

**Alpha Gemini Test**  
**2007 Mu Alpha Theta National Convention**

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For all questions, answer E. "NOTA" means none of the above answers is correct.

1. When "1776" is written out in Roman numerals, what is the letter in the middle?  
A. X    B. L    C. C    D. D    E. NOTA
2. A triangle has sides with lengths  $a-3$ ,  $a$ , and  $a+3$ . The area of this triangle is equal to 6 times the perimeter. What is the perimeter?  
A.  $6\sqrt{145}$     B. 42    C. 126    D.  $18\sqrt{145}$     E. NOTA
3. In a game of poker, Jen is dealt 5 cards completely at random from a standard deck of 52 cards. What is the probability that all of her cards are the same suit?  
A.  $33/16660$     B.  $99/49990$     C.  $33/66630$     D.  $99/199920$     E. NOTA
4. What is the domain of the function  $y = \ln(\ln(\ln(x)))$ ?  
A.  $x > 1$     B.  $x < 1$     C.  $x > e$     D.  $x > 0$     E. NOTA
5. Simplify the following expression for  $a$  rational and not equal to  $-1$ :  
$$(1+a^{x-y})^{-1} + (1+a^{y-x})^{-1}$$
  
A. 0    B.  $a^{x-y} + a^{y-x}$     C.  $a^{x-y} - a^{y-x}$     D. 1    E. NOTA
6. Which of the following equations represents a lemniscate?  
A.  $r^2 = 25 \cos 2\theta$     B.  $4 = 3 + r \cos \theta$     C.  $r = 2(1 + \cos \theta)$     D.  $r = 4\theta$     E. NOTA
7. What is the value of  $\tan(\cos^{-1}(3/5) - \sin^{-1}(5/13))$  if you assume that the  $\cos^{-1}(x)$  and  $\sin^{-1}(x)$  functions both return angles ranging from 0 to 90 degrees.  
A.  $33/56$     B.  $-33/56$     C.  $33/16$     D.  $-33/16$     E. NOTA
8. The force of gravity between two objects varies jointly as the product of their masses and inversely as the square of the distance between them. If the original force between two objects is  $f$ , then what is the force if one of the masses is tripled, the other is quadrupled, and the distance between them is cut in half?  
A.  $3f$     B.  $12f$     C.  $48f$     D.  $24f$     E. NOTA

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9. Which of the following is the inverse matrix of  $\begin{bmatrix} 2 & 5 & -1 \\ 3 & -4 & -1 \\ 5 & 1 & -2 \end{bmatrix}$ ?

- A.  $\begin{bmatrix} 2 & 7 & -2 \\ -1 & -3 & 1 \\ -2 & -1 & 1 \end{bmatrix}$       B.  $\begin{bmatrix} 2/7 & 1 & -2/7 \\ -1/7 & -3/7 & 1/7 \\ -2/7 & -1/7 & 1/7 \end{bmatrix}$
- C.  $\begin{bmatrix} 2 & -5 & 1 \\ -3 & -4 & 1 \\ -5 & -1 & -2 \end{bmatrix}$       D.  $\begin{bmatrix} -1 & 0 & 2 \\ -2 & -1 & -1 \\ 0 & 2 & -2 \end{bmatrix}$       E. NOTA

10. What is the distance between the two polar-coordinate points  $(5, 60^\circ)$  and  $(5\sqrt{2}, 15^\circ)$ ?

- A.  $5\sqrt{5}$     B. 5    C.  $5\sqrt{2}$     D.  $5\sqrt{3}$     E. NOTA

11.  $1/3 + 1/15 + 1/35 + 1/63 + \dots = ?$

- A.  $1/2$     B.  $3/7$     C.  $2/3$     D. 1    E. NOTA

12. A storm chaser at 3 PM notices that an east-west oriented cloud line is 100 miles due south of his position and heading north at 40 miles an hour. At the same time, another such east-west line is 60 miles due north of his position heading south at 20 miles an hour. If he wants to go to the spot closest to his current location where the two lines will collide, assuming they are at the same height above the ground and they keep moving at the same speed, which choice below represents this location relative to his current location?

- A.  $20/3$  miles south                      B.  $40/3$  miles north  
 C. at the same location                      D.  $20/3$  miles north                      E. NOTA

13. In question number 12, at what time will the two cloud lines collide?

- A. 5:40 PM    B. 6:00 PM    C. 5:45 PM    D. 6:20 PM    E. NOTA

14. If  $a$  and  $b$  are positive constants, then solve for  $x$  in the following equation:

$$\begin{vmatrix} a-x & \sqrt{ab} \\ \sqrt{ab} & b-x \end{vmatrix} = 0$$

- A.  $x = \{0\}$     B.  $x = \{0, a-b\}$     C.  $x = \{a-b, a+b\}$     D.  $x = \{0, a+b\}$     E. NOTA

15.  $33_9 \times 44_9 = ?$

- A.  $5500_9$     B.  $1200_9$     C.  $1452_9$     D.  $1563_9$     E. NOTA

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16. What conic section is described by the equation:

$$7x^2 + 3xy - 4y^2 + 5x - 2y + 7 = 0$$

- A. ellipse   B. hyperbola   C. parabola   D. circle   E. NOTA

17. A fast food chain offers customers 10 flavor additions that can be added to whatever soda they desire. You could add one flavor (e.g., chocolate) or you could add up to all 10 flavors. How many different flavor addition combinations can someone add to a drink of coke (adding none is not an option!)?

