Unless otherwise specified, equations are to be solved over the real numbers.

A capitalized inverse trigonometric function (e.g. Arcsin) denotes that the range of the function is restricted to its traditional subset of $\mathbb{R}$ in order that the function return a unique output for each input.

Answer choice (E) NOTA means "none of these answers."

No calculators are permitted on this test.

1. Which of the following is *not* a trigonometric identity (where both sides are defined)?

   (A) $1 - \sin^2\alpha = \cos^2\alpha$ (B) $\cos(\alpha+\beta) - \sin\alpha\sin\beta = \cos\alpha\cos\beta$

   (C) $\sin(\alpha+\beta) - \sin\alpha\cos\beta = \cos\alpha\sin\beta$ (D) $\tan\alpha = \frac{1}{2}(1-\tan^2\alpha)\tan 2\alpha$

   (E) NOTA

2. Which of the following best describes the polar graph of $r = 3 - 28\cos\theta$?

   (A) Limacon with an inner loop (B) Cardioid

   (C) Dimpled limacon (D) Lemniscate

   (E) NOTA

3. How many petals does the polar graph of $r = 7\cos(2007\theta)$ have?

   (A) 7 (B) 14 (C) 2007 (D) 4014 (E) NOTA

4. Which of the following is true?

   (A) $\text{Arctan}(-\sqrt{3}) = \frac{2\pi}{3}$ (B) $\text{Arctan } 0 = \frac{\pi}{2}$

   (C) $\text{Arccot}(-\sqrt{3}) = \frac{5\pi}{6}$ (D) $\text{Arccot}(-\sqrt{3}) = -\frac{\pi}{6}$

   (E) NOTA

5. An angle θ with its terminal side in the second quadrant has a cosine of $-\frac{20}{29}$. If $\tan\theta + \sec\theta = \frac{a}{b}$ with a and b relatively prime integers and $b > 0$, find $a - b$.

   (A) -7 (B) -3 (C) 3 (D) 7 (E) NOTA

6. $\cos 15^\circ$ can be written as $a\sqrt{b+\sqrt{c}}$ with b and c integers with no square factors other than 1. Find the sum of the digits of $(\frac{c}{a})^{\frac{b}{a}}$.

   (A) 9 (B) 18 (C) 19 (D) 27 (E) NOTA

7. In triangle ABC, side a has length 14, side b has length 26, and angle C, opposite side c, has measure 30°. If the area of triangle ABC can be written as $a\sqrt{b}$ with b an integer with no square factors other than 1, find the sum of the digits of the product ab.

   (A) 9 (B) 10 (C) 11 (D) 12 (E) NOTA

8. What is the period of the function $f(x) = \cos(2007\pi x)$?

   (A) $\frac{1}{2007}$ (B) $\frac{2}{2007}$ (C) $\frac{1}{2007\pi}$ (D) $\frac{2}{2007\pi}$ (E) NOTA

9. In parallelogram $ABQT$, the side connecting A to B has length x, the side connecting B to Q has length y, and the sine of angle BAQ is z. Which of the following is equivalent to the area of parallelogram $ABQT$?

   (A) $xy\sin(z + \text{Arcsin}(\frac{xz}{y}))$ (B) $xy\sin(\text{Arcsin } z + \text{Arcsin}(\frac{x\text{Arcsin } z}{y}))$

   (C) $xy\sin(z + \text{Arcsin}(\frac{xy}{z}))$ (D) $xy\sin(z + \text{Arcsin}(\frac{x}{y}))$

   (E) NOTA

10. Where defined, $\cot^2 x - 1$ is equivalent to which of the following?

    (A) $\csc x$ (B) $\csc^2 x$ (C) $1 - \csc x$ (D) $2\cos^2 x$ (E) NOTA

11. If one of the six trigonometric functions $\sin x$, $\cos x$, $\tan x$, $\csc x$, $\sec x$, or $\cot x$ is chosen at random, what is the probability that for the chosen function f, $f(x) + f(-x) = 0$ for any x in the domain of f?

    (A) $\frac{1}{6}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) $\frac{2}{3}$ (E) NOTA

12. If a value of θ is chosen at random from $(0, 2\pi)$, what is the probability that $2007 > 2008 - 3\cot^2\theta$?

    (A) $\frac{1}{6}$ (B) $\frac{1}{3}$ (C) $\frac{2}{3}$ (D) $\frac{5}{6}$ (E) NOTA

13. In which quadrant does the point with polar coordinates $(-2007, 6267^\circ)$ lie?

    (A) I (B) II (C) III (D) IV (E) NOTA

14. What is the minimum value of the function $f(x) = 2007\cos x - 2007\sin x$?

    (A) -4014 (B) $-2007\sqrt{2}$ (C) -2007 (D) 0 (E) NOTA

15. In triangle ABC, side a has length 15, side b has length 16, and the measure of angle C, opposite side c, has measure 120°. What is the sum of the digits in the square of the length of side c?

