

1. Sum of roots is  $-\frac{b}{a} = -\frac{-4}{2} = 2$ .

2.  $\sum_{x=1}^{10} (x(x+1)) = \sum_{x=1}^{10} x^2 + \sum_{x=1}^{10} x = 385 + 55 = 440$

3. Add the second and third equations and get  $v + 2w + x + y = 4$ . Note that the first equation is  $v + 2w + 4x + y = 4$ . Subtracting the two gives  $-3x = 0 \rightarrow x = 0$ .

4. Susan paints  $\frac{1}{2}$  a barn each day and Pam paints  $\frac{1}{4}$  a barn each day, so together they paint  $\frac{3}{4}$  a barn each day so they paint 30 barns in 40 days.

5.  $|x^2 - 1| \geq 0$ , so  $|x^2 - 1| + 4 \geq 4$  so 4 is the minimum.

6.  $3|x-1| - |x-1| + 4|x-1| = 6|x-1| = 18 \rightarrow |x-1| = 3 \rightarrow x = -2, 4 \rightarrow \text{sum} = 2$

7.  $41^2 - 17^2 = (41-17)(41+17)$ , which is divisible by 58

$19^4 - 16 = 361^2 - 4^2 = (361-4)(361+4)$ , which is divisible by 365

$18^2 + 22^2 = 4(9^2 + 11^2)$ , which is divisible by 4

$12^3 + 11^3 = (12+11)(12^2 - (12)(11) + 11^2)$  which is divisible by 23

So none are prime.

8. By definition of inverse, for any function  $g(x)$  with an inverse,  $g(g^{-1}(1)) = 1$ .

9. Let  $x = \sqrt{\pi + \sqrt{\pi + \sqrt{\pi + \dots}}}$  so  $x = \sqrt{\pi + x} \rightarrow x^2 - x - \pi = 0$ . We want the positive root since the radical is obviously positive:  $\frac{1 + \sqrt{1 + 4\pi}}{2}$ .

10. For the sum to be even, there has to be either 0 odd terms, 2 odd terms, or 4 odd terms. This is just as likely as the opposite, which is 5 odd terms (0 even), 3 odd terms (2 even) or 1 odd term (4 even), so the probability  $A$  is just  $\frac{1}{2}$ .

11. By the AM-GM theorem, the geometric mean of 69 and 71 is less than their

arithmetic mean, so  $\sqrt{(69)(71)} < \frac{69+71}{2} = 70$ , and  $\sqrt{(69)(71)} > \sqrt{(69)(69)} = 69$ , so

$\sqrt{(69)(71)}$  is between 69 and 70, so the answer is 70.

12.  $\sum_{x=0}^{10} \sqrt{x^2 y} = \sqrt{y} \sum_{x=0}^{10} \sqrt{x^2} = \sqrt{y} \sum_{x=0}^{10} x = 55\sqrt{y} = 220 \rightarrow y = 16$ .

13. Distance is  $\sqrt{(4-3)^2 + (7-1)^2} = \sqrt{37}$ .

14.  $y = x^2 - 2\sqrt{2}x + 2 = (x - \sqrt{2})^2$ , so there are no rational roots.

15.  $y = 2x$  has slope 2, so any line that is perpendicular to  $y = 2x$  has slope  $-\frac{1}{2}$

16. Say the shoes originally cost  $\$x$ . After the first discount, the cost is down to  $\frac{6x}{10}$ , but after the markup it's back up to  $\frac{72x}{100}$ . After the final discount the price is down to  $\frac{36x}{100} = 9 \rightarrow x = 25$ .

17.  $x + y = \sqrt{48}$ ,  $xy = 4 \rightarrow x^2 + y^2 = (x + y)^2 - 2xy = 48 - 8 = 40$

18.  $(4 + 2i)^2 - (4 - 2i)^2 = (4 + 2i + 4 - 2i)(4 + 2i - 4 + 2i) = (8)(4i) = 32i$

19.  $\frac{x^4 y^2 z x}{xy z^4} = \frac{x^4 y}{z^3}$ .

20. Set the two equal.  $-x^2 = 2x + 1 \rightarrow x^2 + 2x + 1 = 0 \rightarrow x = -1 \rightarrow (-1, -1)$

21. Perimeter is sum of lengths of sides  $(x + 2y - 1) + (y + 4) + (y - x + 4) = 4y + 7$

22. Call the sides  $a, b$   $a > b$ .

$$a + b = 9, ab = 17 \rightarrow b = \frac{17}{a} \rightarrow a + \frac{17}{a} = 9 \rightarrow a^2 - 9a + 17 = 0 \rightarrow a = \frac{9 + \sqrt{13}}{2}$$

23.  $\log 4^{10} = 10 \log 4 = 20 \log 2 = 6$ ,  $\log 5^9 = 9 \log 5 = 9(1 - \log 2) = 6.3$ , so  $5^9$  is larger.

24. Number of gophers goes up by a factor of 13, so 91 gophers can lay 182 eggs in 14 minutes. Now double the time, so 91 gophers can lay 364 eggs in 28 minutes.

25.  $x + 2y - 1 = 0 \rightarrow y = -\frac{1}{2}x + \frac{1}{2}$ , so slope is  $-\frac{1}{2}$ .