Logs/Exponents/Radicals - Alpha
2007 Mu Alpha Theta National Convention
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"For all questions, answer E. "NOTA" means none of the above answers is correct."

1. Simplify: $\left(3^2\right)^3$.

   A. 3^6 B. 3^7 C. 3^8 D. 3^9 E. NOTA

2. If $\log 2 \approx 0.301$, then what is the value of log 500 to the nearest thousandth?

   A. 0.699 B. 1.301 C. 2.301 D. 2.699 E. NOTA

The following information will be used to answer the next three questions:
$\log 375 \approx 2.5740$

3. Which of the following is the best approximation for the mantissa of log 375?

   A. 2 B. 3 C. 375 D. 0.5740 E. NOTA

4. Which of the following is the best approximation for the characteristic of log 375?

   A. 2 B. 3 C. 375 D. 0.5740 E. NOTA

5. Which of the following is the best approximation of antilog 1.5740?

   A. 37.5 B. 187.5 C. 1875 D. 3750 E. NOTA

6. Which of the following gives the number of terms in the full expansion of $\left(a+b\right)^{2007}$?

   A. ${}_{2007}C_2$ B. ${}_{2008}C_1$ C. ${}_{2007}C_1$ D. ${}_{2008}C_2$ E. NOTA

7. In question 6, what is the sum of all of the coefficients in the full expansion of the given expression?

   A. $2^{2007}-1$ B. 2^{2008} C. 2^{2007} D. $2^{2008}-1$ E. NOTA

8. What is the value of $\cot\left(15\right)$, where the argument of the expression is in degrees?

   A. $\sqrt{3}$ B. $2+\sqrt{3}$ C. $\sqrt{3}/3$ D. $2-\sqrt{3}$ E. NOTA

9. The population *P* of a certain type of single-celled organisms at time *t* is modeled by the function $P(t) = I\,e^{kt}$,where I is the initial population and k is a constant. If the population doubles every three hours, then how long, in hours, will it take the population to grow from one organism to 1000 organisms?"

A. 30 B. $9/\ln 3$
C. $\log_2 1000$ D. $9/\log 2$ E. NOTA

10. Which of the following is equivalent to $\ln(-16)$?

A. $i\pi + 4\ln 2$ B. $-4i\pi$ C. $i\pi/2 + 4\ln 2$ D. $4i\ln 2$ E. NOTA

11. What is/are the value(s) of *x* if $\sqrt{x^2 + \sqrt{x^2 + \sqrt{x^2 + \ldots}}} = 9$

A. $6\sqrt{2}$ B. $3\sqrt{10}$ C. $\pm 3\sqrt{10}$ D. $\pm 6\sqrt{2}$ E. NOTA

12. $\sqrt{5.76} = ?$

A. 2.24 B. 2.25 C. 2.42 D. 2.50 E. NOTA

13. Which of the following is equivalent to $\log 2007$?

A. $\ln 9 + \ln 223$ B. $2\log 20.07$ C. $2\log 3 + \log 223$ D. $\log 27 + \log 74$ E. NOTA

14. What is the equation of the line that bisects the acute angle formed by the lines $2x - 3y + 6 = 0$ and $3x - y + 3 = 0$?

A. $\left(2\sqrt{10} - 3\sqrt{13}\right)x + \left(-3\sqrt{10} + \sqrt{13}\right)y + 6\sqrt{10} - 3\sqrt{13} = 0$
B. $\left(2\sqrt{10} + 3\sqrt{13}\right)x - \left(3\sqrt{10} + \sqrt{13}\right)y + 6\sqrt{10} + 3\sqrt{13} = 0$
C. $\left(2\sqrt{13} - 3\sqrt{10}\right)x + \left(-3\sqrt{13} + \sqrt{10}\right)y + 6\sqrt{13} - 3\sqrt{10} = 0$
D. $\left(2\sqrt{13} + 3\sqrt{10}\right)x - \left(3\sqrt{13} + \sqrt{10}\right)y + 6\sqrt{13} + 3\sqrt{10} = 0$
E. NOTA

