

Alpha Individual Test
2007 Mu Alpha Theta National Convention

For all questions, answer E. NOTA means “none of the above answers is correct”. The letter “I” = $\sqrt{-1}$ for the entire test.

1. Find the y-intercept of the line passing through (2, 5) that is perpendicular to the line $x - 2y = 5$.
A. 2 B. 4 C. 9 D. 12 E. NOTA

2. Bo **correctly** solved $2(4^x) - 9^x = -6^x$ and found $x = \log_{\frac{2}{3}} b$. Find the numerical value of b .
A. 0.5 B. 0.65 C. 0.75 D. 1.25 E. NOTA

3. Even silly Sally could simplify ${}_5C_3 + {}_3P_2 - 4!$. She correctly found the expression to be ${}_?_?$.
A. -11 B. 6 C. 39 D. 42 E. NOTA

4. Determine x in the following expression: $\log_x x^{x^2} + \log_x x^{-5x} = \log_x \left(\frac{1}{x^6} \right)$.
A. $x = 1, x = 2$ B. $x = 2, x = 3$ C. $x = 1, x = 3$ D. $x = 4$ E. NOTA

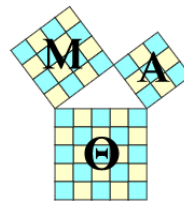
5. For what value of the constant k did Kerry find the inverse of matrix M not to exist?
$$M = \begin{pmatrix} 2 & 4 & k \\ 2 & -1 & 7 \\ 3 & -2 & 11 \end{pmatrix}.$$

A. -1 B. -2 C. 0 D. 2 E. NOTA

6. Erik was excited to see $16 \cos\left(\frac{\pi}{16}\right) \cos\left(\frac{\pi}{8}\right) \cos\left(\frac{\pi}{4}\right) \sin\left(\frac{\pi}{16}\right)$. He knew the answer was ${}_?_?$.
A. 0.5 B. $\frac{\sqrt{2}}{2}$ C. 1 D. 2 E. NOTA

7. Given $\left(-1 + \cos\frac{\pi}{3} + i \sin\frac{\pi}{3}\right)^{36} = x + yi$. Find $x^2 + y^2$.
A. -2 B. 0.5 C. 0 D. 1 E. NOTA

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8. The official Mu Alpha Theta emblem (shown) is the classic three squares adjacent to the sides of a 3-4-5 right triangle. Each original square is composed of unit squares, the side of the original square each has magnitude matching a side of the triangle. How many squares are there in the symbol?
- A. 98 B. 99 C. 100 D. 101 E. NOTA
9. Given the equation $e^{(4x^3-3x^2-2x+4)} = \ln e$. Find the quotient of the sum of all solutions x and the product of all the solutions x .
- A. $\frac{-4}{3}$ B. $\frac{-3}{4}$ C. $\frac{3}{4}$ D. 1 E. NOTA
10. Given an ellipse where the distance between a focus and the closer vertex is 2. If the length of the minor axis is $4\sqrt{5}$, find the length of the major axis.
- A. 6 B. 8 C. 12 D. 24 E. NOTA
11. If $\sin x + \cos x = \frac{-1}{5}$ and $\frac{3\pi}{4} \leq x \leq \pi$, find $\sin 2x$.
- A. $\frac{-24}{25}$ B. $\frac{25}{24}$ C. $\frac{25}{7}$ D. 25 E. NOTA
12. Simplify $\frac{7+3i}{5-2i}$ and give your correct answer in an equivalent polar form.
- A. $-\sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$ B. $\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$ C. $\sqrt{2}\left(\cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4}\right)$
D. $\sqrt{2}\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right)$ E. NOTA
13. Find the value of x which satisfies $f(g(x)) = g(f(x))$ where $g(x) = 9x$ and $f(x) = \frac{3}{x-1}$.
- A. $\frac{-1}{10}$ B. $\frac{1}{10}$ C. $\frac{5}{41}$ D. $\frac{8}{9}$ E. NOTA
14. Find the cosine of the angle between vectors $-4i + 2k$ and $2i + 3j - k$.
- A. $\frac{-\sqrt{70}}{20}$ B. $\frac{-\sqrt{70}}{14}$ C. $\frac{-3\sqrt{14}}{14}$ D. $\frac{\sqrt{70}}{20}$ E. NOTA

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15. If $f(x) = (x + 3)^2 + (x - 3)^2$ for all real numbers, which of the following are true?

