2018 – 2019 Log1 Contest Round 3

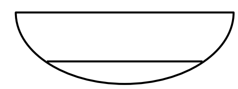
Theta Individual

Name: ____________________

Units do not have to be included.

4 points each		
1	If angles A and B are supplementary, and the measure of angle A is 57º, what is the measure, in degrees, of angle B?	
2	Jessica drove halfway home at 20 miles per hour, then sped up and drove the rest of the way at 30 miles per hour. What was her average speed for the entire trip?	
3	An incredibly large bucket has 24 gallons salted apple juice. 6% of this volume is salt. How many gallons of unsalted apple juice must be added such that the bucket is only 4% salt by volume?	
4	How many ways are there to arrange the letters in the word PIZZA?	
5	How many of the positive integer factors of 4200 are multiples of 4?	

5 points each		
6	100 students were surveyed on what type of movies they liked given the options of sci-fi, horror, and romance. From the results, 72 like sci-fi, 66 like horror, 58 like romance, 49 like sci-fi and horror, 43 like horror and romance, 41 like sci-fi and romance, and 28 like sci-fi, horror, and romance. How many students do not like any of the three genres?	
7	The arithmetic mean of three test scores is a 92. After taking the fifth test, the arithmetic mean of all test scores is now 55.8. Find the score of the fifth test, **in decimal form**, if the fourth score is 1.5x of the fifth score.	
8	Let $a_1 = x, a_2 = y$, and $a_3 = z$ be the first three terms of a geometric sequence. If $x + y + z = \frac{7}{3}$ and $x^2 + y^2 + z^2 = \frac{91}{9}$, determine the sum $x + z$.	
9	Find the sum of all the elements of the first 10 rows of Pascal's triangle, if the first row only contains 1.	
10	Find the arithmetic mean of the 2018 smallest non-negative integers.	

6 points each		
11	Consider the expression $\frac{4m^2+2m+15}{2m+3}$. What is the sum of all possible integer values of m that make the expression also an integer?	
12	Evaluate N mod 60 given that $N = 1! + 2! + 3! + \cdots + 525!$	
13	When $G(x) = x^n + 3x^2 + Ax + 6$ is divided by $(x + 1)$, the remainder is 14. When $G(x)$ is divided by $(x - 1)$ the remainder is 6. Find A and n given that $25 < n < 28$.	
14	The semicircle shown has a radius of r inches, and the chord AB is parallel to diameter CD. If the length of AB is 25% shorter than the length of CD, what is the shortest distance between AB and CD in terms of r? Note: figure not drawn to scale 	
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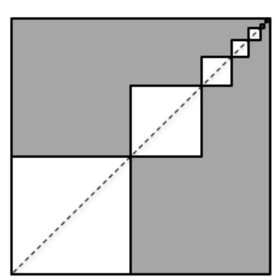
9	The diagram below contains a pattern of squares. Assume the squares are oriented so that all vertical edges are parallel to each other. The largest square is shaded dark gray and has a perimeter of 24. Within it are an infinite sequence of smaller squares, connected by adjacent vertices, as depicted in the diagram. In this sequence of smaller squares, the ratio of the side lengths of consecutive squares is $\frac{7}{12}$. The set of vertices of ALL squares that intersect the dotted line are collinear. Determine the area of the bottom-left square. 	
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14	Let **i** and **j** represent unit vectors of 1 km in the east-west and north-south directions, respectively. The control tower of an airport is at (0,0). At 11:00 AM an aircraft is 300 km west and 200 km south. It is flying parallel to the vector $\mathbf{f} = -2\mathbf{i} - 3\mathbf{j}$ with speed of $30\sqrt{13}\,\frac{\text{km}}{\text{hr}}$. Determine the clock-time, before or after 11:00 AM, of the aircraft's closest approach to the control tower. Please state your answer as a clock-time to the nearest minute (e.g. 6:00 AM).	
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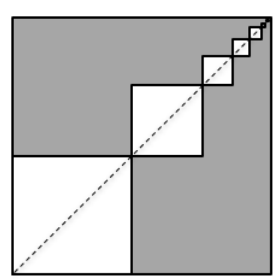
9	The diagram shown contains a pattern of squares. Assume the squares are oriented so that all vertical edges are parallel to each other. The largest square is shaded dark gray and has a perimeter of 24. Within it are an infinite sequence of smaller squares, connected by adjacent vertices, as depicted in the diagram. In this sequence of smaller squares, the ratio of the side lengths of consecutive squares is $\frac{7}{12}$. The set of vertices of ALL squares that intersect the dotted line are collinear. Determine the area of the bottom-left square. 	
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4	How many ways are there to arrange the letters in the word PIZZA?	60
5	How many of the positive integer factors of 4200 are multiples of 4?	24

