Practice Question

Alpha Ciphering 2007 Mu Alpha Theta National Convention

What is the period of the function $f(x)=\cos^4 x-\sin^4 x$?

$(\cos^2 x-\sin^2 x)(\cos^2 x+\sin^2 x)=\cos^2 x-\sin^2 x=\cos 2x$

$P(\cos bx)=\frac{2\pi}{b}=\frac{2\pi}{2}=\pi$

Question #1

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What is the sum of the solutions to the equation: $x^2 \log 2 + 5x = \log 64 + 5x \log 5$?

$x^2 \log 2 + 5x - 5x \log 5 - \log 64 = 0$

$x^2 \log 2 + 5x(1 - \log 5) - \log 64 = 0$

$x^2 \log 2 + 5x(\log 10 - \log 5) - \log 64 = 0$

$x^2 \log 2 + 5x \log 2 - \log 64 = 0$

$\log 2$

$x^2 + 5x - \frac{\log 64}{\log 2} = 0$

$x^2 + 5x - \log_2 64 = 0$

$x^2 + 5x - 6 = 0$

$(x+6)(x-1)$ $x = -6$ or 1 Sum $= -5$

Question #2

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$\sum_{n=6}^{\infty} \frac{4}{n^2 - 8n + 15} = ?$

$\frac{4}{(n-3)(n-5)} = \frac{A}{n-3} + \frac{B}{n-5}$

$4 = A(n-5) + B(n-3)$

$B = 2$

$A = -2$

$\frac{2}{n-5} - \frac{2}{n-3} = \left(\frac{2}{1} - \frac{2}{3}\right) + \left(\frac{2}{2} - \frac{2}{4}\right) + \left(\frac{2}{3} - \frac{2}{5}\right) + \left(\frac{2}{4} - \frac{2}{6}\right) + \cdots$

$\frac{2}{1} + \frac{2}{2} = 3$

Question #3

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Find the sum of the solutions of $\tan x \sin x - 1 - \sin x + \tan x = 0$ over $[0, 2\pi)$.

$\tan x \sin x + \tan x - 1 - \sin x = 0$

$\tan x(\sin x + 1) - 1(\sin x + 1) = 0$

$(\tan x - 1)(\sin x + 1) = 0$

$\tan x = 1 \quad \sin x = -1$

$\frac{\pi}{4}, \frac{5\pi}{4} \qquad \frac{3\pi}{2}$ extraneous

$\frac{\pi}{4} + \frac{5\pi}{4} = \frac{3\pi}{2}$

Question #4

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Find the area of the triangle whose vertices are the 2 foci of the following ellipse. The third vertex is any endpoint of either latus rectum. $3x^2 + 2y^2 - 24x + 12y + 60 = 0$

$3(x^2 - 8x + 16) + 2(y^2 + 6y + 9) = -60 + 48 + 18$

$3(x-4)^2 + 2(y+3)^2 = 6$

$\frac{(x-4)^2}{2} + \frac{(y+3)^2}{3} = 1$

$b^2 \qquad a^2$

$c^2 = a^2 - b^2 = 1$

$c = 1$

$A = 2(\frac{1}{2})(2c)(b)$

$2(\frac{1}{2})(2)\sqrt{2} = 2\sqrt{2}$

Question #5
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Find the limit: $\lim_{x\to 3}\frac{3-\sqrt{x+6}}{3-x} + \lim_{x\to 0}\frac{\frac{1}{x+4}-\frac{1}{4}}{x}$

$\frac{(3-\sqrt{x+6})(3+\sqrt{x+6})}{(3-x)(3+\sqrt{x+6})}$

$\frac{9-(x+6)}{(3-x)(3+\sqrt{x+6})} = \frac{3-x}{(3-x)(3+\sqrt{x+6})}$

$\frac{1}{3+\sqrt{x+6}}$

$\frac{1}{6}$

$4(x+4)\left(\frac{\frac{1}{x+4}-\frac{1}{4}}{x}\right)$

$4(x+4)\left(\frac{4-(x+4)}{4x(x+4)}\right)$

$4(x+4)\left(\frac{-x}{4x(x+4)}\right)$

$4(x+4)\left(\frac{-1}{4(x+4)}\right)$

-1

$\frac{1}{6} - 1 = \frac{8-3}{48} = \frac{5}{48}$

Question #6
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$Matrix\ A = \begin{pmatrix} 1 & 3 & -2 \\ 2 & 2 & 1 \\ -1 & 1 & 4 \end{pmatrix}$ What is the sum of the entries in the third row of A^{-1}?

