2007 National Mu Alpha Theta Convention

Geometry Hustle Solutions

1. 36π $A = \pi\left(\frac{12}{2}\right)^2 = 36\pi$

2. 144 $16^2 + x^2 = (x+2)^2$; $256 + x^2 = x^2 + 4x + 4$; $252 = 4x$; x = 63
Sides are 16, 63, and 65, so perimeter = 16 + 63 + 65 = 144

3. 28 $\frac{n(n-1)}{2} = \frac{8(8-1)}{2} = 28$

4. $12\sqrt{3}$ Trial and error can be used, or if 2x is hypotenuse, then
$2x + x + x\sqrt{3} = 18 + 18\sqrt{3}$; $x(3+\sqrt{3}) = 18 + 18\sqrt{3}$; $x = \frac{18+18\sqrt{3}}{3+\sqrt{3}}$
Rationalize denominator, so $x = \frac{18+18\sqrt{3}}{3+\sqrt{3}}\left(\frac{3-\sqrt{3}}{3-\sqrt{3}}\right) =$
$\frac{54 - 18\sqrt{3} + 54\sqrt{3} - 54}{9-3} = \frac{36\sqrt{3}}{6} = 6\sqrt{3}$. Hypotenuse = 2x = $12\sqrt{3}$.

5. 36 Parts are $\frac{4}{15}$ and $\frac{11}{15}$ of the length of the original segment;
$25\left(\frac{4}{15}\right) = \frac{20}{3} = 6.\overline{6}$ and $25\left(\frac{11}{15}\right) = \frac{55}{3} = 18.\overline{3}$;
$18.\overline{3} - 6.\overline{6} < K < 18.\overline{3} + 6.\overline{6}$, $11.\overline{6} < K < 25$
Since K is an integer, min and max of K is 12 and 24; 12 + 24 = 36

6. 300π Apothem of hexagon = radius of inscribed circle = $\frac{20}{2}\sqrt{3} = 10\sqrt{3}$
Area of circle = 300 π

7. $\frac{1}{4}$ or .25 $KV = \pi\left(\frac{r}{6}\right)^2(9H) = \pi r^2 H\left(\frac{1}{4}\right)$

8. 180 360/n = 2, so n = 180

9. 120 or 120^{o} — Corresponding angles of similar polygons are congruent

10. 98^{o} or 98 — At 4:00, angle is 30(4)=120. Each minute, minute hand moves $\frac{360}{60} = 6^{o}$, and hour hand moves $\frac{30}{60} = 0.5^{o}$. Hands move closer at a rate of 6 – 0.5 = 5.5 deg/min.
At 4:04, hands are 120 – 4(5.5) = 98 degrees apart

11. $\frac{4\pi\sqrt{5}}{3}$ — $h^2 = 3^2 - 2^2$, so $h = \sqrt{5}$, $V = \frac{1}{3}\pi(2)^2(\sqrt{5}) = \frac{4\pi\sqrt{5}}{3}$

12. 3 — T is given, so it is valid. Use it with $S \rightarrow :\ T$ to get the opposite of S by Modus Tollens (MT). Then use $:\ S \rightarrow R$ to get R by Modus Ponens (MP). Next, use $:\ Q \rightarrow :\ R$ to get Q by MT. Finally, use $Q \rightarrow :\ P$ to get the opposite of P by MP. So, T, R, and Q are valid.

13. 62 or 62^{o} — Either two angles are 62 or the vertex angle is 62.
Angles are 62, 62, and 180 – 2(62) = 56 or angles are
62, $\frac{180-62}{2}$, $\frac{180-62}{2}$, or 62, 59, and 59. Largest possible = 62

14. 144^{o} or 144 — Angle of regular pentagon = $\frac{180(5-2)}{5} = 108$. Bisected = 54.
Quadrilateral formed with angles 108, 54, 54, and x.
X = 360 – 108 – 54 – 54 = 144.

15. 32π — Balloon can occupy $\frac{3}{8}$ of a sphere with radius 4,
or $\frac{3}{8}\left(\frac{4}{3}\pi 4^3\right) = 32\pi$.

16. 7/5 — A is opposite side 3. sin A + cos A = 3/5 + 4/5 = 7/5.

17. 27000 — $\sqrt{\frac{4}{9}} = \frac{2}{3}$, $\left(\frac{2}{3}\right)^3 = \frac{8}{27}$, $\frac{8}{27} = \frac{8000}{x}$, x = 27000

18. 200 — $s = \frac{20}{\sqrt{2}}$, $A = s^2 = \left(\frac{20}{\sqrt{2}}\right)^2 = \frac{400}{2} = 200$

19. 30 or 30° $\frac{360}{12} = 30$

20. $\frac{-1}{2}$ or -.5 Sides are 3, 5, 7. Use Law of Cosines to find cosine of largest angle (opposite longest side). So,

$7^2 = 3^2 + 5^2 - 2(3)(5)\cos C$; $49 = 9 + 25 - 30\cos C$; $15 = -30\cos C$;

$\cos C = \frac{-1}{2}$

21. 4π $A = \frac{40}{360}\pi(6)^2 = \frac{1}{9}(36\pi) = 4\pi$

22. 47 or 47° Opposite angles of a parallelogram are congruent

23. -5 Midpt of AB = $\left(\frac{2+8}{2}, \frac{6-4}{2}\right) = (5,1)$

Slope of AB = $\frac{-4-6}{8-2} = \frac{-5}{3}$, so $m_{\perp} = \frac{3}{5}$

Substitute midpoint and $m_{\perp}$ into y = mx + b, yielding $1 = \frac{3}{5}(5) + b$

So, b = -2, and equation of perpendicular bisector is $y = \frac{3}{5}x - 2$.

Since (-5, y) is on the line, plug in to find y = $\frac{3}{5}(-5) - 2 = -5$.

24. incenter Orthocenter, centroid, and circumcenter ALWAYS on Euler line

25. 97.5 or $\frac{195}{2}$ First order vertices consecutively: (1, 5), (-7, 10), (-3, -8), (4, -3)

A = $\frac{1}{2}\begin{vmatrix} 1 & -7 & -3 & 4 \\ 5 & 10 & -8 & -3 \end{vmatrix} = \frac{1}{2}\left[(10 + 56 + 9 + 20) - (-35 - 30 - 32 - 3)\right]$

$= \frac{1}{2}(95 + 100) = \frac{195}{2}$