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9	Find the sum of all the elements of the first 10 rows of Pascal's triangle, if the first row only contains 1.	1023
10	Find the arithmetic mean of the 2018 smallest non-negative integers.	$\frac{2017}{2}$

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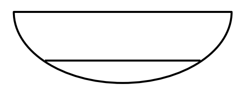
14	The semicircle shown has a radius of r inches, and the chord AB is parallel to diameter CD. If the length of AB is 25% shorter than the length of CD, what is the shortest distance between AB and CD in terms of r? Note: figure not drawn to scale 	$\frac{\sqrt{7}}{4}r$
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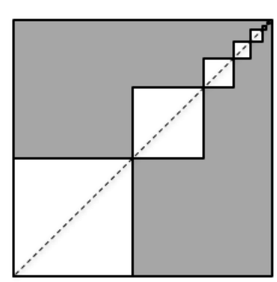
9	The diagram shown contains a pattern of squares. Assume the squares are oriented so that all vertical edges are parallel to each other. The largest square is shaded dark gray and has a perimeter of 24. Within it are an infinite sequence of smaller squares, connected by adjacent vertices, as depicted in the diagram. In this sequence of smaller squares, the ratio of the side lengths of consecutive squares is $\frac{7}{12}$. The set of vertices of ALL squares that intersect the dotted line are collinear. Determine the area of the bottom-left square. 	$\frac{25}{4}$
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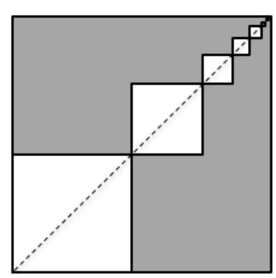
9	The diagram shown contains a pattern of squares. Assume the squares are oriented so that all vertical edges are parallel to each other. The largest square is shaded dark gray and has a perimeter of 24. Within it are an infinite sequence of smaller squares, connected by adjacent vertices, as depicted in the diagram. In this sequence of smaller squares, the ratio of the side lengths of consecutive squares is $\frac{7}{12}$. The set of vertices of ALL squares that intersect the dotted line are collinear. Determine the area of the bottom-left square. 	$\frac{25}{4}$
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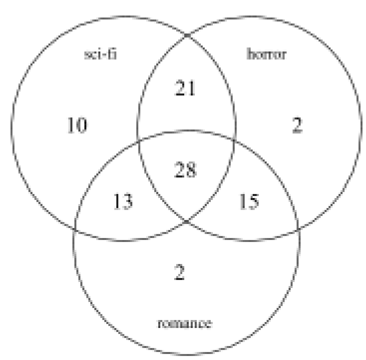
15	Evaluate $\int_0^4 x^2 e^x \, dx$	$10e^4 - 2$

