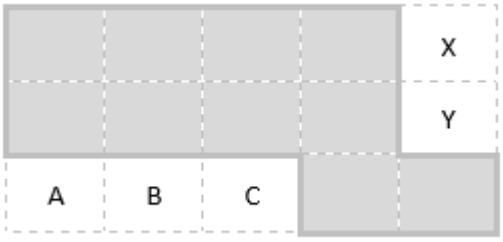
2018 – 2019 Log1 Contest Round 2

Theta Applications

Name: _____

Units do not have to be included.

4 points each		
1	On his morning commute, Eduardo's drive to work with an average speed of 35 mph for 15 miles. The afternoon commute home takes significantly longer at an average speed of 20 mph. Calculate the total time that Eduardo is on the road for his morning and afternoon commutes. Express your answer as an improper fraction.	
2	Bruce bought a 9-inch x 15-inch rectangular pizza that is cut into 15 square slices of equal area, as shown in the diagram below. Thor eats 2 slices, X and Y, and Bruce eats 3 slices, A, B, and C. Calculate the exterior perimeter of the shape formed by the leftover slices. 	
3	Jeffrey is given the number 5^{54}. James is given the number 9^{64}. Jaqueline is given the number 2^{15}. Their teacher tells them to find the units' digit when all the numbers are added together. What number should Jeffrey, James, and Jacqueline get?	
4	Natasha has a cylindrical bowl. The radius of the bowl, on its inner surface, is 5 cm. In the bowl, there is a hemispherical scoop of vanilla ice cream with a radius of 3 cm. Natasha must run an errand and forgets to put her bowl of ice cream in the freezer. When she returns, the ice cream has completely melted. If all the ice cream from the original scoop has melted and it forms a cylindrical volume of liquid in the bowl, calculate the depth of the melted ice cream.	
5	Lisa is taking a Music Theory class. On her first quiz, she received a grade of 75%. On her second quiz, she received a 55%. She has her upcoming final exam and wants to obtain a passing grade. This class requires a 70% average on all coursework to pass. What percentage must she achieve on her final if quizzes all hold the same weight, and her final is 60% of her grade? Express the answer as a mixed number.	

5 points each		
6	Jennie has a two-digit number. The units digit of this number is 4 more than the tens digit. Jennie decides to reverse the digits causing the new number to be 21 more than twice the original number. What is the largest prime factor of the product of the original number and the reversed number?	
7	A company called Banana says that their phones only fail once every million years. If there is a probability of phone failure of one in one-million per year, and if it costs $100 to replace a phone, and your company has 20,000 phones, how much should you expect to spend to replace phones in the first 6 years?	
8	Gerald loves to watch YouTube video shorts. On October 1st, he watched one video. On even days, he watches four times the number of videos as the previous day. Except for October 1st, on odd days, he watches half the number of videos as the previous day. If he keeps watching videos with this pattern, how many movies will Gerald watch during the month of October?	
9	On planet Cindy, sound travels at a rate of 40 "nancys" per second. 4 "nancys" equal 8 "kims", and 80 "kims" are equal to 10 "jenns". How many seconds will it take for Hanh to hear the echo from her yelling at a wall that is 100 "jenns" away?	
10	Irene, Joy, and Katy won a pile of candy at a math competition. They decide to divide the candy in a ratio of 3: 2: 1 respectively. However, because of poor planning, they come at different times to claim their candy, and each assumes she is the first to arrive. If each takes what she believes to be the correct amount of candy, what fraction of the candy will go unclaimed?	

