For all questions, answer E. "NOTA" means that none of the above answers is correct.

1. Three light bulbs are sitting in a row. The first one flashes regularly every 30 minutes, the next one flashes regularly every 40 minutes, and the last one flashes regularly every 25 minutes. They all flash at the same time at 12:00 A.M. When is the next time that they will all flash simultaneously?

a) 12:05 A.M. b) 12:40 A.M. c) 10:00 A.M. d) 8:00 P.M. e) NOTA

2. A roller coaster is shaped like a polynomial curve. The cars start off at the top, going down, then they go up, and then they go down again until the ride is over. Which of the following curves could describe the shape of the roller coaster?

a) $y = x^3 + 9$ b) $y = x^3 - 3x^2 + 3x + 5$ c) $y = 2x^3 + 6x^2 + 6x + 5$ d) $y = x^2 - 3x + 3$ e) NOTA

3. Vectors <1, 2, 2>, <0, 1, -1> and <x, -1, z> are all mutually perpendicular to each other. What is $x + z$?

a) 1 b) 3 c) 5 d) cannot be determined e) NOTA

4. An isosceles triangle ABC has sides $\overline{AC}$ and $\overline{BC}$ both of length 1. A graph is plotted where the independent variable is the measure of the angle c as it goes from 0 to 2π, and the dependent variable is the length of side $\overline{AB}$. What kind of curve will be graphed? (Note: the graph contains the points (π, 2) and (2π, 0), although the triangles at these points are degenerate cases.)

a) a straight line b) a portion of a sine curve with period π
c) a portion of a sine curve with period 2π d) a portion of a sine curve with period 4π e) NOTA

5. Jeff is becoming steadily more addicted to the card game FreeCell. During the month of June, the number of games he plays per day is an arithmetic sequence, so that each day he plays 2 more games than he played the day before. If he plays 12 games on June 1st, how many games does he play total in the month of June (which has 30 days)?

a) 930 b) 942 c) 1,170 d) 1,290 e) NOTA

6. Solve this product: $\prod_{s=1}^{8}(\cos\left(\frac{\pi s}{4}\right) + i\sin\left(\frac{\pi s}{4}\right)) = ?$

a) 0 b) 1 c) –1 d) i e) NOTA

7. According to Kepler, the planets of the solar system move around the sun in elliptical orbits, with the sun at one of the focal points. A planet orbits a star in this way in a hypothetical solar system. At one point, when the planet is lying on the major axis of the ellipse and is closer to the star than to the other focus, it is 1 billion kilometers away from the star. At another point in its orbit, when the line from it to the star is parallel to the minor axis, it is 1.4 billion kilometers from the star. What is the distance between the star and the other focus of the ellipse?

a) 3 billion kilometers b) 4 billion kilometers c) 6 billion kilometers
d) 8 billion kilometers e) NOTA

8. Fruit flies are breeding in a small, abandoned house. No fly which is in the house leaves, and no outside fly comes in. At the beginning ($t = 0$), there are 100 fruit flies. The maximum carrying capacity is 300 flies. Which of the following functions $P(t)$, where t is in weeks, could model the population of the fruit flies?

a) $100e^{t/2}$ b) $300-100e^{-t}$ c) $\frac{300}{2+e^{t/2}}$ d) $\frac{300e^{t/2}}{2+e^{t/2}}$ e) NOTA

9. Aaron Skysocki has a height of exactly 2 meters. At a particular time of day when Aaron is out in the sun, the angle of elevation from the top of Aaron's head to the sun is $\pi / 3$. How long in meters is Aaron's shadow?

a) 2 b) 1 c) $\sqrt{3}$ d) $\frac{4}{\sqrt{3}}$ e) NOTA

10. Alan likes to drive his shiny, new car around a racetrack every day. The racetrack has a diameter of 60 meters. One day he manages to go all the way around the track in 12π seconds. What is his average speed in meters per second?