2018 – 2019 Log1 Contest Round 3
Individual Solutions

Mu	Al	Th	Solution
1	1	1	The sum of supplementary angles is 180 degrees. Therefore, $$180° - 57° = 123°$$
2	2	2	If Jessica drove for 60 miles, she would have driven 20 miles for 1.5 hours and 30 miles for 1 hour. This gives us 60 miles in 2.5 hours. $$\mathrm{v} = \frac{\text{distance}}{\text{time}} = \frac{60\text{ miles}}{2.5\text{ hours}} = 24\text{ mph}$$
3	3	3	If the bucket is 6% salt by volume, then the volume of salt is $\mathrm{V} = 24 * \frac{3}{50} = \frac{72}{50} = \frac{36}{25}$ gallons. To make the salted juice 4% by adding more unsalted apple juice, add juice until the proportion equates to 4% $$\mathrm{V}' = \frac{\frac{36}{25}}{24 + \mathrm{x}} = \frac{2}{50} \to (24 + \mathrm{x})2 = \frac{36}{25}(50) = 72$$ $$\mathrm{V}' = 48 + 2\mathrm{x} = 72 \to 2\mathrm{x} = 24 \to \mathrm{x} = 12\text{ gallons}$$
4	4		Use the Law of Sines $$\frac{\mathrm{XR}}{\sin \mathrm{W}} = \frac{\mathrm{WR}}{\sin \mathrm{X}} \to \frac{\mathrm{XR}}{\sin 30°} = \frac{8}{\sin 45°} \to \mathrm{XR} = \frac{8 \sin 30°}{\sin 45°}$$ $$\mathrm{XR} = \frac{4}{1/\sqrt{2}} = 4\sqrt{2}$$
		4	The permutations of n letters with m repeating letters is $\frac{\mathrm{n}!}{\mathrm{m}!}$ Thus, $\frac{5!}{2!} = \frac{5*4*3*2*1}{2*1} = 5 * 4 * 3 = 60$

5			If the point of inflection is also a stationary point, this implies the following. $$\frac{dy}{dx} = 3x^2 - 2ax - b = 0$$ $$\frac{d^2y}{dx^2} = 6x - 2a = 0$$ Via substitution, $$\frac{dy}{dx} = 3x^2 - 2(3x)x - b = 0 \rightarrow b = -3x^2$$ Via substitution again, $$b = -3\left(\frac{a}{3}\right)^2 \rightarrow b = -\frac{1}{3}a^2$$
	5	5	Expressed as a product of prime powers, $4200 = 2^3 * 3 * 5^2 * 7$. For a factor to be a multiple of 4, there must be at least a factor of 4 in its prime factorization. Therefore, we are led to the following realizations after examining the form of the prime factorization for $4200 = 2^a 3^b 5^c 7^d$. For the exponent, a, we must choose 2 or 3 since these would produce a factor of 4. Multiplying any other possible factor by these choices ensures that the positive integer factor of 4200 is a multiple of 4. For the other exponents, b, c, and d, their choices are {0,1}, {0,1,2}, and {0,1}, respectively. Thus, the number of factors that are a multiple of 4 is $(2)(2)(3)(2) = 24$
6	6	6	Let S = set of students that like sci-fi, H = set of students that like horror, R = set of students that like romance, and A = the union of sets S, H, and R, which is the set of students that like at least one of the movie genres. $$S = 72, H = 66, R = 58$$ $$A = S + H + R - S \cap H - H \cap R - R \cap S + S \cap H \cap R$$ $$A = 72 + 66 + 58 - 49 - 43 - 41 + 28 = 91$$ Thus, the number of students that like none of the genres is the complement of A. $$\overline{A} = 9$$ Similarly, a Venn Diagram also reveals the solution. These numbers add to 91. If there are 100 students, then 9 must like neither of the genres. 