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11	Heinrich and Gustavo are working together to trim a garden. Heinrich can trim a garden alone in 10 hours. Working together, they can trim the garden in 7 hours. How many hours would it take Gustavo to trim the garden alone?	
12	The total amount of digits used to number a book is 1,188. If the pages are numbered consecutively beginning with 1, how many pages does the book have?	
13	A party is being planned for eight friends; among the eight attendees, Jo is dating Mo and Ro is dating Vo. If all eight people are assigned random seats around a circular table, what is the probability that both Jo and Ro get to sit next to their dates?	
14	Murphy is knitting a blanket. He wants the blanket to cover the bottom of his sister's crib and the inside vertical surface. The crib's is 2 feet deep and the horizontal cross-sectional area is in the shape of an ellipse with a circumference of approximately $\frac{33}{8}\pi$ feet and is given by the following equation. $$36x^2 + 100y^2 = 225$$ If x and y are measured in feet, what is the area that the blanket must cover, in square feet.	
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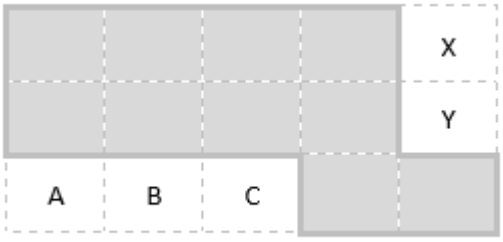
2	Bruce bought a 9-inch x 15-inch rectangular pizza that is cut into 15 square slices of equal area, as shown in the diagram below. Thor eats 2 slices, X and Y, and Bruce eats 3 slices, A, B, and C. Calculate the exterior perimeter of the shape formed by the leftover slices. 	
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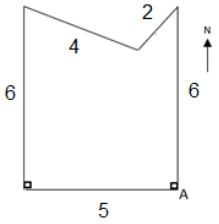
9	Consider a farm house with the following cross-sectional profile from above. The dimensions given in the diagram are in meters and the bottom two vertices form right angles. A cow is tied to the corner of the farm house, at point A, with a rope that is 11 meters long. If the cow is free to walk around the farm, limited by the length of the rope, determine the maximum amount of grazing area for the cow. Assume that the cow cannot enter the farm house. 	
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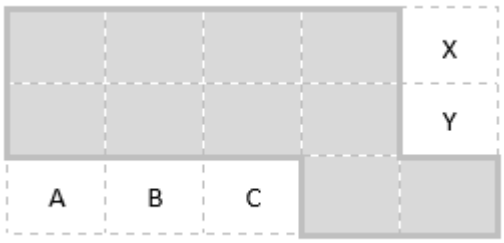
2	Bruce bought a 9-inch x 15-inch rectangular pizza that is cut into 15 square slices of equal area, as shown in the diagram below. Thor eats 2 slices, X and Y, and Bruce eats 3 slices, A, B, and C. Calculate the exterior perimeter of the shape formed by the leftover slices. 	
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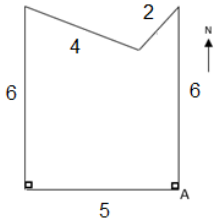
9	Consider a farm house with the following cross-sectional profile from above. The dimensions given in the diagram are in meters and the bottom two vertices form right angles. A cow is tied to the corner of the farm house, at point A, with a rope that is 11 meters long. If the cow is free to walk around the farm, limited by the length of the rope, determine the maximum amount of grazing area for the cow. Assume that the cow cannot enter the farm house. 	
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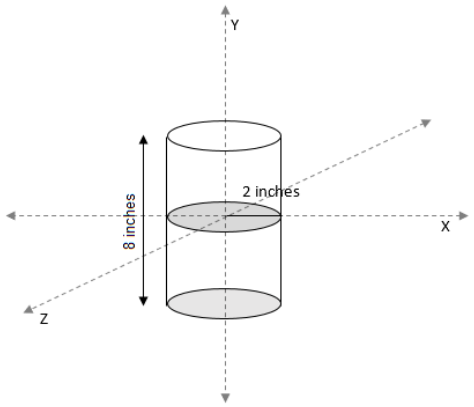
15	A sealed cylinder, with a radius of 2 inches and a height of 8 inches, is half-filled with water. The cylinder is rotated, clockwise starting from rest, in the XY plane with an axis of rotation that is the Z axis with an angular acceleration of $\alpha = \frac{\pi}{6}$ radians per second-squared. The top surface remains completely parallel to the XZ plane as depicted the diagram. If the angular velocity, $\omega = \frac{d\theta}{dt}$, is given by the equation $\omega^2 = 2\alpha\theta$, what is the rate of change, $\frac{dA}{dt}$, of the top surface of the water when the cup has rotated through $\theta = \frac{\pi}{6}$ radians? At this angle, the surface is an ellipse. 	

