

ALPHA INDIVIDUAL SOLUTIONS  
2007 Mu Alpha Theta National Convention

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1

1.  $x - 2y = 5 \Rightarrow \text{slope} = \frac{1}{2}$   
 Perpendicular line through  
 $(2,5) \Rightarrow (y-5) = -2(x-2)$   
 $y = -2x + 9$   
 $y$ -intercept = 9      C

2.  $2g^{2x} + 2^x g^x - 3^{2x} = 0$   
 $(2g^x - 3^x)(2^x + 3^x) = 0$   
 $2g^x = 3^x ; 2^x \neq -3^x$   
 $\left(\frac{2}{3}\right)^x = \frac{1}{2}$   
 $\log_{\frac{2}{3}} \frac{1}{2} = x$       (A)

3.  ${}_5C_3 + {}_3P_2 - 4!$   
 $10 + 6 - 24 = -8$       (E)

4.  $\log_x x^{x^2} + \log_x x^{-5x} = \log_x \left(\frac{1}{x^6}\right)$   
 $x^2 - 5x = -6$   
 $x = 2 \text{ or } 3$       (B)

5.  $\det M = \det \begin{pmatrix} 2 & 4 & k \\ 2 & -1 & 7 \\ 3 & -2 & 11 \end{pmatrix} = (-22 + 84 - 4k) - (-3k - 28 + 88)$   
 $= 2 - k = 0; k = 2$       (D)

6.  $16 \cos \frac{\pi}{16} \sin \frac{\pi}{16} \cos \frac{\pi}{8} \cos \frac{\pi}{4}$   
 $8 \sin \frac{\pi}{8} \cos \frac{\pi}{8} \cos \frac{\pi}{4}$   
 $4 \sin \frac{\pi}{4} \cos \frac{\pi}{4}$   
 $2 \sin \frac{\pi}{2}$   
 $2$       (D)

7.  $\left(-1 + \cos \frac{\pi}{3} + i \sin \frac{\pi}{3}\right)^{36} = \left(\frac{-1}{2} + i \frac{\sqrt{3}}{2}\right)^{36}$   
 $= \left(1 \text{cis} \frac{2\pi}{3}\right)^{36}$   
 $= 1^{36} (\cos 24\pi)$   
 $= 1 + 0i$   
 $\Rightarrow 1^2 + 0^2 = 1$       (D)

8.  $S_n = \frac{n(n+1)(2n+1)}{6}; S_3 = \frac{3 \cdot 4 \cdot 7}{6} = 14$   
 $S_4 = \frac{4 \cdot 5 \cdot 9}{6} = 30$   
 $S_5 = \frac{5 \cdot 6 \cdot 11}{6} = 55$

Squares total =  $14 + 30 + 55 = 99$       (B)

9.  $e^{(4x^3 - 3x^2 - 2x + 4)} = \ln e = 1 = e^0$   
 $\Rightarrow 4x^3 - 3x^2 - 2x + 4 = 0$   
 Sum roots =  $\frac{3}{4}$ , Product = -1  
 Quotient =  $\frac{\text{sum}}{\text{product}} = \frac{-3}{4}$       (B)

10. By defn focus is on the major axis.  
 Major Axis =  $2a$ ; Minor axis =  $2b = 4\sqrt{5}$   
 let  $c = x \Rightarrow a = x + 2$   
 $\Rightarrow (x+2)^2 = x^2 + (2\sqrt{5})^2$   
 $\Rightarrow x^2 + 4x + 4 = x^2 + 20$   
 $\Rightarrow 4x = 16$   
 $\Rightarrow x = 4 \quad 2a = 2(x+2) = 12$       (C)

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$$11. (\sin x + \cos x)^2 = \left(\frac{-1}{5}\right)^2$$

$$\sin^2 x + 2 \sin x \cos x + \cos^2 x = \frac{1}{25}$$

$$\sin 2x = \frac{-24}{25} \quad (A)$$

$$12. \frac{7+3i}{5-2i} \cdot \frac{5+2i}{5+2i} = \frac{29+29i}{29}$$

$$= 1+i$$

$$= \sqrt{2} \left( \cos \frac{\pi}{4} + i \sin \frac{\pi}{4} \right) \quad (B)$$

$$13. 9 \left( \frac{3}{x-1} \right) = \frac{3}{9x-1}$$

$$27(9x) - 27 - 3x = -3$$

$$240x = 24, \quad x = \frac{1}{10} \quad (B)$$

$$14. \cos \theta = \frac{-4(2) + 2(-1)}{\sqrt{20} \sqrt{14}}$$

$$= \frac{-\sqrt{70}}{14} \quad (B)$$

$$15. f(-x) = (-x+3)^2 + (-x-3)^2$$

$$= (-1)^2(x-3)^2 + (-1)^2(x+3)^2$$

$$= f(x)$$

$$f(x) = |f(x)| \quad (C)$$

$$16. x = \frac{5y-2}{3y+4}$$

$$3xy + 4x = 5y - 2$$

$$4x - 2 = 5y - 3xy$$

$$f^{-1}(x) = \frac{4x-2}{5-3x}, \quad x \neq \frac{5}{3} \quad (E)$$

$$17. 5 > |4 - x^2|$$

$$5 - (x^2 - 4) > 0, \quad x < -2 \text{ or } x > 2$$

$$9 - x^2 > 0 \Rightarrow -3 < x < 3$$

$$5 > |4 - x^2|; \quad 5 - (4 - x^2) > 0, \quad -2 < x < 2$$

$$1 + x^2 > 0 \text{ for all } x \in \mathbb{R} \Rightarrow \text{intersection: } -3 < x < 3 \quad (C)$$