7	7	7	The sum of the first three test scores is $$x_1 + x_2 + x_3 = 92 \times 3 = 276$$ The sum of all five test scores is $$x_1 + x_2 + x_3 + x_4 + x_5 = 55.8 \times 5 = 279$$ The sum of the last two test scores must be $$x_4 + x_5 = 3$$ Thus, $$x_4 + x_5 = 1.5x_5 + x_5 = 2.5x_5 = 3$$ $$x = \frac{30}{25} = \frac{6}{5} = 1.2$$ **The answer, as stipulated in the problem, must be in decimal form.**
8	8	8	Determine $(x + y + z)^2$. $$x^2 + xy + xz + yx + y^2 + yz + zx + zy + z^2 = \frac{49}{9}$$ $$x^2 + y^2 + z^2 + 2(xy + xz + yz) = \frac{49}{9}$$ $$\frac{91}{9} + 2(xy + xz + yz) = \frac{49}{9} \rightarrow 2(xy + xz + yz) = -\frac{42}{9}$$ $$xy + xz + yz = -\frac{21}{9}$$ Since the sequence is geometric, there is a common ratio between terms. That is, $\frac{y}{x} = \frac{z}{y} \rightarrow y^2 = xz$. Therefore, $$xy + y^2 + yz = -\frac{21}{9} \rightarrow y(x + y + z) = -\frac{21}{9}$$ $$y\left(\frac{7}{3}\right) = -\frac{21}{9} \rightarrow y = -1$$ Furthermore, since $xy + y^2 + yz = -\frac{21}{9}$, it follows that $$xy + 1 + yz = -\frac{21}{9} \rightarrow xy + yz = -\frac{30}{9}$$ $$y(x + z) = -\frac{10}{3} \rightarrow -1(x + z) = -\frac{10}{3}$$ $$x + z = \frac{10}{3}$$

9	9		Given a perimeter of 24, the side length of the largest, shaded square is 6. If we let A be the bottom left vertex of the largest square and B its top right vertex, the length of this diagonal, AB, would be $6\sqrt{2}$. Let us assume that the side length of the largest, white square is x. Its diagonal length would be $x\sqrt{2}$. Given that subsequent white squares have $\frac{7}{12}$ the side lengths of the previous larger white square, the second largest white square has side lengths equal to $\frac{7}{12}x$. Therefore, its diagonal length is $\frac{7}{12}x\sqrt{2}$. The set of diagonal lengths for all white squares, starting from point A and moving up and to the right towards point B, form an infinite, converging geometric sequence. Let g_n represent the diagonal length of the nth white square. If g_1 represents the diagonal length of the largest white square, then, $$g_n = x\sqrt{2}\left(\frac{7}{12}\right)^{n-1}$$ This sequence forms a geometric series that converges. Since the vertices of these diagonals are collinear with the points A and B, the sum must equal the diagonal length, AB, of the largest, shaded square. $$S = x\sqrt{2}\left(1 + \left(\frac{7}{12}\right)^1 + \left(\frac{7}{12}\right)^2 + \cdots\right) = 6\sqrt{2}$$ $$x\left(\frac{1}{1-\frac{7}{12}}\right) = 6 \to x\left(\frac{12}{5}\right) = 6 \to x = \frac{30}{12} = \frac{5}{2}$$ Thus, the area of the largest, white square is $\frac{25}{4}$
		9	The elements of the n-th row consist of the coefficients of the expansion of $(1+x)^{n-1}$. The sum of all entries is $2^0 + 2^1 + \ldots + 2^9 = 2^{10} - 1 = 1023$

10			Determine the equation for line JK. $$y = mx + b = \left(-\frac{6}{8}\right)x + b = -\frac{3}{4}x + b$$ Determine the y-intercept for the line JK. $$3 = -\frac{3}{4}(-2) + b \rightarrow 3 = \frac{3}{2} + b \rightarrow b = \frac{3}{2}$$ **Equation 1: The equation for line JK is** $$y = -\frac{3}{4}x + \frac{3}{2}$$ Evaluate the derivative of y and equate with the slope of y. $$\frac{dy}{dx} = -2c(x+1)^{-3} = -\frac{3}{4} \rightarrow c = \frac{3}{8}(x+1)^3$$ **Equation 2: Equate the equations for both curves.** $$-\frac{3}{4}x + \frac{3}{2} = \frac{c}{(x+1)^2} = \frac{\frac{3}{8}(x+1)^3}{(x+1)^2} = \frac{3}{8}(x+1)$$ Solve to determine the value of the x coordinate of their point of intersection. $$\frac{3}{8}(x+1) = \frac{3}{4}x + \frac{3}{2} \rightarrow \frac{3}{8}x + \frac{3}{8} = -\frac{3}{4}x + \frac{3}{2}$$ $$\frac{9}{8}x = \frac{9}{8} \rightarrow x = 1$$ Evaluate c. $$c = \tfrac{3}{8}(x+1)^3 = \tfrac{3}{8}(2^3) = 3$$
	10	10	The set of the first 2018 non-negative integers is represented as {0, 1, 2,..., 2017}. Since it is inclusive of 0, their sum is given by the formula $S_{2017} = \frac{n(n+1)}{2}$. The arithmetic mean of this series is $\overline{S} = \frac{S_{2017}}{2018} = \frac{2017(2018)}{2(2018)} = \frac{2017}{2}$