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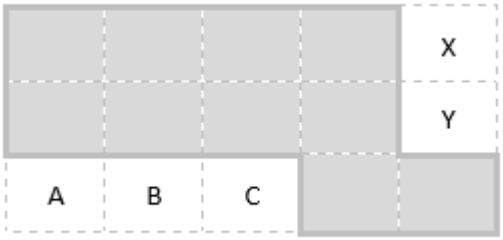
2	Bruce bought a 9-inch x 15-inch rectangular pizza that is cut into 15 square slices of equal area, as shown in the diagram below. Thor eats 2 slices, X and Y, and Bruce eats 3 slices, A, B, and C. Calculate the exterior perimeter of the shape formed by the leftover slices. 	48
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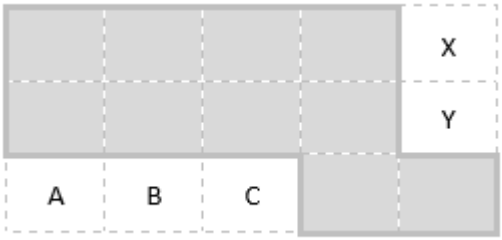
2	Bruce bought a 9-inch x 15-inch rectangular pizza that is cut into 15 square slices of equal area, as shown in the diagram below. Thor eats 2 slices, X and Y, and Bruce eats 3 slices, A, B, and C. Calculate the exterior perimeter of the shape formed by the leftover slices. 	48
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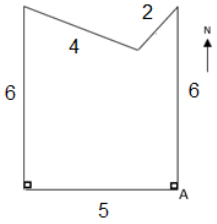
9	Consider a farm house with the following cross-sectional profile from above. The dimensions given in the diagram are in meters and the bottom two vertices form right angles. A cow is tied to the corner of the farm house, at point A, with a rope that is 11 meters long. If the cow is free to walk around the farm, limited by the length of the rope, determine the maximum amount of grazing area for the cow. Assume that the cow cannot enter the farm house. 	$106\pi + \frac{\sqrt{231}}{4}$
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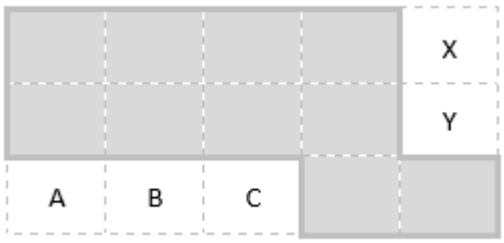
2	Bruce bought a 9-inch x 15-inch rectangular pizza that is cut into 15 square slices of equal area, as shown in the diagram below. Thor eats 2 slices, X and Y, and Bruce eats 3 slices, A, B, and C. Calculate the exterior perimeter of the shape formed by the leftover slices. 	48
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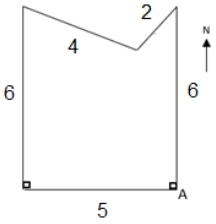
9	Consider a farm house with the following cross-sectional profile from above. The dimensions given in the diagram are in meters and the bottom two vertices form right angles. A cow is tied to the corner of the farm house, at point A, with a rope that is 11 meters long. If the cow is free to walk around the farm, limited by the length of the rope, determine the maximum amount of grazing area for the cow. Assume that the cow cannot enter the farm house. 	$106\pi + \frac{\sqrt{231}}{4}$
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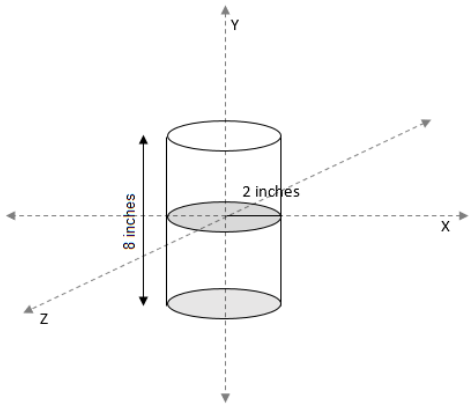
12	The total amount of digits used to number a book is 1,188. If the pages are numbered consecutively beginning with 1, how many pages does the book have?	432
13	A party is being planned for eight friends; among the eight attendees, Jo is dating Mo and Ro is dating Vo. If all eight people are assigned random seats around a circular table, what is the probability that both Jo and Ro get to sit next to their dates?	$\frac{2}{21}$
14	Eight coins fit exactly on a rectangular sheet of paper when lined up in 4 columns and two rows. Each coin is in the shape of a regular dodecagon, a polygon containing 12 sides. What fraction of the paper sheet will not be covered?	$\frac{1}{4}$
15	A sealed cylinder, with a radius of 2 inches and a height of 8 inches, is half-filled with water. The cylinder is rotated, clockwise starting from rest, in the XY plane with an axis of rotation that is the Z axis with an angular acceleration of $\alpha = \frac{\pi}{6}$ radians per second-squared. The top surface remains completely parallel to the XZ plane as depicted the diagram. If the angular velocity, $\omega = \frac{d\theta}{dt}$, is given by the equation $\omega^2 = 2\alpha\theta$, what is the rate of change, $\frac{dA}{dt}$, of the top surface of the water when the cup has rotated through $\theta = \frac{\pi}{6}$ radians? At this angle, the surface is an ellipse. 	$\frac{4\pi^2\sqrt{2}}{9}$