- A. 3628801   B. 3628800   C. 1024   D. 1023   E. NOTA

18. A running track is made in the standard shape of two semicircles attached to opposite ends of a rectangle. One such track has straightaways that are 150 yards each, and a person walking the entire inner lap (lane 1) would walk a quarter mile. Each additional outside lane is one yard wide. If a person walks 4 laps around the track on the farthest out lane (lane 6 for this track), then how many extra yards did he walk over someone who did 4 laps on lane 1?

- A.  $24\pi$    B.  $20\pi$    C.  $10\pi$    D.  $12\pi$    E. NOTA

19. Solve for  $x$  in the equation below:

$$\langle x^2 + 3x, 4x \rangle \bullet \langle x - 3, x + 3 \rangle = 0,$$

where the dot represents the dot product of the two vectors.

- A.  $x = \{0, -1, -3\}$    B.  $x = \{1, 3\}$    C.  $x = \{0, 1, 3\}$    D.  $x = \{-1, -3\}$    E. NOTA

20. A snail wants to get up a wall 100 feet in height. During the day it climbs 5 feet, but each night it slips back 4 feet. How many days would it take to reach the top if it starts at the bottom?

- A. 95   B. 96   C. 99   D. 100   E. NOTA

21. How many solutions are there to the equation below within the domain  $x = [0, 2\pi]$ ?

$$2\sin^2(2x) + \sin(2x) - 1 = 0$$

- A. 3   B. 4   C. 6   D. 8   E. NOTA

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22. Which of the following choices is a unit vector parallel to the vector sum of the vectors  $\langle 3, 2, -1 \rangle$  and  $\langle -10, 4, -5 \rangle$ ?
- A.  $\langle 7/11, 6/11, 6/11 \rangle$       B.  $\langle 13\sqrt{21}/63, 2\sqrt{21}/63, 4\sqrt{21}/63 \rangle$   
C.  $\langle -7/11, 6/11, -6/11 \rangle$       D.  $\langle 13\sqrt{21}/63, -2\sqrt{21}/63, 4\sqrt{21}/63 \rangle$       E. NOTA
23. Which of the following choices is equal to  $i^i$ , where  $i = \sqrt{-1}$ ?
- A.  $-1$     B.  $e^{\pi/2}$     C.  $1$     D.  $e^{-\pi/2}$     E. NOTA
24. A certain Mu Alpha Theta student is trying to decide on a 4-digit PIN for a password on a website. He wants to make it easy to remember, so he wants it to either be a combination of 2 different 2-digit square numbers (e.g., 1649, or  $4^2$  and  $7^2$ ) or a 4-digit square by itself (e.g., 1024 is  $32^2$ ). How many distinct PINs are available?
- A. 103    B. 97    C. 98    D. 104    E. NOTA
25. For the equation  $x^2 + x - pq = 0$ ,  $p$  and  $q$  are prime numbers. How many different sets of roots are possible if both roots must be rational?
- A. 0    B. 1    C. 2    D. Infinite    E. NOTA
26. Eddie travels 100 miles at the rate of 50 mph, and then on a freeway the next 100 miles at the rate of 70 mph. What is his average speed (in miles per hour)?
- A.  $10\sqrt{35}$     B.  $175/3$     C. 60    D.  $185/3$     E. NOTA
27. What is the area of the region enclosed by the graph of  $9x^2 + y^2 - 36x - 2y + 28 = 0$ ?
- A.  $81\pi$     B.  $27\pi$     C.  $3\pi$     D.  $9\pi$     E. NOTA
28. What is the period of the graph of  $y = \tan(8x - 10)$ ?
- A.  $\pi/16$     B.  $\pi/8$     C.  $8\pi$     D.  $16\pi$     E. NOTA
29. If  $f(x) = 3x - 5$  and  $g(x) = 2x^2 + 5x - 3$ , then what values of  $x$  satisfy  $f(g(x)) = 0$ ?
- A.  $1/2, -3$     B.  $-1/2, 3$     C.  $\frac{-15 \pm \sqrt{561}}{12}$     D.  $\frac{15 \pm \sqrt{561}}{12}$     E. NOTA
30. An open rectangular box has height 4 and a square base. The volume of the box is 48. What is the total area of all exposed surfaces (assume that the thickness of the walls is 0)?
- A.  $32\sqrt{3} + 24$     B.  $32\sqrt{3} + 12$     C.  $64\sqrt{3} + 24$     D.  $64\sqrt{3} + 48$     E. NOTA