1. The time between them simultaneously flashing and them simultaneously flashing again is the least common multiple of 30, 40, and 25 minutes, which is 600 minutes (which is easy to compute with prime factorization). So, 600 minutes after midnight is 10:00 A.M.
Answer: **C**

2. The curve has to turn around twice. This is impossible for a second degree polynomial (as is choice D). We know that the x^3 function never increases at any time. Choice A moves that curve up by 9 units, choice B is $(x-1)^3+6$, and choice C is $2(x+1)^3+3$. So, choices B and C shift the curve to the right and left by 1 unit as well as move it up by 6 or 3 units. Therefore, none of them could describe the roller coaster.
Answer: **E**

3. The dot product of any two of the vectors has to be zero. This is true of <1, 2, 2> and <0, 1, -1> by inspection. It also means that $x-2+2z=0$ and $-1-z=0$. Solving the latter equation gives us $z=-1$, and plugging that into the first equation yields $x=4$. Therefore, $x+z=4+(-1)=3$.
Answer: **B**

4. The side length of c in terms of the variable c is $\sqrt{2-2\cos(c)}=\sqrt{4\sin^2\left(\frac{c}{2}\right)}=2\sin\left(\frac{c}{2}\right)$, which will plot a sine curve of period 4π (even though the graph only shows c from 0 to 2π.
Answer: **D**

5. It’s 12 + 14 + 16 + … + 70 = 10*30 + (2 + 4 + 6 +…+ 60) = 300 + 2*(1 + 2 + 3 +…+30) =300 + 2*(30*31/2) = 300 + 930 = 1,230.
Answer: **E**

6. We have cis(π/4) * cis(2π/4) * cis(3π/4) *…* cis(8π/4) = cis(π/4) * cis(π/4)^2 *…* cis(π/4)^8 = cis(π/4)^(1 + 2 + 3 +…+ 8) = cis(π/4)^36 = cis(36π/4) = cis(9π) = -1.
Answer: **C**

7. The ellipse is described by $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, or, for the top half, $y(x)=b\sqrt{1-\frac{x^2}{a^2}}$, and the focal points should be $\sqrt{a^2-b^2}$ from the center. A point on the end of the ellipse’s major axis is a units from the center, and therefore, $a-\sqrt{a^2-b^2}$ units from the nearest focal point. At the point where the line from it to the focal point is perpendicular to the major axis, the distance is $y(\sqrt{a^2-b^2})=\frac{b^2}{a}$, so we have the equations $a-\sqrt{a^2-b^2}=1$ and $\frac{b^2}{a}=\frac{9}{5}$. These can be solved to yield a = 5 and b = 3. So the distance between the two focal points is $2\sqrt{a^2-b^2}=8$.
Answer: **D**

8. The function in choice D has an output of 100 when $t = 0$ and it goes to 300 as t goes to infinity. It is the standard equation for a population with an initial size of 100 and a carrying capacity of 300. The other choices don't meet all of these criteria.
Answer: **D**

9. A right triangle is made with an angle of interest $\pi/3$ radians. The vertical leg is Aaron's body, which stands 2 meters tall, and the horizontal leg is the shadow. Therefore, 2 is equal to the hypotenuse of the triangle times $\sqrt{3}/2$ (sine of the angle of interest), so the hypotenuse must be $4/\sqrt{3}$ meters. Then the length of the horizontal leg is the hypotenuse times ½ (the cosine of the angle), so we end up with $2/\sqrt{3}$ as our answer.
Answer: **E**

10. Because the diameter is 60 meters, the circumference is 60π meters. This divided by the time taken to move that distance is $60\pi/12\pi = 5$ meters per second.
Answer: **B**

11. $rA = r \cdot \pi r^2 = \pi r^3$ and $V = \frac{1}{3}\pi r^2 h$, so $V \propto r^2 h$ and it is not proportional to r^3.
Answer: **B**

12. For Jimmy, $x = 2 - 2t$ and $y = -2t$. For Joey, $x = 0$ and $y = 1 - t$. The distance between them is the square root of the sum of the difference in x squared and the difference in y squared. Therefore, it is $\sqrt{(2-2t)^2 + (-1-t)^2}$. By inspection, $t = 5$ yields a distance of $\sqrt{(-8)^2 + 6^2} = 10$, or we could set the distance formula to 10, square both sides, and solve the quadratic for the positive solution.
Answer: **A**

13. Statement I: Counterexample: $b_5 = b_6 = 60$; FALSE.
Statement II: If p is prime, then b_{p-1} must not have p as a factor, since all integers up to $p - 1$ are relatively prime to p. So p must be multiplied in to make b_p divisible by p; TRUE.
Statement III: For the composite numbers that are prime powers, such as 8, 9, and 25, this does not hold; FALSE.
Answer: **C**

14. Let's start scoring a sample test by first giving the competitor the maximum 150 points, then subtracting off 5 for every question skipped and 7 for every question answered wrong. The smallest number that is not a positive linear combination of 5's and 7's is given by $5 \cdot 7 - 5 - 7 = 23$. So the lowest impossible score is 150 – 23 = 127.
Answer: **B**

15. We are looking for the number of *combinations* of cards one can choose from the twelve cards to get the number of possible outcomes (because it doesn't matter what order you picked them in). Therefore, we have 12-choose-3, or 220, possibilities. Only one of those combinations gives us a jack of diamonds, a queen of hearts, and a king of spades. Therefore, the probability is 1 / 220.
Answer: **D**