2018 – 2019 Log1 Contest Round 2
Applications Solutions

Mu	Al	Th	Solution
1	1	1	The travel times are calculated using $t = \frac{d}{v}$ $t_{morning} = \frac{15}{35}\text{hour} = \frac{3}{7}\text{ hour}$ $t_{afternoon} = \frac{15}{20}\text{hour} = \frac{3}{4}\text{ hour}$ $t_{total} = \frac{3}{7} + \frac{3}{4} = \frac{12}{28} + \frac{21}{28} = \frac{33}{28}\text{ hours}$
2	2	2	Because each slice is a square, each side must be 3 inches in length. The perimeter of the shaded area is $16 \text{ x } 3 \text{ inches} = 48 \text{ inches}$
3	3	3	The units' digit for 5^{54} is 5 because 5 to the power of any number has a units' digit of 5. The unit's digit for 9^{64} is 1 because 9 to the power of an even number has a units' digit of 1. The units' digit for 2^{15} is 8 because 2 to the power of any number will have a units' digit of 2, 4, 8, or 6 for $2^1, 2^2, 2^3,$ and $2^4,$ respectively. Starting with 2^5, this pattern repeats. Thus, with 2^{15}, the remainder after dividing the exponent by 4 is 3. Thus, 2^{15} ends in 8. Adding the units' digit for the three numbers yields 5 + 1 + 8 = 14. Therefore, the units' digit would be 4.
4	4		In completing 180 revolutions in 1 minute, the pulley completes 3 revolutions in 1 second. The belt travels a distance equal to 3 times the circumference of the pulley in 1 second. Thus, $3(\pi d) = 36$ $d = \frac{12}{\pi}$

		4	The volume of the scoop is equal to that of a hemisphere. $$V_{scoop} = \frac{2}{3}\pi r^3 = \frac{2}{3}\pi(3)^3 = \frac{54}{3}\pi$$ After melting, the volume is a cylinder. $$V_{melted} = \pi r^2 h = \pi(5)^2 h$$ Since the volume of ice cream before and after melting should not change, $V_{scoop} = V_{melted}$ $$\pi r^2 h = \frac{54}{3}\pi$$ $$h = \frac{54}{3r^2} = \frac{54}{3(5^2)} = \frac{54}{75} = \frac{18}{25}$$
5			Take the derivative of the position function. $$s'(t) = 4 - 2t$$ The instantaneous speed at t = ¾ hour hours is $$s'(t) = 4 - 2\left(\frac{3}{4}\right) = 4 - \frac{3}{2} = \frac{5}{2} \text{ kph}$$
	5	5	Lisa's average for the first two quizzes is $\frac{75+55}{2} = 65\%$ Combined with Lisa's final, which is 60% of her grade, the first two quizzes must be 40% of her grade. The weighted average of these quizzes and final must equal 70% to pass the class. Let x be the grade she needs on her final: $$(0.4)(65) + 0.6x = 70$$ $$26 + \frac{3}{5}x = 70$$ $$\frac{3}{5}x = 44$$ $$x = \frac{220}{3} = 73\frac{1}{3}$$

