_{48}) + \mathrm{Im}(z_{49}) + \mathrm{Im}(z_{50})$$ $$S' = 0 + 3i^{49} + 3i^{50}$$ $$S' = 0 + 3i - 3 = -3 + 3i$$ It follows that, $$S_{50} = \mathrm{Re}(S_{50}) + S'$$ $$S_{50} = 12350 - 3 + 3i = 12347 + 3i$$
13	13	13	To find the total distance of the rebound portions of the motion, this is an infinite, converging geometric series. $$S = 2(4)\sum_{n=0}^{\infty}\left(\frac{7}{10}\right)^{n+1} = 8\left(\frac{7}{10}\right)\sum_{n=0}^{\infty}\left(\frac{7}{10}\right)^{n}$$ $$S = \frac{56}{10}\left(\frac{1}{1-\frac{7}{10}}\right) = \frac{56}{10}\left(\frac{10}{3}\right) = \frac{56}{3} \text{ meters}$$ Add in the original drop height of 4 meters. $$S_{\mathrm{Total}} = \frac{12}{3} + \frac{56}{3} = \frac{68}{3}$$
14	14		A four digit palindrome is a number of the form abba where (a,b) are natural numbers such that $0 \le b < 10$ and $1 \le a < 10$, whose value is $1001a + 110b$ Summing such values across the ninety combinations for (a,b) gives the result. The sum, expressed algebraically, is $$10(1001)\sum_{1}^{9} a + 9(110)\sum_{0}^{9} b$$ $$\text{The sum equals } 10010\frac{9*10}{2} + 990\frac{9*10}{2} = 495{,}000$$

		14	$$Let\ S = \sum_{n=10}^{\infty} [\log_3(n-1) - \log_3 n]$$ Then, $S = (\log_3 9 - \log_3 10)$ + $(\log_3 10 - \log_3 11)$ + $(\log_3 11 - \log_3 12)$ + … One might presume that the interior terms cancel all the way to infinity, leaving the first term, which is 2. However, associative and commutative laws will lead to inconsistencies when expressing the series in this fashion. If we express the infinite series as a finite sum, only the first and last terms remain. Therefore, $$S = \lim_{n\to\infty} (2 - \log_3(n+9))$$ It is evident that the infinite sum diverges to $-\infty$.

15			Evaluate the derivative of the function, f. $$f'(x) = 6x + 2$$ The derivative defines an arithmetic sequence. $$a_n = 8 + 6(n - 1)$$ As an ordered list, this becomes $$a_n = \{8,14,20,\ldots,152\}$$ The sine factor in the summation may also be expressed as an ordered list, which becomes $$b_n = \{1,0,-1,0,\ldots,1\}$$ The sine function in the summation stipulates that for every value of n that is even, the combined term equals 0. Thus, we only need to add the odd terms of the sum. Furthermore, for the set of odd numbers {3, 7, 11, ..., 23}, the sine factor equals -1. For the set of odd numbers {1, 5, 9, …, 25}, the sine factor equals +1. We can define arithmetic sequences for the former set (6 elements) and latter set (7 elements) of listed odd numbers. $$c_n = 20 + 24(n - 1);\text{ for } n = 1 \text{ to } 6$$ $$d_n = 8 + 24(n - 1);\text{ for } n = 1 \text{ to } 7$$ Thus, $$S_{25} = \sum_{n=1}^{25} \sin\left(\frac{n\pi}{2}\right) f'(n) = S_{d(7)} - S_{c(6)}$$ $$S_{d(7)} = \frac{7}{2}(d_1 + d_7) = \frac{7}{2}(8 + 152) = 560$$ $$S_{c(6)} = \frac{6}{2}(c_1 + c_6) = \frac{6}{2}(20 + 140) = 480$$ Therefore, $S_{25} = 560 - 480 = 80$

	15	15	There are 2 sets of squares formed with the following perimeters. A: 2048, 1024, 512, 256, … B: $1024\sqrt{2}, 512\sqrt{2}, 256\sqrt{2}, ...$ The general formula for a geometric series; $$S_n = a\left(\frac{1-r^n}{1-r}\right)$$ For set A; excluding the largest square of 2048 There will be 11 squares in this set. $$S_{11-A} = 1024\left(\frac{1-\left(\frac{1}{2}\right)^{11}}{1-\frac{1}{2}}\right) = 1024(2)\left(1-\frac{1}{2048}\right) = 2048\left(\frac{2047}{2048}\right) = 2047$$ For set II; There will also be 11 squares in this set. The only difference between this set and the set calculated above is a factor of $\sqrt{2}$ $$S_{11-B} = 2047\sqrt{2}$$ The sum of all 23 squares will be the sum of these two sets plus the largest square. $$S_{Total} = 2048 + 2047\left(1+\sqrt{2}\right) = 4095 + 2047\sqrt{2}$$