    (A) 7 (B) 8 (C) 9 (D) 10 (E) NOTA

16. If $\sin\theta = \frac{3}{5}$ and the terminal side of θ lies in the second quadrant, find $\cot\theta$.

    (A) $-\frac{4}{3}$ (B) $-\frac{4}{5}$ (C) $\frac{3}{4}$ (D) $\frac{4}{3}$ (E) NOTA

17. If α and β are distinct values in the interval $[0, 2\pi)$ and $\cos\alpha = \cos\beta$, which of the following must be true?

    (A) $\alpha - \beta = \pi$ (B) $|\alpha - \beta| = \pi$ (C) $\alpha - \beta = 2\pi$ (D) $\alpha + \beta = 2\pi$ (E) NOTA

18. What is the range of the function $f(x) = -2007\sin(2007x + 4014) + 2007$?

(A) $[-4014, 4014]$ (B) $[-2007, 2007]$ (C) $[0, 2007]$ (D) $[0, 4014]$ (E) NOTA

19. Which of the following is equivalent to $\cos(3\theta)\cos(28\theta) - \sin(3\theta)\sin(28\theta)$?

(A) $\sin(25\theta)$ (B) $\cos(25\theta)$ (C) $\sin(31\theta)$ (D) $\cos(31\theta)$ (E) NOTA

20. If $f(x) = \cos x - \sin x$, find $f(\frac{3\pi}{4}) - f(\frac{7\pi}{6})$.

(A) $\frac{\sqrt{3}}{2} - \frac{1}{2}$ (B) $\frac{1}{2} - \frac{\sqrt{3}}{2}$ (C) $\frac{\sqrt{3}}{2} - \sqrt{2} - \frac{1}{2}$ (D) $\frac{1}{2} - \sqrt{2} - \frac{\sqrt{3}}{2}$ (E) NOTA

21. What is the distance between the polar coordinates $(\sqrt{2}, 45°)$ and $(\sqrt{2}, 315°)$?

(A) $\sqrt{2}$ (B) 2 (C) $2\sqrt{2}$ (D) 270 (E) NOTA

22. If the square of the cosine of the smaller angle between the vectors $i + 2j$ and $3i - 5j$ is $\frac{a}{b}$ with a and b relatively prime natural numbers, find the sum of the digits of $a + b$.

(A) 12 (B) 13 (C) 14 (D) 15 (E) NOTA

23. What is the period of the function $f(x) = \sin(\frac{4x}{5}) + \cos(\frac{x}{3})$?

(A) 5π (B) 15π (C) 30π (D) 60π (E) NOTA

24. How many values of x are there in $[0, 2\pi)$ such that $\sin x = \frac{3}{2}$?

(A) 0 (B) 2
(C) 4 (D) Infinitely many
(E) NOTA

25. What is the domain of $f(x) = \text{Arctan } x$?

(A) Real numbers (B) $\{x|\ x \neq \frac{(2n+1)\pi}{2}, \text{for all integers } n\}$
(C) $\{x|\ x \neq \frac{n\pi}{2}, \text{for all integers } n\}$ (D) $\{x|\ |x| \leq \frac{\pi}{2}\}$
(E) NOTA

26. Where defined, $\cot x \cos 2x + \sin 2x$ is equivalent to which of the following?

(A) $\cot x$ (B) $\tan x$ (C) $\cos x$ (D) $\csc x$ (E) NOTA

27. If α and β are the measures of two consecutive angles of a parallelogram, which of the following is *not* true?

(A) $\cos\alpha + \cos\beta = 0$ (B) $\sin\alpha + \sin\beta = 0$
(C) $\sin^2\alpha - \sin^2\beta = \cos^2\alpha - \cos^2\beta$ (D) $\sin 2\alpha = \sin\beta\cos\alpha - \sin\alpha\cos\beta$
(E) NOTA

28. Which of the following is equivalent to $(\cos\theta + \sin\theta)^3$?

(A) $\cos\theta + \sin\theta$ (B) $\cos^3\theta + \sin^3\theta$

(C) $(\cos\theta + \sin\theta)(1 - \frac{1}{2}\sin 2\theta)$ (D) $(\cos\theta + \sin\theta)(1 + \sin 2\theta)$

(E) NOTA

29. Suppose $\cos\alpha = \frac{7}{8}$, $\sin\beta = \frac{12}{13}$, and $0 < \alpha < \frac{\pi}{2} < \beta < \pi$. If $\cos(\alpha - \beta)$ is expressed as $\dfrac{a\sqrt{b} - q}{t}$, with a, b, q, t natural numbers with no common divisor and $a\sqrt{b}$ in its most simplified form, find the sum of the digits of $a + b + q + t + 90$.

(A) 12 (B) 13 (C) 14 (D) 15 (E) NOTA

30. Evaluate: $\sin\pi$

(A) -1 (B) 0 (C) $\frac{\sqrt{2}}{2}$ (D) 1 (E) NOTA