Matrices and Vectors - Alpha
Mu Alpha Theta National Convention 2007

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Directions: There are no calculators permitted on this test. In each question, the choice E. NOTA stands for "None Of These Answers"

1. If $A = \begin{bmatrix} 3 & 4 \\ 1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 5 & 6 \\ 4 & 2 & 0 \\ 12 & -3 & -7 \end{bmatrix}$, and $C = \begin{bmatrix} 3 & 4 \\ -5 & -5 \\ 0 & 8 \end{bmatrix}$, which of the following can be computed?

I. BCA II. (BC)*(CA) III. BC + CA IV. CAB

A. I, III, and IV only B. I only C. II and III only. D. I and III only E. NOTA

2. What is the trace of the matrix $\begin{bmatrix} 4 & 4 & 5 \\ 0 & 1 & 2 \\ 2 & 3 & 3 \end{bmatrix}$?

A. 4 B. 5 C. 6 D. 7 E. NOTA

3. If $A = \begin{bmatrix} 2 & -2 \\ 4 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 1.5 & 5 \\ -4 & -1 \end{bmatrix}$, what is AB – BA?

A. $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ B. $\begin{bmatrix} 12 & 0 \\ 6 & -12 \end{bmatrix}$ C. $\begin{bmatrix} -12 & 0 \\ 6 & 12 \end{bmatrix}$ D. $\begin{bmatrix} -12 & 0 \\ 0 & 12 \end{bmatrix}$ E. NOTA

4. Which of the following statements is/are true?

I. The sum of two singular N-by-N matrices is always singular.
II. The product of one singular N-by-N matrix and one non-singular N-by-N matrix is always singular.
III. The product of two singular N-by-N matrices is always singular.
IV. The sum of one singular N-by-N matrix and one non-singular N-by-N matrix is always singular.

A. III only B. II and III only C. I, II, and III only D. I, II, III, and IV E. NOTA

5. Suppose matrix $D = \begin{bmatrix} 1 & 7 \\ 42 & -6 \end{bmatrix}$. If x is the larger of D's eigenvalues and y is the smaller of D's eigenvalues, what is 3x +2y?

A. 5 B. 30 C. 90 D. 114 E. NOTA

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6. If matrix M = $\begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}$, and n is a positive integer, which of the following expressions represents the sum of the elements of M^n?

A. $2+2^n$ B. 2^{n+1} C. $4n$ D. $2(n+1)!$ E. NOTA

7. If A = $\begin{bmatrix} 4 & -1 \\ x & 6 \end{bmatrix}$, and x is an integer, what is the product of all values of x such that A^{-1} also contains only integral values?

A. 575 B. 99 C. 143 D. No such x exist E. NOTA

8. Find the product of all real values *b* such that there is no solution to the system:

$$2x + 5y + z = 19$$
$$-4x + by + 6z = -42$$
$$-3y - bz = 81$$

A. -30 B. -48 C. -24 D. -18 E. NOTA

9. What is the sine of the smaller of the two angles created by the intersection of the vectors <4, 1, 8> and <8, 24, 6> ?

A. $\frac{1}{9}$ B. $\frac{4}{9}$ C. $\frac{\sqrt{5}}{3}$ D. $\frac{\sqrt{65}}{9}$ E. NOTA

10. If a = <0, –4, 5> , b = <–2, 3, –2>, and c = <–11, –10, 1>, what is $\sqrt{|(a \times b) \bullet c|}$?

A. 10 B. $\sqrt{31}$ C. 0 D. 13 E. NOTA

11. Which of the following matrices is symmetric?

A. $\begin{bmatrix} 0 & 5 & 3 \\ 5 & 0 & -1 \\ 3 & -1 & 0 \end{bmatrix}$ B. $\begin{bmatrix} 0 & -3 & 2 \\ 1 & 2 & -3 \\ 2 & 1 & 0 \end{bmatrix}$ C. $\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 3 & 6 & 3 \end{bmatrix}$ D. $\begin{bmatrix} 0 & 2 & -2 \\ -2 & 0 & 3 \\ 2 & -3 & 0 \end{bmatrix}$ E. NOTA

12. Consider the matrix A = $\begin{bmatrix} 2W & X \\ 4 & 1 \end{bmatrix}$. If the determinant of A is 10, and W + X = 8, What is the product WX?