6	6	6	Let $t = $ tens digit and $u = $ units digit Therefore, $10t + u = $ original number and $10u + t = $ reversed number. We establish a system of equations. $$u = 4 + t$$ $$10u + t = 2(10t + u) + 21.$$ $$10u + t = 20t + 2u + 21$$ $$8u - 19t = 21$$ $$8(4 + t) - 19t = 21$$ $$32 + 8t - 19t = 21$$ $$-11t = -11$$ $$t = 1$$ $$u = 5$$ The original number is $10(1) + 5 = 15$, with prime factors of 3 and 5. The reversed number is 51, with prime factors of 3 and 17. Although the problem asks for the largest prime factor of the product of the original and reversed numbers, the largest prime factor will come from the reversed number. The answer is 17.
7	7	7	This is an expectation value problem. Let X = event of phone failure in one year. Once in a million years is equivalent to a failure of one-millionth of a phone every year, or $$P(X) = 10^{-6}$$ If the company has 20,000 phones, the expectation value for one year is given by $$E(X) = nP(X) = 20000x10^{-6}$$ Since every failed phone is replaced, the total population of phones remains 20,000 for all 6 years. Let T = total number of phone failures in 6 years. $$T = 6E(x) = 120000x10^{-6}$$ Let C = total cost to replace all failed phones. $$C = T(\$100) = \$12000000x10^{-6} = \$12$$

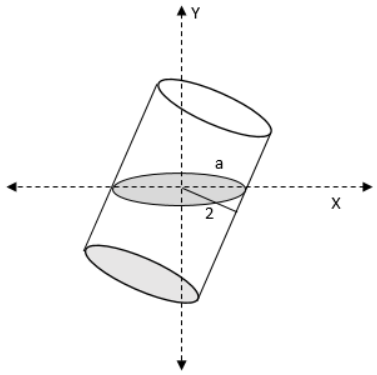
8	8	8	The sequence is described below. $a_1 = 1, a_2 = 4, a_3 = 2, a_4 = 8, a_5 = 4, a_6 = 16, \ldots.$ $a_{odd} = \{1,2,4,8, \ldots\}$ $a_{even} = \{4,8,16,32, \ldots\}$ These partial sequences are geometric with a common ratio of 2. $S = \frac{a_1(1 - r^n)}{1 - r}$ $S_{odd} = \frac{1(1 - 2^{16})}{1 - 2} = \frac{1 - 2^{15}}{-1} = \frac{1 - 65536}{-1} = 65535$ $S_{even} = \frac{4(1 - 2^{15})}{1 - 2} = \frac{4(1 - 2^{15})}{-1}$ $S_{even} = \frac{4(1 - 32768)}{-1} = 4(32767) = 131068$ $S = S_{odd} + S_{even} = 196603$
9	9		The cow can reach the corner opposite of point A because the rope is 11 meters, matching the length of adjacent sides of the farm house. If the cow starts at the corner opposite of point A and walks counter-clockwise with the rope maximally stretched, it traces a quarter-circle of radius 6 meters until the rope points towards a west bearing. The area of this region is given below. $A_1 = \frac{1}{4}(6^2\pi) = 9\pi \text{ m}^2$ The cow continues its counter-clockwise path, tracing three-quarters of a circle until its rope points towards a north bearing. The area of this region is given below. $A_2 = \frac{3}{4}(11^2\pi) = \frac{363}{4}\pi \text{ m}^2$ The cow continues its counter-clockwise path, tracing a final, one-quarter circle until it reaches the opposite corner of the farm house again. Since the bottom two vertices are right angles and the two N-S oriented sides are both equal in length, the distance between the top two vertices is also 5 m. The area of this region is given below. $A_3 = \frac{1}{4}(5^2\pi) = \frac{25}{4}\pi \text{ m}^2$ The cow can also graze in the triangular region on the north end of the farm house. This area is found using Heron's formula. $A_4 = \sqrt{\frac{5 + 4 + 2}{2}\left(\frac{11}{2} - 5\right)\left(\frac{11}{2} - 4\right)\left(\frac{11}{2} - 2\right)}$ $A_4 = \sqrt{\frac{11}{2}\left(\frac{1}{2}\right)\left(\frac{3}{2}\right)\left(\frac{7}{2}\right)} = \sqrt{\frac{231}{16}} = \frac{\sqrt{231}}{4} \text{ m}^2$ $A = A_1 + A_2 + A_3 + A_4$ $A = \left(9 + \frac{363}{4} + \frac{25}{4}\right)\pi + \frac{\sqrt{231}}{4} \text{ m}^2 = 106\pi + \frac{\sqrt{231}}{4} \text{ m}^2$

