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For all questions, answer E. "NOTA" means none of the above answers is correct.

1. Evaluate: $\displaystyle\int_0^{\pi/3} \cos^2 x \, dx$

   A) $\dfrac{4\pi + 3\sqrt{3}}{24}$  B) $\dfrac{2\pi + 3\sqrt{3}}{12}$  C) $\dfrac{4\pi + 3\sqrt{3}}{12}$

   D) $\dfrac{2\pi + 3\sqrt{3}}{6}$  E) NOTA

2. What is the area enclosed by the polar curve $r = \cos 3\theta$ ?

   A) $\dfrac{\pi}{12}$  B) $\dfrac{\pi}{8}$  C) $\dfrac{\pi}{4}$  D) $\dfrac{\pi}{2}$  E) NOTA

3. Let $f(x, y) = xe^y + xy^2$. What is $\nabla f(2,1)$ (that is, the gradient of $f$ at $(2,1)$)?

   A) $(e+1)\hat{x} + (2e+4)\hat{y}$  B) $(2e+1)\hat{x} + (e+4)\hat{y}$  C) $(e+4)\hat{x} + (2e+1)\hat{y}$
   D) $(2e+4)\hat{x} + (e+1)\hat{y}$  E) NOTA

4. What is the surface area formed by rotating the curve $y = x^2$ on the domain $x \in [0,2]$ around the $y$-axis?

   A) $\dfrac{17\sqrt{17} - 1}{12}$  B) $\dfrac{17\sqrt{17}}{12}$  C) $\dfrac{34\sqrt{17}}{3}$

   D) $\dfrac{34\sqrt{17} - 2}{3}$  E) NOTA

5. What is the volume of the solid formed by taking a sphere of radius 3 and excluding any volume within 60 degrees of a plane passing through the sphere's center?

   A) $18\pi\left(2 - \sqrt{3}\right)$  B) $9\pi\sqrt{3}$  C) $18\pi$
   D) $18\pi\sqrt{3}$  E) NOTA

6. Given that $\vec{F}(x, y, z) = (x + y)\hat{x} + z^2\hat{y} + xy\sqrt{z}\,\hat{z}$, what is $\nabla \cdot \vec{F}$ (a.k.a. div $\vec{F}$) evaluated at the point $(2,-1,1)$ ?

   A) 0  B) 3  C) $\hat{x} + \hat{y} - 2\hat{z}$
   D) $2\hat{x} + \hat{z}$  E) NOTA

7. Let $\vec{f}(t) = \sin t \hat{x} + \cos t \hat{y} + t \hat{z}$. What is $\vec{f}''(-\pi/4)$?

A) $\dfrac{\sqrt{2}}{2}\hat{x} - \dfrac{\sqrt{2}}{2}\hat{y}$

B) $\dfrac{\sqrt{2}}{2}\hat{x} + \dfrac{\sqrt{2}}{2}\hat{y}$

C) $\dfrac{\sqrt{2}}{2}\hat{x} + \dfrac{\sqrt{2}}{2}\hat{y} - \hat{z}$

D) $\dfrac{\sqrt{2}}{2}\hat{x} + \dfrac{\sqrt{2}}{2}\hat{y} + \hat{z}$

E) NOTA

8. What is the volume of the solid formed by taking an ellipse with semimajor axes of length 4 and semiminor axes of length 2 centered at $(-4,6)$ with its major axis parallel to the line $y = 5x + 17$ and rotating it around the line $y = x$?

A) $20\pi^2 \sqrt{2}$          B) $40\pi^2 \sqrt{2}$          C) $60\pi^2 \sqrt{2}$
D) $80\pi^2 \sqrt{2}$          E) NOTA

9. What is the approximate value of $\ln 2$ found using the first 4 nonzero terms of the Taylor polynomial for $\ln x$ centered at $x = 1$ (round to the nearest thousandth)?

A) .583          B) .693          C) .783          D) .833          E) NOTA

10. Evaluate: $\displaystyle\int_{\pi/2}^{\pi} x^2 \cos x \, dx$

A) $\dfrac{\pi^2 - 8\pi - 8}{4}$

B) $\dfrac{\pi^2 + 4\pi - 4}{4}$

C) $\dfrac{-\pi^2 + 8\pi + 8}{4}$

D) $\dfrac{\pi^2 + 8\pi - 8}{4}$

E) NOTA

11. Let $f(x, y) = x^2 y$ and consider the triangular region A with vertices $(0,0)$, $(1,0)$, and $(0,1)$. Evaluate: $\displaystyle\iint_A f(x, y)\,dA$

A) $\dfrac{1}{60}$          B) $\dfrac{1}{30}$          C) $\dfrac{1}{10}$          D) $\dfrac{1}{5}$          E) NOTA

12. Evaluate: $\displaystyle\int_0^1 \sqrt{1 - x^2}\,dx$

A) $\dfrac{\pi}{4}$          B) 1          C) $\dfrac{\pi + 2}{4}$          D) $\dfrac{\pi}{2}$          E) NOTA

13. Let the position function of a particle, $f(t)$, be defined as follows:

$f(t) = t\hat{x} + t^2\hat{y} + \cos t\hat{z}$. What is the speed of the particle at time $t = \dfrac{\pi}{2}$?

A) $\sqrt{\pi^2 - 1}$  B) $\pi$  C) $\sqrt{\pi^2 + 1}$

D) $\sqrt{\pi^2 + 2}$  E) NOTA

14. For positive integers $a$ and $b$ evaluate: $\displaystyle\lim_{x\to\infty} \dfrac{x\log_a x}{x\log_b x}$

A) $\dfrac{1}{\log_a b}$  B) $\log_a b$  C) $\log_b a$  D) $a\log_b a$  E) NOTA

15. The centroid of the region bound by the curves $y = \sqrt{x}$ and $y = x$ is the point $(a,b)$.

What is $\dfrac{a}{b}$?