a) $5 / \pi$ b) 5 c) 10 d) 75 e) NOTA

11. A cone has volume V has height h. Its base has radius r, circumference c, and area A. In general, which of the following is NOT directly proportional to V (as V, h, r, c, and A vary)?

a) r^2h b) rA c) hc^2 d) hA e) NOTA

12. Jimmy is standing at point (2, 0), and his brother Joey is standing at point (0, 1) on the xy plane. They both begin walking at the same time. Each one walks in a straight line at some constant speed. After 1 second, Jimmy is at point (0, -2) and Joey is at point (0, 0). After how many seconds since they started walking will the distance between them be exactly 10 units?

a) 5 b) 6 c) 9 d) never will occur e) NOTA

13. Let $b_1, b_2, b_3, \ldots$ be a sequence such that b_n, for all integral $n > 0$, b_n is the smallest positive integer that's divisible by all positive integers less than or equal to n. Which of the following statements are true?

I. For all n, $\frac{b_n}{b_{n-1}} > 1$.

II. For every prime p, $\frac{b_p}{b_{p-1}} = p$.

III. For every composite number c, $\frac{b_c}{b_{c-1}} = 1$.

a) None are true b) I only c) II only d) II and III only e) NOTA

14. In the imaginary state of Louisibama, the MAΘ individual tests have the usual 30 multiple-choice questions with 5 choices each. A question answered correctly gives you 5 points, a question answered incorrectly subtracts 2 points from your score, and a question skipped does not change your score. What is the lowest positive score that is IMPOSSIBLE to obtain on the test?

a) 115 b) 127 c) 144 d) 146 e) NOTA

15. You take a regular deck of 52 cards and take out all aces through tens, so that all you have left are the kings, queens, and jacks of all four suits. Then, after shuffling, you deal yourself a hand of three cards. What is the probability that your hand will consist of a jack of diamonds, a queen of hearts, and a king of spades?

a) $\frac{3}{52}$ b) $\frac{1}{1320}$ c) $\frac{64}{1320}$ d) $\frac{1}{220}$ e) NOTA

16. If $\frac{x^2+5x+6}{x^2-2x-8}=0$, which of the following could be the value of x?

a) -2 b) -3 c) 4 d) all of the above e) NOTA

17. The pendulum of a clock is swinging back and forth. The period of its motion is 4 seconds. At time t = 0 seconds, it is pointing straight down and moving to the right. At time t = 3 seconds, where is it pointing?

a) pointing straight down
b) pointing to the far left of its arc
c) pointing a bit to the right, but still moving right
d) pointing to the far right of its arc
e) NOTA

18. Joe Christmastree likes to take his MAΘ individual tests by bubbling in random answers to all 30 questions, with the one constraint that he makes sure there are an equal number of A's, B's, C's, D's, and E's bubbled in on his scantron. In the answer key to a particular individual test that Joe takes, there actually are 6 A's, 6 B's, 6 C's, 6 D's, and 6 E's. What is the probability that Joe gets a perfect score on the test?

a) $\frac{1}{30!}$ b) $\left(\frac{30!}{24!}\right)^{-5}$ c) $\left(\frac{30!}{6!\,24!}\right)^{-5}$ d) $\frac{(6!)^5}{30!}$ e) NOTA

19. A triangle has a side with length a and a side with length b, and the two sides are at right angles to each other. At first, a is 10 centimeters, and b is 20 centimeters. a begins increasing at a rate of x centimeters per second, and b at y centimeters per second. After ten seconds, the triangle's area is 1,200 square centimeters, and after five more seconds, it is 2,125 square centimeters. What is the product of x and y?

a) 10 b) 12 c) $\frac{17}{3}$ d) 11 e) NOTA

20. Cody is on a beach, standing 216 meters away from the water. He starts walking towards the water, but is getting sleepier as he walks, so that it takes 1 minute for him to walk the first 81 meters, 1 minute for the next 27 meters, 1 minute for the next 9, etc. If he is left to walk this way for an infinite period of time, approximately how many meters from the water will he end up? (Assume that the water never moves.)