		9	This is a unit conversion problem. $$\text{Let "nancys"} = \text{n, "kims"} = \text{k, and "jenns"} = \text{j}$$ $$100\text{ j}\left(\frac{80\text{ k}}{10\text{ j}}\right)\left(\frac{4\text{ n}}{8\text{ k}}\right)\left(\frac{1\text{ sec}}{40\text{ n}}\right) = 10\text{ sec}$$ Because the echo must travel back to Hanh, the total distance is double, so the true answer is 20 sec.
10			The length and width of the square base can be defined by x and the height can be defined by y. $$V = \text{length x width x depth} \rightarrow x^2 y \rightarrow 1372 = x^2 y$$ (1 litre $= 1000\text{ cm}^3$, therefore 1.372 liters = 1372 cm^3) The surface area can be defined as: $A = x^2 + 4xy$ From the volume equation, we know that $y = \frac{1372}{x^2}$ Therefore, $A = x^2 + 4x\left(\frac{1372}{x^2}\right) \rightarrow A(x) = x^2 + 5488x^{-1}$ To find the minimum, we need to take the derivative and make it equal to 0. $$A'(x) = 2x - 5488x^{-2}$$ $$0 = 2x - 5488x^{-2} \rightarrow 2x = 5488x^{-2}$$ $$2x^3 = 5488 \rightarrow x^3 = 2744 \rightarrow x = 14$$ Using the sign diagram test to check: $$A"(x) = 2 + 10976\,x^{-3} = 2 + \frac{10976}{x^3}$$ $$A"(14) = 2 + \frac{10976}{14^3} = 6$$ $6 > 0$ which indicates that we have a local minimum. Therefore, the dimensions of the container are 14, 14, and 7. $$14 + 14 + 7 = 35$$
	10	10	The ratios are 3: 2: 1 so Irene, Joy, and Katy believe that they need to take $\frac{1}{2}, \frac{1}{3}, \text{and}\,\frac{1}{6}$ of the pile, respectively. Consider that Irene, Joy, and Katy come to get their candy, in this order. After Irene leaves, one-half the candy pile remains. After Joy leaves, two-thirds of the previous pile remains, or one-third of the original pile of candy. After Katy leaves, five-sixths of the previous pile remains, or five-eighteenths of the original pile. The sequence of candy claims follows this pattern. $$\text{Remaining candy} = 1\left(\frac{1}{2}\right)\left(\frac{2}{3}\right)\left(\frac{5}{6}\right) = \frac{5}{18}$$ There is no reason to check every combination of the three persons as this just re-orders the multiplication of factors given above. Since multiplication is a commutative operation, the results will always be the same.

11	11	11	Working alone, Heinrich will trim one-tenth of the garden in 1 hour. His rate of working is $R_H = \frac{1}{10}$ For Gustavo, let T = be the number of hours it would take him to trim the garden alone. His rate of working is $R_G = \frac{1}{T}$ When they work together, they will trim one-seventh of the garden in 1 hour. Their combined rate of working is $R = \frac{1}{7}$ Combining rates, $\frac{1}{10} + \frac{1}{T} = \frac{1}{7} \rightarrow \frac{1}{T} = \frac{1}{7} - \frac{1}{10} = \frac{10}{70} - \frac{7}{70} = \frac{3}{70}$ $$T = \frac{70}{3} \text{ hours}$$
12	12	12	Pages 1 through 9 contain a total of 9 digits. Pages 10 through 99 contain 180 digits (2 digits for 90 numbers, therefore 2*90 = 180). Therefore, the total number of digits between 1-99 would 189. That leaves (1188 – 189) or 999 digits, which would be 333 3-digit numbers. Starting with 100, the 333rd three-digit number is 432.
13	13	13	The sample space is the number of ways the 8 people can be situated around the table which is (8-1)! = 7! The number of satisfying situations can be found by initially treating the couples as single people. That gives 5! arrangements which is multiplied by four as each couple can be seated in two ways. $\frac{4*5!}{7!} = \frac{2}{21}$