A. -33 B. 7 C. 15 D. 35 E. NOTA

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13. Matrix B is orthogonal. Therefore:

A. $B = B^{-1}$ B. $B^T = B^{-1}$ C. $B = B^T$ D. $|B| = 0$ E. NOTA

14. If $A = \begin{bmatrix} 4 & 3 \\ 1 & 5 \end{bmatrix}$ and the determinant of the product AB is 170, which of the following matrices could be the matrix B?

A. $\begin{bmatrix} -1 & 2 \\ -3 & 4 \end{bmatrix}$ B. $\begin{bmatrix} -1 & 2 \\ 3 & 4 \end{bmatrix}$ C. $\begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix}$ D. $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ E. NOTA

15. What is the area of the triangle with vertices at (0,2,1), $(-2,0,3)$, and $(-4, 1, -2)$?

A. $\frac{\sqrt{74}}{4}$ B. $2\sqrt{74}$ C. $\frac{\sqrt{74}}{2}$ D. $\sqrt{74}$ E. NOTA

16. Which of the following matrices has the greatest determinant?

A. $\begin{bmatrix} 1 & 8 & -2 & 11 \\ 0 & 5 & -9 & 5 \\ 0 & 0 & 2 & 3 \\ 0 & 0 & 0 & 10 \end{bmatrix}$ B. $\begin{bmatrix} 3 & -6 \\ 8 & 8 \end{bmatrix}$ C. $\begin{bmatrix} 4 & 0 & 3 \\ 5 & 0 & 9 \\ 10 & 0 & 8 \end{bmatrix}$ D. $\begin{bmatrix} 7 & 10 & 5 \\ 0 & 3 & -2 \\ 0 & 0 & -5 \end{bmatrix}$ E. NOTA

17. Which of the following matrices has a rank of 2?

A. $\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$ B. $\begin{bmatrix} -1 & 3 \\ 2 & -6 \end{bmatrix}$ C. $\begin{bmatrix} 2 & 0 \\ 1 & 0 \end{bmatrix}$ D. $\begin{bmatrix} 3 & 1 \\ -1 & 0 \end{bmatrix}$ E. NOTA

18. Consider vector $\vec{u}$ = <4,5,2> and vector $\vec{v}$ = <12, – 3,4>. What is the length of the projection of $\vec{u}$ onto $\vec{v}$?

A. $\frac{71}{13}$ B. $\frac{56}{13}$ C. $\frac{41}{13}$ D. 2 E. NOTA

19. Assume a certain plane always flies at the same airspeed. If a plane starting at the origin flies for 2 hours at 120^o clockwise of north and then promptly switches its direction to 270^o clockwise of north, how many hours will it take the plane to reach the y-axis again?

A. 1 B. $\sqrt{2}$ C. $\sqrt{3}$ D. $\frac{\sqrt{3}}{2}$ E. NOTA

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20. Given matrix M = $\begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$, which of the following represents M^n?

A. $\begin{bmatrix} \cos^n(\theta) & -\sin^n(\theta) \\ \sin^n(\theta) & \cos^n(\theta) \end{bmatrix}$ B. $\begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$ C. $\begin{bmatrix} \cos(n\theta) & (-1)^n \sin(n\theta) \\ (-1)^{n+1} \sin(n\theta) & \cos(n\theta) \end{bmatrix}$

D. $\begin{bmatrix} \cos(n\theta) & -\sin(n\theta) \\ \sin(n\theta) & \cos(n\theta) \end{bmatrix}$ E. NOTA

21. If both <4,3,6> • <x,2,y> and <y,x> • <3,4> are both equal to 18, what is x – y?

A. 4 B. 8 C. 10 D. 27 E. NOTA

22. If the equation of the plane through the points (1,0,3) , (4,-2,5) , and (7,7,1) is written as Ax + By + Cz + D = 0, with A > 0, what is A + B + C + D?

