Alpha Equations and Inequalities Test
2007 Mu Alpha Theta National Convention

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For all questions, answer E. "NOTA" means that none of the above answers is correct.
The use of calculators is not permitted.

1. For what real value(s) of x on the interval $[0, \pi]$ is it true that $|4\sin^2(x)| > 1$?

a) $x \in (0, \frac{\pi}{6})$ b) $x \in (\frac{\pi}{6}, \frac{5\pi}{6})$ c) $x \in (\frac{5\pi}{6}, \pi)$ d) no real values e) NOTA

2. Which of the following functions is NOT equal to its own inverse?

a) $f(x) = x$ b) $f(x) = 2 - x$ c) $f(x) = 2x^{-1}$ d) $f(x) = 2^x$ e) NOTA

3. Three particles are sitting on the x-axis at $x = 0$ at time $t = 0$ and start moving along the x-axis, each following a different function of time. The first one's position on the x-axis can be expressed by $x_1 = |\sin(\frac{t}{2})|$, the next one by $x_2 t + x_2 - t = 0$, and the third by $e^{x_3} - t - 1 = 0$. At time t_0, where $\frac{t_0}{\pi}$ is an odd, positive integer, which of the following statements is valid?

a) $x_3 > x_1 > x_2$ b) $x_3 > x_2 > x_1$ c) $x_2 > x_3 > x_1$ d) cannot be determined e) NOTA

4. If for some real a, $a^{a^{a^{\mathrm{N}}}} = -3$, solve for a.

a) $-\frac{1}{27}$ b) 1 c) $\log_3(3)$ d) $(-3)^{-\frac{1}{3}}$ e) NOTA

5. Let c be a complex number and $\bar{c}$ denote its complex conjugate. If $c \cdot \bar{c} = 8$ and $c + \bar{c} = 5$, then what is the absolute value of the imaginary part of c?

a) 0 b) $\frac{5}{2}$ c) 3 d) $\sqrt{39}$ e) NOTA

6. Let a, b, c, and d be real numbers such that $a = b^2$, $b = c^3$, and $c = d^2$. If $a = 64$, then how many real solutions are there for d?

a) 1 b) 2 c) 4 d) 12 e) NOTA

7. A polynomial is said to "split" over **R** (the field of real numbers) if all of its solutions are real. Which of the following equations splits over **R**?

a) $x^2+x+1=0$ b) $x^3+x^2-2=0$ c) $x^3+3x^2+3x+1=0$ d) $x^3+x^2+x+1=0$
e) NOTA

8. What is the domain of the function $f(x)=\tan^{-1}(\sqrt{-\log_2(x^2)})$?

a) $\{x \mid 0<x\le 1\}$ b) $\{x \mid -1\le x\le 1,\ x\ne 0\}$
c) $\{x \mid 0<x\le \frac{\sqrt{2}}{2}\}$ d) $\{x \mid -\frac{\sqrt{2}}{2}\le x\le \frac{\sqrt{2}}{2}, x\ne 0\}$ e) NOTA

9. If $\sqrt[3]{\ln(\sin(2x))}$ is equal to 0, then which of the following is equal to $\cos^4(x)+\sin^4(x)-2\cos^2(x)\sin^2(x)$?

a) 0 b) 1 c) –1 d) ± 1 e) NOTA

10. For the real numbers *p*, *q*, *r*, *s*, and *t*, it is known that $p=q$, $s\le r$, and $t<p$. Which of the following inequalities is necessarily false?

a) $s+t\ge r+p$
b) $r+t\ge s+p$
c) $2t<p+q$
d) $s\ge r$
e) NOTA

11. Let *b* be a real number such that $b>1$ and $\log_b 2-\log_{2b} 2=\frac{1}{6}$. Solve for *b*.

a) 2 b) 4 c) 8 d) 64 e) NOTA

12. A right triangle has side lengths *a*, *b*, and *c*, where *a*, *b*, and *c* are positive integers, $|a-b|\le 1$, and at least one of the sides has length 7. What is the perimeter of the triangle?

a) $13+\sqrt{13}$ b) 50 c) 56 d) 1250 e) NOTA

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13. What is the area of the shape defined by the polar graph of the solution set of $r < \sin\theta$?

a) $\frac{\pi}{4}$ b) $\frac{\pi}{2}$ c) π d) 4π e) NOTA

14. A second-degree polynomial equation, $p(x) = ax^2 + bx + c = 0$, where *a*, *b*, and *c* are real numbers, has two distinct real integer solutions for *x*. Which of the following statements is (are) guaranteed to be true?

I. *a*, *b*, and *c* are integers
II. There is no integer *d* such that $\frac{c}{a} = d^2$.
III. If $a > 0$, the *y*-coordinate of the vertex of the parabola *p*(*x*) is less than 0.

a) III only b) I and II only c) I and III only d) II and III only e) NOTA

15. Which of the following pairs are not necessarily equal for any real number z?

a) sin(z) and cos(π/2 – z) b) sin(z)cos(z) and $\frac{1}{2}\sin(2z)$ c) $\sin(\frac{z}{2})$ and $\sqrt{\frac{1}{2}(1-\cos(z))}$
d) $2\cos^2(z)$ and 1 + cos(2z) e) NOTA

16. Let *n* be a positive integer such that $n = 9a = 223b + 2 = 2007c + x$ for some integers *a*, *b*, and *c*. If $0 \le x < 2007$, solve for *x*.