16. The roots of the polynomial in the numerator are -2 and -3, but -2 would make the denominator 0 as well as the numerator, so only -3 will work.
Answer: **B**

17. The function of its motion to the right and left is the sine curve $x = k\sin(\pi t/2)$ for some constant *k*. So when *t* = 3, *x* is –*k*, its minimum value (which means that the pendulum is all the way to the left).
Answer: **B**

18. First Joe chooses which 6 questions will be A's. There are 30-choose-6 combinations of questions to choose from, and only one of those combinations is correct. Then he chooses the 6 B's out of the remaining 24 questions. There are 24-choose-6 combinations to choose from now. For the C's, there are only 18-choose-6 combinations, for the D's, only 12-choose-6, and for the E's only one choice. Therefore, the number of possibilities of where to put the A's, B's, C's, D's, and E's is $\left(\frac{30!}{6!\,24!}\right)\left(\frac{24!}{6!\,18!}\right)\left(\frac{18!}{6!\,12!}\right)\left(\frac{12!}{6!\,6!}\right) = \frac{30!}{(6!)^5}$. Because only one choice will be correct, the probability is the multiplicative inverse of the above fraction.
Answer: **D**

19. $\frac{1}{2}(10+10x)(20+10y) = 1{,}200$ and $\frac{1}{2}(10+15x)(20+15y) = 2{,}125$ so
$100 + 50y + 100x + 50xy = 1{,}200$ and $100 + 75y + 150x + 112.5xy = 2{,}125$ so
$2y + 4x + 2xy = 44$ and $2y + 4x + 3xy = 54$ so, subtracting the first from the second,
$xy = 10$.
Answer: **A**

20. His overall walking distance is a geometric series starting at 81 with a ratio of 1/3.
$81\left(\frac{1}{1-1/3}\right) = 121.5$, so he walks 121.5 meters. 216 – 121.5 = 94.5.
Answer: **C**

21. The number of three-person combinations is 6-choose-3, which is 20. The number of these which include Seyhun is 5-choose-2, which is 10. Therefore, the probability is 10/20 = ½.
Answer: **B**

22. The parabolic cross-sections are described by the curve $y = (x-1)^2/8$, so the focus is 8/4 = 2 units away from the vertex.
Answer: **A**

23. The implicit formula can be simplified to $(x-1)^2 + (y-2)^2 = 4^2$, so it is a circle of radius 4 centered at (1, 2). Therefore, when rotated about the line y = 2, a sphere of radius 4 is created. Thus, $A = 4\pi \cdot 4^2$ and $V = \frac{4}{3}\pi \cdot 4^3$ and so $\frac{V}{A} = \frac{256\pi/3}{64\pi} = \frac{4}{3}$.
Answer: **A**

24. $\tan(x) = \sin(x)/\cos(x) = \frac{e^{ix} - e^{-ix}}{2i} \div \frac{e^{ix} + e^{-ix}}{2} = \frac{e^{ix} - e^{-ix}}{e^{ix} + e^{-ix}} \cdot \frac{2}{2i} = \frac{e^{ix} - e^{-ix}}{e^{ix} + e^{-ix}} \cdot -i$

$\frac{i(e^{-ix} - e^{ix})}{e^{ix} + e^{-ix}}$.
Answer: **D**

25. When converting to rectangular coordinates, we have $x = r\cos\theta = \sec\theta\cos\theta = 1$ and $y = r\sin\theta = \sec\theta\sin\theta = \tan\theta$. Since x is fixed at 1 while y can be anywhere between positive and negative infinity, it maps out the vertical line x = 1.
Answer: **A**

26. When it hits the ground, h will be 0, so solve $-5t^2 + 24t + 5 = 0$ and get $t = -\frac{1}{5}$ or 5.

Taking the positive solution yields $d = 3t = 15$.
Answer: **C**

27. The first curve is a circle centered at (1, 1), the second curve is an ellipse centered at (1, 1), and the third curve is an ellipse centered at (1, 3). The point (1, 3) lies in the diameter of the circle, the major axis of the first ellipse, and the center of the second ellipse.
Answer: **C**

28. The denominator of the fraction can be factored to (x + 3)(x – 2). The (x + 3)'s can be canceled out and we are left with $\lim_{x\to -3} \frac{1}{x-2}$. This is 1 / -5 = -.2
Answer: **B**

29. The number of pebbles in pile number n would be 2 ^ (n – 1). Therefore, the number of pebbles in the tenth pile would be 2 ^ 9. The number of pebbles in the first nine piles would be 1 + 2 + 2 ^ 2 + … + 2 ^ 8 = 2 ^ 9 – 1. Therefore, the difference is 1.
Answer: **E**

30. We could set up the three equations $a - b + 2c = 2$, $3a + 3b = -2$, and $2b - 2c = 1$. Adding three times the first equation to the second equation gives us $6a + 6c = 4$. Adding that to three times the third equation gives us $6a + 6b = 7$. However, multiplying the second equation by 2 gives us $6a + 6b = -4$, yielding a contradiction. Therefore, there is no solution for *a*, *b*, and *c*. (The three vectors are not linearly independent; in fact, the third equals the first minus the second.)
Answer: **D**

Answer Key:

1. C
2. E
3. B
4. D
5. E
6. C
7. D
8. D
9. E
10. B
11. B
12. A
13. C
14. B
15. D
16. B
17. B
18. D
19. A
20. C
21. B
22. A
23. A
24. D
25. A
26. C
27. C
28. B
29. E
30. D