A. 108 B. 48 C. 66 D. 52 E. NOTA

23. If A = $\begin{bmatrix} 2 & -3 \\ 6 & -8 \end{bmatrix}$ and B = $\begin{bmatrix} 4 & -1 \\ 3 & 6 \end{bmatrix}$, what is |2B+A| ?

A. 86 B. 52 C. 185 D. 100 E. NOTA

24. If $\begin{bmatrix} x+y & 8 \\ x & -2 \end{bmatrix}\begin{bmatrix} x \\ 3 \end{bmatrix} = \begin{bmatrix} 18 \\ x \end{bmatrix}$, what is the sum of all possible values for y?

A. 0 B. -1 C. 4 D. 1 E. NOTA

25. Consider the points in polar coordinates $(2\sqrt{2}, 3\pi/4)$ and $(2, \pi/3)$. What is the dot product of the vectors from the origin to each of these two points?

A. $\frac{15\sqrt{3}}{4}$ B. $2-2\sqrt{3}$ C. $2\sqrt{3}-2$ D. $2+2\sqrt{3}$ E. NOTA

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26. Given the following data about how many stocks John and Fred have, as well as how much money one share of that stock gained or lost today, which of the following expressions will yield a matrix whose only entry is the total amount gained/lost by the two men combined?

Stock Name	John Amount	Fred Amount	Stock Performance
W	40 shares	100 shares	+ \$25
X	90 shares	30 shares	- \$70
Y	150 shares	60 shares	+ 50
Z	75 shares	80 shares	+ 110

A. $\begin{bmatrix}25 & -70 & 50 & 110\end{bmatrix}\begin{bmatrix}40 & 100\\ 90 & 30\\ 150 & 60\\ 75 & 80\end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix}$ B. $\begin{bmatrix}40 & 100\\ 90 & 30\\ 150 & 60\\ 75 & 80\end{bmatrix}\begin{bmatrix}25 & -70 & 50 & 110\\ 25 & -70 & 50 & 110\end{bmatrix}\begin{bmatrix}1\\1\\1\\1\end{bmatrix}$

C. $\begin{bmatrix}40 & 100\\ 90 & 30\\ 150 & 60\\ 75 & 80\end{bmatrix}\begin{bmatrix}25 & -70 & 50 & 110\\ 25 & -70 & 50 & 110\end{bmatrix}$ D. $\begin{bmatrix}40 & 100\\ 90 & 30\\ 150 & 60\\ 75 & 80\end{bmatrix}\begin{bmatrix}25 & -70 & 50 & 110\end{bmatrix}$ E. NOTA

27. What is the magnitude of the vector $5i - 6j + 4k$?

A. 5 B. $\sqrt{5}$ C. $\sqrt{77}$ D. 15 E. NOTA

28. What vector results when the vector $\vec{u} = \begin{bmatrix}1\\1\end{bmatrix}$ is rotated 90^o clockwise about the origin, and then reflected over the line y = x ?

A. $\begin{bmatrix}1\\1\end{bmatrix}$ B. $\begin{bmatrix}-1\\1\end{bmatrix}$ C. $\begin{bmatrix}1\\-1\end{bmatrix}$ D. $\begin{bmatrix}-1\\-1\end{bmatrix}$ E. NOTA

29. Where do the two lines $\begin{matrix}x = 3t + 2\\ y = 4 - t\end{matrix}$ and $\begin{matrix}x = 4t + 5\\ y = 2t - 2\end{matrix}$ intersect in the x-y plane?

A. $(-3, 2)$ B. (11, 1) C. $(-7, 2)$ D. They do not intersect E. NOTA

30. If $x\begin{bmatrix}x\\y\end{bmatrix} + y\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}13\\5\end{bmatrix}$, what is x+y?

A. $3\sqrt{2}$ B. 9 C. 18 D. Cannot be determined E. NOTA