- I. $f(-x) = f(x)$
- II. $f(x) = f(x - 1)$
- III. $f(x) = |f(x)|$

A. I B. I, II only C. I, III only D. I, II and III E. NOTA

16. If $f(x) = \frac{5x-2}{3x+4}$, what value of x must be excluded from the domain of $f^{-1}(x)$?

A. $-\frac{4}{3}$ B. $-\frac{2}{3}$ C. $\frac{2}{5}$ D. $\frac{3}{5}$ E. NOTA

17. Solve $5 > |4 - x^2|$ for $x \in \mathbb{R}$.

A. $x < -3$ or $x > 3$ B. $x < \frac{4}{5}$ or $x > \frac{5}{4}$ C. $-3 < x < 3$ D. $x > 3$ E. NOTA

18. Find A given that $\log(\log_A(\log_3 9)) = 0$.

A. 2 B. 3 C. 6 D. 10 E. NOTA

19. Simplify: $\sqrt{9+4\sqrt{5}} - \sqrt{9-4\sqrt{5}}$.

A. 4 B. 4.5 C. 5 D. 6 E. NOTA

20. $\text{Arccos}(\sin(\text{Arccot}(\sec 0))) = \underline{\quad? \quad}$

A. 0 B. $\frac{\pi}{6}$ C. $\frac{\pi}{4}$ D. $\frac{\pi}{3}$ E. NOTA

21. Simplify: $\frac{1}{2i} \left(e^{\frac{\pi}{2}i} - e^{-\frac{\pi}{2}i} \right) + \frac{3\pi}{2}i - 3!$.

A. $\frac{3\pi}{2}i - 6$ B. $\frac{3\pi}{2}i - 4$ C. $\frac{-3\pi}{2}i - 6$ D. $\frac{3\pi}{2}i - 5$ E. NOTA

22. Find the value of the constant k in the equation $27x^3 - kx^2 + 38x - 8 = 0$ given the solutions form a geometric sequence of real numbers.

A. -57 B. -33 C. 33 D. 57 E. NOTA

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23. Evaluate the series: $-2 + 2 + \frac{4}{3} + \frac{2}{3} + \frac{-8}{9} + \frac{2}{9} + \frac{16}{27} + \frac{2}{27} \dots$

- A. $\frac{4}{5}$ B. $\frac{6}{5}$ C. $\frac{7}{5}$ D. $\frac{9}{5}$ E. NOTA

24. Vikram found the correct coefficient of the term containing x^4y^3z in the expansion of $(2x - 3y + z)^8$ to be:

- A. -241,920 B. -220,960 C. -120,960 D. 241,920 E. NOTA

25. The probability of Denis passing a test is p . However, the probability of Martin passing the same test is q . What is the probability that if both Denis and Martin are taking this test that at least one of them passes it?

- A. $1 - pq$ B. $p^2 + q^2 - 2pq$ C. $p + q - pq$ D. $2pq - p - q$ E. NOTA

26. Suzie and Jon in Mr. Baxter's precalc class have been arguing about a problem. Suzie says

that $\lim_{x \rightarrow \sqrt{3}} \frac{x - \sqrt{3}}{x^2 - 3}$ cannot be found and Jon says that it can. Jon shows Suzie the correct limit to be ?

- A. undefined B. $-\sqrt{3}$ C. $-\frac{1}{2}$ D. $\frac{\sqrt{3}}{6}$ E. NOTA

27. If x is real and $x + xy - 5y + 2 = 0$, then y cannot be ? .

- A. -2 B. -1 C. $\frac{2}{5}$ D. 1 E. NOTA

28. Given: $\frac{x^2 + 3x - 1}{(x^2 + 2)(x - 1)} = \frac{a}{x^2 + 2} + \frac{b}{x - 1}$. Find $a^2 + b^2 =$? .

- A. 8 B. 9 C. 10 D. 11 E. NOTA

29. Anisha just can't get enough of analytic geometry and set out to find the distance between the planes $P_1: 6x - 4y + 2z + 10 = 0$ and $P_2: 3x - 2y + z = 4$. She correctly found the distance to be ? .

- A. $\frac{\sqrt{14}}{14}$ B. $\frac{9\sqrt{14}}{14}$ C. $\sqrt{14}$ D. 14 E. NOTA

30. Billy, having nothing else to wonder about, decided to find the number of digits in 3^{2007} . He memorized in precalc class that $\log 3 \approx 0.47712$. He then found the number of digits to be ? .

- A. 956 B. 958 C. 960 D. 962 E. NOTA