a) 0 b) 54 c) 94.5 d) 121.5 e) NOTA

21. Seyhun and five other people all like to talk to each other on the phone every week in three-way conversations. What is the probability that a randomly chosen three-person conversation will include Seyhun, assuming that each person is equally likely to be included in any given conversation?

a) $\frac{1}{6}$ b) $\frac{1}{2}$ c) $\frac{3}{5}$ d) $\frac{3}{4}$ e) NOTA

22. The parabola $8y = x^2 - 2x + 1$ is drawn on a graph. Light rays parallel to the *y*-axis travel downwards, hit the curve, and all bounce into the same point. What is the distance between this point and the vertex of the parabola?

a) 2 b) $\sqrt{2}$ c) $\sqrt{8}$ d) 4 e) NOTA

23. The curve described by $x^2 + y^2 - 2x - 4y = 11$ is rotated about the line $y = 2$. Let A be the surface area of the surface formed, and V be the volume of the surface formed. What is V divided by A?

a) $\frac{4}{3}$ b) $\frac{8}{3}$ c) $\frac{16}{3}$ d) $\frac{\sqrt{21}}{3}$ e) NOTA

24. The sine and cosine functions can be defined in terms of *e* and *i*, where $i = \sqrt{-1}$. Given that $\sin(x) = \frac{e^{ix} - e^{-ix}}{2i}$ and $\cos(x) = \frac{e^{ix} + e^{-ix}}{2}$, then which of the following is equivalent to tan(*x*)?

a) $\frac{e^x - e^{-x}}{i}$ b) $\frac{i(e^{ix} + e^{-ix})}{e^{ix} - e^{-ix}}$ c) $\frac{i(e^{ix} - e^{-ix})}{e^{ix} + e^{-ix}}$ d) $\frac{i(e^{-ix} - e^{ix})}{e^{ix} + e^{-ix}}$ e) NOTA

25. If the equation $r = \sec\theta$ is drawn in polar coordinates, what is the shape of the resulting curve?

a) a straight vertical line b) a straight horizontal line c) a circle d) an ellipse e) NOTA

26. I throw a ball so that its height above the ground (in meters) is given by the formula $h = -5t^2 + 24t + 5$, and its horizontal distance from me is given by the formula $d = 3t$, where *t* is time. Time *t* equals 0 when I release the ball. When the ball hits the ground, how far (in meters) is it from me horizontally?

a) $\frac{72}{5}$ b) $\frac{3}{5}$ c) 15 d) $\frac{15}{2}$ e) NOTA

27. I draw on a graph the shapes described by the three equations $x^2 - 2x + y^2 - 2y - 7 = 0$, $9x^2 - 18x + y^2 - 2y + 1 = 0$, and $x^2 - 2x + 4y^2 - 24y + 33 = 0$. Which of the following points lies in the interior of all three shapes?

a) (0, 0) b) (1, 1) c) (1, 3) d) (1, 4) e) NOTA

28. What is $\lim_{x \to -3} \frac{x+3}{x^2+x-6}$?

a) undefined b) – 0.2 c) 0 d) -5 e) NOTA

29. I set out piles of pebbles on the ground, so that the first pile has 1 pebble in it, the second pile has 2 pebbles in it, the third pile has 4 pebbles in it, and so on, with each pile having twice as many pebbles as the last one. What is the difference between the number of pebbles in the tenth pile and the number of pebbles in the first nine piles combined?

a) $2^{10}-1$ b) 2^9 c) 2^9-1 d) 2^8 e) NOTA

30. If the vector <2, -2, 1> equals a<1, 3, 0> + b<-1, 3, 2> + c<2, 0, -2> for some real numbers a, b, and c, find $a + b + c$.

a) 0 b) 1 c) 2 d) there is no such a, b, and c e) NOTA