11	11	11	Re-write the expression as follows. $$\frac{4m^2+2m+15}{2m+3}=\frac{(2m+3)(2m-2)+21}{2m+3}$$ $$\frac{(2m+3)(2m-2)+21}{2m+3}=2m-2+\frac{21}{2m+3}$$ In the above expression, $2m-2$ will always be an integer. Therefore, we must determine the value of m for which the last term is also an integer. $$2m+3=\{-21,-7-3-1,1,3,7,21\}$$ $$m=\{-12,-5,-3,-2,-1,0,2,9\}$$ Adding all possible values of m results in $\Sigma m=-12$
12	12	12	Consider that the first factorial that is divisible by 60 is $6!=720$. Thus, $7!=720*7$, $8!=720(7*8)$, $9!=720(7*8*9)$, etc. This means that by adding n! for $n\geq 6$ to the sum, every term has a factor of 720 so each of these must be divisible of 90 and will produce no effect on the remainder. Let $M=1!+2!+3!+4!+5!=153$. Then $N \bmod 60 = M \bmod 60 = 153 \bmod 60 = 33$
13	13	13	Given the function $G(x)=\ x^n+3x^2+Ax+6$, the Remainder Theorem states that if $(x-a)$ divides $G(x)$ and leaves a remainder, then $G(a)=$ the remainder. Therefore, $$G(-1)=14\rightarrow(-1)^n+3-A+6=14$$ $$(-1)^n-A+9=14$$ $$A=-5+(-1)^n$$ Also, $$G(1)=6=(1)^n+3+A+6=6$$ $$1+A+9=6$$ $$A=-4$$ Thus, $$-4=-5+(-1)^n$$ $$1=(-1)^n$$ $$\therefore\ n\text{ is even}$$ $$\therefore\ n\ =\ 26$$

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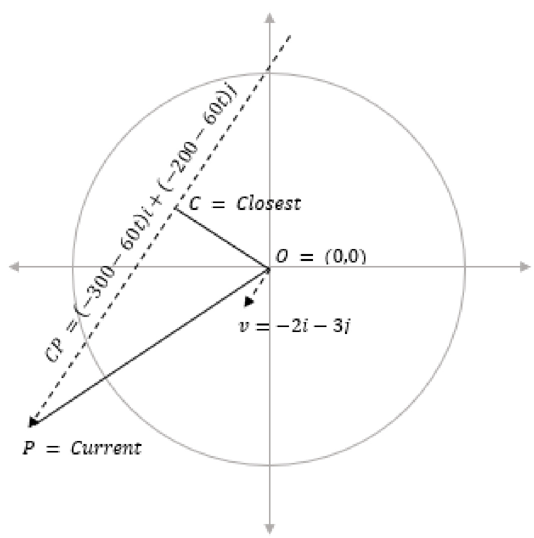
The magnitude of vector v is given by the Pythagorean Theorem. $|f| = \sqrt{(-2)^2 + (-3)^2} = \sqrt{13}$
Since the speed is $30\sqrt{13}\,\frac{\text{km}}{\text{hr}}$, the velocity, in vector form is
$$\vec{v} = 30\vec{f} = \begin{pmatrix} -2 \\ -3 \end{pmatrix} = \begin{pmatrix} -60 \\ -90 \end{pmatrix}$$
A general point on the path is $P = (300 - 60t, 200 - 90t)$

The displacement vector for the aircraft is thus determined to be $\overrightarrow{OP} = \begin{pmatrix} -300 - 60t \\ -200 - 90t \end{pmatrix}$.