a) 2 b) 225 c) 446 d) 450 e) NOTA

17. What are all the real values of $x \in [0, 2\pi]$ for which $\cos(x) \ge \sin(x)$?

a) $x \in [0, \frac{\pi}{4}]$ b) $x \in [\frac{\pi}{4}, \frac{5\pi}{4}]$ c) $x \in [0, \frac{\pi}{4}] \cup [\pi, \frac{5\pi}{4}]$ d) $x \in [0, \frac{\pi}{4}] \cup [\frac{5\pi}{4}, 2\pi]$
e) NOTA

18. A sequence of positive numbers, $a_1, a_2, a_3, \ldots$ is defined by $a_n = \frac{2007^n}{n!}$. At first, for smaller values of *n*, $a_{n+1} > a_n$. What is the first positive integer *n* for which $a_{n+1} < a_n$?

a) 6 b) 7 c) 2006 d) 2007 e) NOTA

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19. You are given the following set of simultaneous equations for the unknowns *x*, *y*, and *z*:

$$ax + by + cz = 0$$
$$dx + ey + fz = 0$$
$$gx + hy + iz = 0$$

If *a*, *b*, *c*, *d*, *e*, *f*, *g*, *h*, and *i* are real constants, but you don't know their values, which of the following scenarios do you know is (are) NOT possible for the solution set (*x*, *y*, *z*)?

I. There is no consistent solution to the equations.
II. There is exactly one consistent solution to the equations.
III. There is a finite number of solutions to the equations, but more than one.
IV. There is an infinite number of solutions to the equations.

a) III only b) I and III only c) IV only d) I, III, and IV only e) NOTA

20. The equation $p(x) = 0$, where $p(x)$ is an eighth degree polynomial of *x*, has its eight solutions for *x* in the form $x = \text{cis}\left(\frac{2\pi n}{9}\right)$ for $n = 1, 2, 3, \ldots, 8$. If $p(0) = 1$, then what is $p(1)$?

a) 0 b) 8 c) 9 d) undefined e) NOTA

21. Given the system of equations below:

$$x + y + z = 10$$
$$x^2 - y^2 = 8$$
$$-x - y + z = 2$$

Which of the following third-degree polynomials equations has solutions equal to *x*, *y*, and *z*?

a) $t^3 - 10t^2 + 27t - 18 = 0$ b) $t^3 - 2t^2 - 21t - 18 = 0$ c) $t^3 + 10t^2 + 27t + 18 = 0$
d) $t^3 + 2t^2 - 21t + 18 = 0$ e) NOTA

22. The ellipses described by the implicit functions $x^2 + 4y^2 + 8(y - x) = -16$ and $4x^2 + y^2 - 32x + 2y = -61$ are drawn on a graph. Connecting the four focal points makes a quadrilateral of what area?

a) 1 b) $4\sqrt{3}$ c) $\frac{3}{2}$ d) 6 e) NOTA

23. Let matrix $A=\begin{bmatrix}4 & 5\\ 3 & 4\end{bmatrix}$, and B be another 2x2 matrix such that $AB=BA$. Which of the following matrices could NOT be B?

a) $\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ b) $\begin{bmatrix}4 & 5\\ 3 & 4\end{bmatrix}$ c) $\begin{bmatrix}4 & -5\\ -3 & 4\end{bmatrix}$ d) $\begin{bmatrix}1 & 5\\ 3 & 1\end{bmatrix}$ e) NOTA

24. What is the area of the triangle defined by the graph of the solutions set of $2y<x$ and $x\geq -y$, and $x\leq 2$?

a) 3 b) 4 c) 6 d) does not form a triangle e) NOTA

25. What is the volume of the prism graphed in the *x*-*y*-*z* space by the inequalities $|x|\leq \frac{1}{2}$, $2<y<4$, and $0\leq z\leq \frac{3}{2}$?

a) $\frac{3}{4}$ b) $\frac{3}{2}$ c) 3 d) 6 e) NOTA

26. Find the product of the six non-real solutions of
$2x^7-4x^6+14x^5-22x^4+28x^3-26x^2+16x-8=0$.

a) –8 b) 4 c) –4 d) 2 e) NOTA

27. I roll two fair, standard six-sided dice. Let *S* be the sum of the numbers they land on. Which of the following equations and inequalities is has the GREATEST probability of being true?

a) $S<3$ b) $S\leq 6$ c) $S\geq 7$ d) $S=7$ e) NOTA

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28. A piece of ice is sitting in a funnel and is melting at a constant rate of $\pi/3$ cubic centimeters per second. The ice starts out as a perfect sphere with a radius of $\sqrt[3]{16}$, and as it melts, it maintains the shape of a perfect sphere. The melted ice drips down the funnel and lands on an extremely cold surface, where it freezes instantly into a cone shape. As this ice cone grows, it keeps the shape of a cone with a height 4 times the radius. After how many seconds since the sphere started melting will the radius of the sphere be equal to the radius of the cone below it?

a) 32 b) 56 c) 8 d) $16 \cdot 17^{-1/3}$ e) NOTA

29. The function $f(x) = \sinh(x) = \dfrac{e^x - e^{-x}}{2}$ is known as the hyperbolic sine function, and the function $f(x) = \cosh(x) = \dfrac{e^x + e^{-x}}{2}$ is known as the hyperbolic cosine function. Which of the following trigonometric identities with sine and cosine does NOT hold true for their hyperbolic counterparts?

a) sin(-x) = -sin(x) b) cos(-x) = cos(x) c) sin(2x) = 2sin(x)cos(x)
d) $\sin^2(x) + \cos^2(x) = 1$ e) NOTA

30. For any x, which of the following is always greater than or equal to $\sin^2(x)$?

a) $\cos^2(x)$ b) cos(2x) c) sin(x) d) |sin(x)| e) NOTA