14	14		The area of a regular dodecagon is given by the formula $A_{12} = 3x^2$, where x is the circumradius. This can be derived as follows. Consider that the dodecagon can be divided into 12 isosceles triangles with a 30-degree angle at the center of the dodecagon. The area of this triangle is $$A_1 = \frac{1}{2} x * x * \sin 30 = \frac{x^2}{4}$$ Multiply by 12 for each triangle in the dodecagon. $$A_{12} = 3x^2$$ Let A = total area of 8 coins $$A = 8 \times 3x^2 = 24x^2$$ Let B = the area of the rectangular sheet $$B = 8x \times 4x = 32x^2$$ The fraction covered is $\frac{A}{B} = \frac{24x^2}{32x^2} = \frac{3}{4}$ Therefore, the fraction not covered is $\frac{1}{4}$

		14	The area of an ellipse is given by the formula $$A = \pi ab$$ Where (a) and (b) are the semi-major and semi-minor axes, respectively. To find the values of (a) and (b), we must convert the equation into standard form, $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ $$\frac{36}{225}x^2 + \frac{100}{225}y^2 = 1$$ $$\frac{x^2}{\frac{225}{36}} + \frac{y^2}{\frac{225}{100}} = 1$$ Thus, $$a^2 = \frac{225}{36} \text{ and } b^2 = \frac{225}{100}$$ And $$a = \frac{5}{2} \text{ and } b = \frac{3}{2}$$ $$A_1 = \frac{15}{4}\pi \text{ ft}^2$$ The area of the interior, vertical surface is just a rectangle of width 2 feet and length $\frac{33}{8}\pi$ feet. Its area is $$A_2 = \frac{33}{4}\pi \text{ ft}^2$$ $$A = A_1 + A_2 = \frac{48}{4}\pi \text{ ft}^2 = 12\pi \text{ ft}^2$$

15			The surface forms an ellipse whenever it stays away from the top and bottom surfaces of the cylinder. At $\frac{\pi}{6}$ radians, the cup is oriented as described in the picture. During this rotation, the surface remains an ellipse. To verify, the angle of a diagonal of the cylinder is $\tan^{-1}\left(\frac{4}{2}\right)$. By inspection, this angle is greater than $\frac{\pi}{4}$ radians. If the cylinder rotates less than this angle, which it does, the semi-minor axis will not extend to either the top or bottom surface.  Through the rotation, the semi-minor axis will remain a constant 2 inches. The triangle formed by the semi-major axis and the radius of the cup is a right triangle. Therefore, $a = \frac{2}{\cos\theta}$ The area of an ellipse is $A = \pi ab = \frac{4\pi}{\cos\theta}$. Furthermore, $$\frac{dA}{dt} = 4\pi(\cos\theta)^{-2}\sin\theta\frac{d\theta}{dt} = 4\pi\sec\theta\tan\theta\frac{d\theta}{dt}$$ Since $\omega = \frac{d\theta}{dt}$, determine the angular velocity. $$\omega^2 = 2\alpha\theta = 2\left(\frac{\pi}{6}\right)\left(\frac{\pi}{6}\right) = \frac{\pi^2}{18} \rightarrow \omega = \frac{\pi\sqrt{2}}{6} = \frac{d\theta}{dt}$$ $$\frac{dA}{dt} = 4\pi\sec\theta\tan\theta\frac{d\theta}{dt} = 4\pi\left(\sec\frac{\pi}{6}\right)\left(\tan\frac{\pi}{6}\right)\left(\frac{\pi\sqrt{2}}{6}\right)$$ $$\frac{dA}{dt} = 4\pi\left(\frac{2}{\sqrt{3}}\right)\left(\frac{1}{\sqrt{3}}\right)\left(\frac{\pi\sqrt{2}}{6}\right) \rightarrow 4\pi\left(\frac{2}{3}\right)\left(\frac{\pi\sqrt{2}}{6}\right) = \frac{4\pi^2\sqrt{2}}{9}$$
	15	15	Let $N = \overline{abcd} = 1000a + \overline{bcd}$, such that $\frac{N}{9} = \overline{bcd}$ Then $$1000a + \overline{bcd} = 9\overline{bcd}$$ $$1000a = 8\overline{bcd}$$ $$125a = \overline{bcd}$$ Since $100 \leq \overline{bcd} < 1000$, this implies that $100 \leq 125a < 1000$ Thus, $a = \{1,2,3,4,5,6,7)$