Referring to the diagram, the closest point occurs when the displacement vector is perpendicular to the flight vector. In other words, the cross-product of these two vectors equals zero.



$$\overrightarrow{OP} \cdot \vec{f} = (-300 - 60t \quad -200 - 90t) \cdot \begin{pmatrix} -2 \\ -3 \end{pmatrix} = 0$$
$$\begin{aligned} \text{Therefore, } -2(-300 - 60t) - 3(-200 - 90t) &= 0 \\ +1200 + 390t &= 0 \\ t &= -\frac{40}{13}\text{hours} = -3\frac{1}{13}\text{hours} \end{aligned}$$
Since $\frac{1}{12}$ hr $= 5$ min, $\frac{4.5}{60}$ hr $= 4.5$ min, and $\frac{1}{15}$ hr $= 4$ min, it is easy to show that $\frac{1}{12} > \frac{1}{13} > \frac{4.5}{60}$. Therefore, $\frac{1}{13}$ hr will round to 5 minutes.

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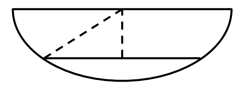
We know that AB is 25% shorter than CD, so $AB = \frac{3}{4}CD$. Because the radius is half the diameter, $r = \frac{1}{2}CD$, and consequently, $\frac{3}{4}r = \frac{1}{2}AB$. We now have two of the three sides of the right triangle and can solve for the third using the Pythagorean theorem.

$$AB = \frac{3}{4}CD$$

$$r = \frac{1}{2}CD \rightarrow \frac{3}{4}r = \frac{1}{2}AB$$



$$(\frac{3}{4}r)^2 + x^2 = r^2$$

$$\frac{9}{16}r^2 + x^2 = r^2$$

$$x^2 = \left(\frac{16}{16} - \frac{9}{16}\right)r^2$$

$$x^2 = \frac{7}{16}r^2$$

$$x = \pm\frac{\sqrt{7}}{4}r$$

A distance cannot be negative, therefore the answer is

$$x = \frac{\sqrt{7}}{4}r$$

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This requires a two-step integration by parts methodology.

$$u = x^2 \quad v = e^x$$

$$du = 2xdx \quad dv = e^x dx$$

$$\int_0^4 x^2 e^x dx = x^2 e^x - \int_0^4 e^x 2xdx$$

$$y = 2x \quad z = e^x$$

$$dy = 2dx \quad dz = e^x dx$$

$$\int_0^4 x^2 e^x dx = x^2 e^x - (2xe^x - \int_0^4 e^x 2dx)$$

$$\int_0^4 x^2 e^x dx = x^2 e^x - 2xe^x + 2e^x \Big|_0^4$$

$$e^x(x^2 - 2x + 2)\Big|_0^4 = 10e^4 - 2$$

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	15	15	Expand using the Binomial Theorem or through Pascal's Triangle. $$(1 + x)^5 = 1 + 5x + 10x^2 + 10x^3 + 5x^4 + x^5$$ Now consider multiplying each term by $(1 + \alpha x)$ $$(1 + \alpha x)(1) = 1 + \alpha x$$ $$(1 + \alpha x)(5x) = 5x + 5\alpha x^2$$ $$(1 + \alpha x)(10x^2) = 10x^2 + 10\alpha x^3$$ Subsequent terms will not have any x^2 terms. Only two x^2 terms appear above. Thus, $$5\alpha x^2 + 10x^2 = 25x^2 \rightarrow (5\alpha+10)x^2 = 25x^2$$ $$5\alpha + 10 = 25 \rightarrow \alpha = 3$$