15. $\sqrt{4132_7} = ?$

A. 36_7 B. 41_7 C. 53_7 D. $2_7\sqrt{1034_7}$ E. NOTA

16. If $f(x) = \ln\left(x + \sqrt{x^2+1}\right)$, then what is the value of $f^{-1}(2)$?

A. $11\sqrt{5} - 22$ B. $\dfrac{e^4+1}{2e^2}$ C. $(1/2)\left(e^2 - e^{-2}\right)$ D. $(1/2)\left(e^{-2} - e^2\right)$ E. NOTA

17. Which function below is a reflection of the graph of $y = 4^x$ through the line $y = x$?

A. $y = \log_4 x$ B. $y = (1/4)\ln x$ C. $y = (1/4)\log x$ D. $y = 4\log_4 x$ E. NOTA

18. The expression $\dfrac{\left(\sqrt{x}\right)\left(\sqrt[4]{x}\right)}{\sqrt[3]{x}}$ simplifies to which of the following for $x > 0$?

A. $x^{11/12}$ B. $x^{5/12}$ C. x D. x^{-1} E. NOTA

19. What is the largest prime factor of the value represented by the expression $3^{10} - 2^{10}$?

A. 101 B. 157 C. 211 D. 919 E. NOTA

20. $\prod_{n=2}^{100} \log_n (n+2) = ?$

A. $(\log_2 101)(\log_3 102)$ B. $(\log_3 101)(\log_3 102)$
C. $\log_6 10302$ D. $\log_2 102$ E. NOTA

21. Solve for x in the equation $\ln(2x-1) + \ln(3x-2) = \ln 7$.

A. $x = -1/2$ or $5/3$ B. $x = -1/2$ or $-5/3$
C. $x = 1/2$ or $-5/3$ D. $x = 1/2$ or $5/3$ E. NOTA

22. \$3,000 is put into a certificate of deposit at 4.8% interest compounded monthly. What is the value 1 year after the start of the investment?

A. $\$3000(1.004)^{12}$ B. $\$3000(1.048)$
C. $\$3000(1.0048)^{12}$ D. $\$3000(1.04)$ E. NOTA

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23. What is the domain of the function $y = \frac{1}{\sqrt{4-x}}$?

A. $x \le 4$ B. $x > -4$ C. $x < -4$ D. $x \ge -4$ E. NOTA

24. $4^0 + 4^{-2} + 4^{-4} + 4^{-6} + \text{L} = ?$

A. 16/17 B. 16/15 C. 64/17 D. 64/15 E. NOTA

25. What is the sum of the solutions for *x*, $x \in \text{¡}$, in the equation $\sqrt[3]{x^2 + 2x - 7} = 2$.

A. 2 B. –8 C. –2 D. 8 E. NOTA

26. Which of the following is the exponential form of the equation $\log_{7x} y = 42x^2$ for $x > 1$?

A. $y = 7x^{42x^2}$ B. $y = 42x^{2^{7x}}$ C. $y = (7x)^{42x^2}$ D. $y = \left(42x^2\right)^{7x}$ E. NOTA

27. If $[x]$ represents the greatest integer function on the argument *x*, then what is $\left[\sqrt{2007}\right]$?

A. 44 B. 45 C. 46 D. 47 E. NOTA

28. Which of the choices below is the largest (Hint: $\log 2 \approx 0.3010$, $\log 3 \approx 0.4771$, and $\log 7 \approx 0.8451$)?

A. 2^{40} B. 3^{25} C. 7^{15} D. 9^{12} E. NOTA

29. Which of the following is equivalent to $\left(\cos(3\pi/8) + i\sin(3\pi/8)\right)^{10}$?

A. $\left(\frac{\sqrt{2}}{2}\right)(1+i)$ B. $-\left(\frac{\sqrt{2}}{2}\right)(1+i)$ C. $-\left(\frac{\sqrt{2}}{2}\right)(1-i)$ D. $\left(\frac{\sqrt{2}}{2}\right)(1-i)$ E. NOTA

30. Which of the following is equivalent to $\log_{100} 10000$?

A. $\log 2007$ B. $\log_{101} 12221$ C. $\log_{20.07} 2007$ D. $\log_{200} 40000$ E. NOTA