3rd column adjoint

$\begin{vmatrix} 2 & 2 \\ -1 & 1 \end{vmatrix} \quad \begin{vmatrix} 1 & 3 \\ -1 & 1 \end{vmatrix} \quad \begin{vmatrix} 1 & 3 \\ 2 & 2 \end{vmatrix}$

$2 - -2 \quad 1 - -3 \quad 2 - 6$

$4 \quad 4 \quad -4$

negate

$4 - 4 - 4 = -4$

$\begin{matrix} 1 & 3 & -2 & 1 & 3 \\ 2 & 2 & 1 & 2 & 2 \\ -1 & 1 & 4 & -1 & 1 \end{matrix}$

$8 - 3 - 4 - 4 - 1 - 24$

-28

$\left(\frac{1}{-28}\right)(-4) = \frac{1}{7}$

Question #7

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5,9,19,35,... is a quadratic sequence that can be expressed in the form $a_n = An^2 + Bn + C$ for $n = 1, 2, \ldots$: What is the value of $A - B + C$?

5 9 19 35

4 10 16

6 6

$A+B+C$ $4A+2B+C$ $9A+3B+C$

$3A+B$ $5A+B$

$2A$

$2A=6$ $3A+B=4$ $A+B+C=5$

$A=3$ $B=-5$ $C=7$

$3--5+7=15$

Question #8

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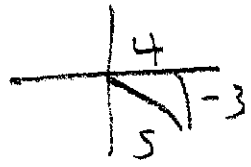
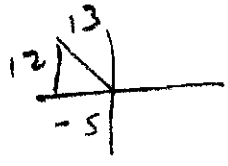
$\tan\left(\sin^{-1}\frac{-3}{5} - \cos^{-1}\frac{-5}{13}\right) = ?$

$\tan(A-B) = \frac{\tan A - \tan B}{1+\tan A \tan B}$





$$\frac{-\frac{3}{4} - -\frac{12}{5}}{1+\left(\frac{3}{4}\right)\left(\frac{12}{5}\right)} = \frac{\frac{33}{20}}{1+\frac{36}{20}} = \frac{33}{56}$$

Question #9

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The three cube roots of 8 form a triangle centered about the origin of the complex plane. What is the area of this triangle?

$x^3 - 8 = 0$

$(x-2)(x^2+2x+4) = 0$

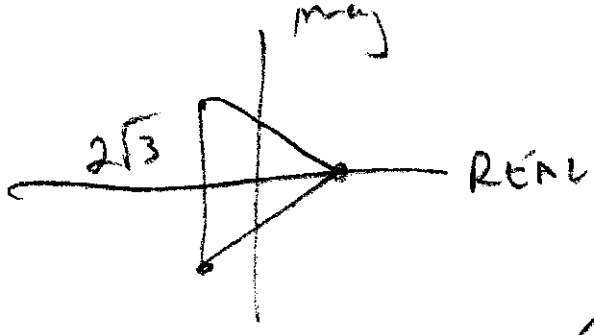
$2 \qquad \frac{-2 \pm \sqrt{4-4(4)}}{2}$

$\frac{-1 \pm \sqrt{-12}}{2}$

$\frac{-1 \pm 2i\sqrt{3}}{2}$

$-1 \pm i\sqrt{3}$

$(2,0)\ (-1,\sqrt{3})\ (-1,-\sqrt{3})$



$\frac{1}{2}bh = \frac{1}{2}(2\sqrt{3})(3) = 3\sqrt{3}$

Question #10

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If $\cos x + \cos y = \frac{1}{3}$ and $\sin x - \sin y = \frac{1}{6}$, find the value of $\cos(x+y)$.

$\cos^2 x + 2\cos x\cos y + \cos^2 y = \frac{1}{9}$

$\sin^2 x - 2\sin x\sin y + \sin^2 y = \frac{1}{36}$

$\hookrightarrow \cos x\cos y - \sin x\sin y$

$1 + 1 + 2\cos x\cos y - 2\sin x\sin y = \frac{1}{9} + \frac{1}{36}$

$2 + 2(\cos x\cos y - \sin x\sin y) = \frac{4+1}{36}$

$2\cos(x+y) = \frac{5}{36} - \frac{72}{36}$

$2\cos(x+y) = -\frac{67}{36}$

$\cos(x+y) = -\frac{67}{72}$