A) $\dfrac{1}{5}$  B) $\dfrac{2}{5}$  C) $\dfrac{5}{2}$  D) 5  E) NOTA

16. Evaluate: $\displaystyle\lim_{(x,y)\to(2,1)} \dfrac{xy}{xy^3 + xy}$

A) $\dfrac{1}{5}$  B) $\dfrac{2}{5}$  C) $\dfrac{1}{2}$  D) 1  E) NOTA

17. Let $L = \sqrt[3]{\dfrac{\pi}{3}}$. Evaluate: $\displaystyle\int_0^L\int_0^z\int_0^2 xy\cos z^3 dxdydz$

A) $\dfrac{\sqrt{3}}{6}$  B) $\dfrac{\sqrt{3}}{4}$  C) $\dfrac{\sqrt{3}}{3}$  D) $\dfrac{\sqrt{3}}{2}$  E) NOTA

18. Evaluate: $\displaystyle\int_1^2 \ln x\,dx$

A) $\ln 2 - 2$  B) $2\ln 2 - 2$  C) $\ln 2 - 1$
D) $2\ln 2 - 1$  E) NOTA

19. The coordinates of a particle are given as a function of time: $x(t) = t^2$, $y(t) = t^3$. What distance does the particle traverse from time $t = 0$ to $t = \dfrac{\sqrt{5}}{3}$ ?

A) $\dfrac{37}{54}$    B) $\dfrac{19}{27}$    C) $\dfrac{19}{18}$    D) $\dfrac{38}{3}$    E) NOTA

20. Evaluate: $\displaystyle\int_0^1 \int_0^{z^2} \int_1^{2y} \frac{z^3 e^{y^2}}{x}\,dxdydz$

A) $\dfrac{(e-2)\ln 2}{4}$    B) $\dfrac{(e-2)\ln 2}{2}$    C) $\dfrac{(4e-5)\ln 2}{8}$

D) $\dfrac{(4e-5)\ln 2}{4}$    E) NOTA

21. Consider the function $f(x, y) = (x-1)^2 + y^2 + 1$ with a domain consisting of a disk of radius two centered at the origin. What is the absolute maximum value of $f$ minus the absolute minimum value of $f$?

A) 0    B) 1    C) 8    D) 9    E) NOTA

22. A square with sides of length 3 and centered at $(5,9)$ is rotated about the line $y = b$ to form a solid with a volume of $72\pi$. What is the sum of all possible values of $b$?

A) 5    B) 10    C) 18
D) Cannot be determined    E) NOTA

23. Evaluate: $\displaystyle\int_0^1 x\sqrt{1-x}\,dx$

A) $\dfrac{1}{5}$    B) $\dfrac{4}{15}$    C) $\dfrac{14}{15}$    D) $\dfrac{16}{15}$    E) NOTA

24. A cylinder's base is a circle of radius 2 centered at the origin in the $x$-$y$ plane extending a height of 4 in the positive $z$ direction. The density of the cone is given: $\rho(x, y, z) = (x^2 + y^2)z$. What is the mass of the cylinder?

A) $\dfrac{128\pi}{3}$    B) $64\pi$    C) $\dfrac{256\pi}{3}$    D) $256\pi$    E) NOTA

25. Consider the curve $y = x^2$ from $(0,0)$ to $(3,9)$ denoted $C$ and the function $f(x, y) = \dfrac{y}{x}$.

What is $\int_C f(x, y)\,ds$ ?

A) 3

B) $\dfrac{37\sqrt{37} - 1}{12}$

C) $\dfrac{54\sqrt{37}}{3}$

D) $\dfrac{54\sqrt{37} - 1}{3}$

E) NOTA

26. What is the area of the cardioid described by the polar equation $r = 1 + \cos\theta$ ?

A) $\pi$      B) $2\pi$      C) $3\pi$      D) $4\pi$      E) NOTA

27. Let $z = xy^2 \sin xy$. What is $\dfrac{\partial z}{\partial y}$ evaluated when $x = \dfrac{\pi}{3}$ and $y = 2$?

A) $\dfrac{2\left(\sqrt{3} - 2\pi\right)}{3}$

B) $\dfrac{2\pi\left(\sqrt{3} - \pi\right)}{3}$

C) $\dfrac{2\left(3\sqrt{3} - 2\pi\right)}{3}$

D) $\dfrac{2\pi\left(3\sqrt{3} - \pi\right)}{3}$

E) NOTA

28. A particle's coordinates are described as a function of time: $x(t) = e^{t^2}$, $y(t) = 2e^{3t}$. Let $\theta$ be defined as the acute angle between the $x$-axis and the tangent line to the particle's trajectory at time $t = 2$. What is $\tan\theta$?

A) $\dfrac{4e^{-2}}{3}$      B) $\dfrac{3e^2}{4}$      C) $\dfrac{3e^2}{2}$      D) $3e^2$      E) NOTA

29. Let $f(x) = \dfrac{(x^2 - 4)(x^2 - x - 12)}{(x^2 - 5x + 6)(x + 4)}$. How many asymptotes does $f(x)$ have?

A) 1      B) 2      C) 3      D) 4      E) NOTA

30. Given the relation $V = \pi r^2 h$, what is $\dfrac{\partial V}{\partial r}$ when $r = 2$ and $h = 3$?

A) $4\pi$      B) $8\pi$      C) $12\pi$      D) $16\pi$      E) NOTA