$$18. \log_A(\log_3 9) = 10^0 = 1$$

$$\log_A(2) = 1$$

$$A = 2 \quad (A)$$

$$19. \sqrt{a} + \sqrt{b} = \sqrt{9+4\sqrt{5}}$$

$$a + 2\sqrt{a}\sqrt{b} + b = 9 + 4\sqrt{5}$$

$$a + b = 9; ab = 20 \Rightarrow a = 4, b = 5$$

$$\sqrt{9+4\sqrt{5}} = 2 + \sqrt{5}$$

$$\text{Similarly } \sqrt{9-4\sqrt{5}} = 2 - \sqrt{5}$$

$$\Rightarrow \sqrt{9+4\sqrt{5}} - \sqrt{9-4\sqrt{5}} = 2 + \sqrt{5} + 2 - \sqrt{5}$$

$$= 4 \quad (A)$$

$$20. \text{Arccos}(\sin(\text{Arccot}(\sec 0)))$$

$$\text{Arccos}(\sin(\text{Arccot}(1)))$$

$$\text{Arccos}\left(\sin \frac{\pi}{4}\right)$$

$$\frac{\pi}{4} \quad (C)$$

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$$\begin{aligned}
 21. \quad & \frac{1}{2i} \left( e^{\frac{\pi}{2}i} - e^{-\frac{\pi}{2}i} \right) + \frac{3\pi}{2}i - 3! \\
 & \Rightarrow \sin \frac{\pi}{2} + \frac{3\pi}{2}i - 3! \\
 & \Rightarrow 1 + \frac{3\pi}{2}i - 6 \\
 & \Rightarrow \frac{3\pi}{2}i - 5 \qquad (D)
 \end{aligned}$$

$$\begin{aligned}
 23. \quad & -2 + 2 + \frac{4}{3} + \frac{2}{3} + \frac{-8}{9} + \frac{2}{9} + \dots \\
 & \left( -2 + \frac{4}{3} + \frac{-8}{9} + \dots \right) + \left( 2 + \frac{2}{3} + \frac{2}{9} + \dots \right) \\
 & \frac{-2}{1 - \left( \frac{-2}{3} \right)} + \frac{2}{1 - \frac{2}{3}} = -\frac{6}{5} + \frac{15}{5} \\
 & = \frac{9}{5} \qquad (D)
 \end{aligned}$$

$$\begin{aligned}
 25. \quad & P(\text{at least one passes}) = 1 - P(\text{neither passes}) \\
 & = 1 - (1-p)(1-q) \\
 & = 1 - (1-p-q+pq) \\
 & = p+q-pq \qquad (C)
 \end{aligned}$$

$$\begin{aligned}
 26. \quad & \lim_{x \rightarrow \sqrt{3}} \frac{x - \sqrt{3}}{(x + \sqrt{3})(x - \sqrt{3})} = \frac{1}{2\sqrt{3}} \\
 & = \frac{\sqrt{3}}{6} \qquad (D)
 \end{aligned}$$

$$\begin{aligned}
 29. \quad & \text{pt on } P_1(-1, 1, 0) \\
 d &= \frac{|3(-1) - 2(1) - 4|}{\sqrt{9+4+1}} \\
 &= \frac{9\sqrt{14}}{14} \qquad (B)
 \end{aligned}$$

$$22. \quad 27x^3 - kx^2 + 38x - 8 = 0 ;$$

3 roots  $\frac{x}{r}, x, rx$

$$\text{product } x^3 = \frac{8}{27}, x = \frac{2}{3}$$

$$27 \left( \frac{8}{27} \right) - k \left( \frac{4}{9} \right) + 38 \left( \frac{2}{3} \right) - 8 = 0$$

$$k = 38 \left( \frac{2}{3} \right) \left( \frac{9}{4} \right) = 57 \quad (D)$$

$$\begin{aligned}
 24. \quad & \binom{8}{4} \binom{4}{3} (2x)^4 (-3y)^3 (z) \\
 & \Rightarrow \binom{8}{4} \binom{4}{3} (2x)^4 (-3y)^3 (z) \\
 & \Rightarrow -120,960x^4y^3z \qquad (C)
 \end{aligned}$$

$$\begin{aligned}
 27. \quad & x(1+y) = 5y - 2 \\
 & x = \frac{5y-2}{1+y}, y \neq -1 \qquad (B)
 \end{aligned}$$

$$\begin{aligned}
 28. \quad & \frac{x^2+3x-1}{(x^2+2)(x-1)} = \frac{a}{x^2+2} + \frac{b}{x-1} \\
 & x^2+3x-1 = a(x-1) + b(x^2+2) \\
 & x=1 \Rightarrow 3=3b, b=1 \\
 & x=0 \Rightarrow -1=-a+2, a=3 \\
 & \therefore a^2+b^2=10 \qquad (C)
 \end{aligned}$$

$$\begin{aligned}
 30. \quad & 3^{2007} = 10^{\log 3^{2007}} \\
 & = 10^{2007 \log 3} \\
 & = 10^{957.57984} \\
 & \Rightarrow \text{no. of digits } 957 + 1 \qquad (B)
 \end{aligned}$$

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1. C
2. A
3. E
4. B
5. D
6. D
7. D
8. B
9. B
10. C
11. A
12. B
13. B
14. B
15. C
16. E
17. C
18. A
19. A
20. C
21. D
22. D
23. D
24. C
25. C
26. D
27. B
28. C
29. B
30. B