Mu Applications Solutions
2007 Mu Alpha Theta National Convention

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1. **B** - The velocity of the rock in the x-direction is $\frac{dx}{dt} = 50$ and the y-direction is $\frac{dy}{dt} = 40t + 40$.
At t = 2 the total magnitude of velocity is $v = \sqrt{50^2 + 120^2} = 130$.

2. **B** - Average rate at which the pages are viewed is $f(x) = \frac{\int_0^5 4x^2 + 5x + 2}{5} = \frac{\frac{1435}{6}}{5} = \frac{287}{6}$

3. **C** - Volume = $\pi\int_0^5 \left(\left(x^2+2\right)-(-1)\right)^2 - (0--1)^2 = \pi\int_0^5 \left(x^2+3\right)^2 - 1 = 915\ \pi$

4. **A** - $\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{\sec^2\theta}{\sec\theta\tan\theta} = \csc\theta$, $\frac{d^2y}{dx^2} = \frac{d\left(\frac{dy}{dx}\right)}{\frac{dx}{d\theta}} = \frac{-\csc\theta\cot\theta}{\sec\theta\tan\theta} = -\cot^3\theta$

5. **C** – Base of roof has equation $\frac{x^2}{3600} + \frac{y^2}{900} = 1 \rightarrow y = \sqrt{\frac{3600 - x^2}{4}}$.
Base of triangle = 2y, height of triangle is y
Volume of roof = $2\int_0^{60} \frac{1}{2}bh = 2\int_0^{60} \frac{1}{2}2y \cdot y = 2\int_0^{60}\left(\sqrt{\frac{3600 - x^2}{4}}\right)^2 = 72000$

6. **C** - $f(x) = e^{2x} + 2e^x + 1 \rightarrow f'(x) = 2e^{2x} + 2e^x$, $g'(4) = \frac{1}{f'(0)} = \frac{1}{4}$

7. **D** – Total Area = 77/60

x	0	1/2	1	3/2
f(x)	1	2/3	1/2	2/5
Area of Rectangle	1/2	1/3	1/4	1/5

8. **A** - $V = \frac{4}{3}\pi r^3 \rightarrow dV = 4\pi r^2 dr$, $dV = 4\pi\left(\frac{3}{2}\right)^2 \cdot \frac{1}{16} = \frac{9\pi}{16}$

9. **D** - The particles speed is increasing when both velocity and acceleration are both positive or both negative. $s(t) = t^3 - 12t^2 + 21t + 5$, $v(t) = 3t^2 - 24t + 21, a(t) = 6t - 24$. Velocity is positive $0 \le t < 1 \cup t > 7$ and negative on $1 < t < 7$. Acceleration is negative $0 \le t < 4$ and positive when $t > 4$. Speed will be increasing on the intervals $(1,4) \cup (7,\infty)$.

10. **B** - $V = \pi r^2 h$, $dV = 2\pi rh\frac{dr}{dt} + \pi r^2\frac{dh}{dt}$. $\frac{dr}{dt} = 0 \therefore \frac{dV}{dt} = \pi r^2\frac{dh}{dt}$

$6=\pi\cdot 8^2\cdot\frac{dh}{dt}$, $\frac{dh}{dt}=\frac{3}{32\pi}$

11. **C** - $V=\pi r^2h=81\pi$, $h=\frac{81}{r^2}$. Surface Area $=2\pi r^2+2\pi rh=2\pi r^2+\frac{162\pi}{r}$

Cost = $3(2\pi r^2)+2\left(\frac{162\pi}{r}\right)$ Minimize cost by taking derivative and setting equal to 0:

$12\pi r-\frac{324\pi}{r^2}=0$, r = 3. Cost = 162π

12. **A** - $a(t)=32\rightarrow v(t)=32t+v_0\rightarrow s(t)=16t^2+v_0t+s_0$.
1st 64 feet; $v_0=0, s_0=0$, $64=16t^2$, $t=2$.
2nd 64 feet; $v_0=64, s_0=64$, $128=16t^2+64t+64$, $16(t^2+4t-4)=0\rightarrow t=-2+\sqrt{2}$
1st – 2nd $=2-(-2+2\sqrt{2})=4-2\sqrt{2}$

13. **B** - $W=\int F(x)dx$ $\quad F=kx$, $600=3k\rightarrow k=200$

$$W=\int_3^9 200x\,dx=100x^2\Big|_3^9=7200$$

14. **B**- Average rate will just be slope from $0\le t\le 5$. $\frac{s(5)-s(0)}{5-0}=\frac{40}{5}=8$

15. **C** - Volume = $2\pi\int_0^1 x\left(x^{1/3}-x^2\right)=2\pi\left[\frac{3x^{7/3}}{7}-\frac{x^4}{4}\right]_0^1=\frac{5\pi}{14}$

16. **D** - $\frac{dP}{dt}=kP \quad \frac{dP}{P}=kdt \quad \ln P=kt+C \quad e^{kt+C}=P \quad Ce^{kt}=P$
when $t=0$, $C=2$ $\quad P=2e^{kt}$
when $t=2$, $P=50$ $\quad 50=2e^{2k}\rightarrow 25=e^{2k}\rightarrow \ln 25=2k\rightarrow k=\frac{1}{2}\ln 25=\ln 5$
$P=2e^{\ln 5\cdot t}$ when $t=4$ $\quad P=2e^{4\ln 5}=2e^{\ln 625}=2\cdot 625=1250$

17. **E** - $f'(x)=\tan x$, $f'(\pi/3)=\tan(\pi/3)=\sqrt{3}$ slope of tangent line is $\sqrt{3}$ which makes an angle of 60 degrees with the horizontal.
$g'(x)=\cot x, g'(\pi/3)=\frac{1}{\sqrt{3}}$ slope of $\frac{1}{\sqrt{3}}$ makes a slope of 30 degrees with the horizontal. The difference in the angles is 30 degrees and cos (30) is $\frac{\sqrt{3}}{2}$.

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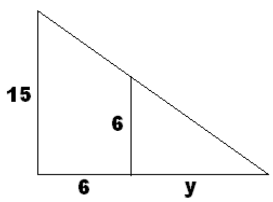
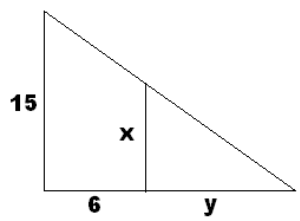
18. **<u>C</u>**- SA $= 2\pi\int_3^8 2\sqrt{x}\cdot\sqrt{1+\left(\frac{1}{\sqrt{x}}\right)^2} = 4\pi\int_3^8 \sqrt{x}\cdot\sqrt{1+\frac{1}{x}} = 4\pi\int_3^8 \cancel{\sqrt{x}}\cdot\sqrt{\frac{x+1}{\cancel{x}}} = 4\pi\int_3^8 \sqrt{x+1}$

$\rightarrow 4\pi\left[\frac{2}{3}(x+1)^{3/2}\right]_3^8 = \frac{152\pi}{3}$

19. **<u>C</u>** -



. $\frac{y}{6} = \frac{y+6}{15} \rightarrow y = 4$



$\frac{y}{x} = \frac{y+6}{15} \rightarrow 15y = xy + 6x \rightarrow 15\frac{dy}{dt} = x\frac{dy}{dt} + y\frac{dx}{dt} + 6\frac{dx}{dt}$

$15\frac{dy}{dt} = 6\frac{dy}{dt} + 4(-2) + 6(-2) \rightarrow 9\frac{dy}{dt} = -20 \rightarrow \frac{dy}{dt} = -\frac{20}{9}$ speed = $\frac{20}{9}$

20. **<u>A</u>** - $P(x) = R(x) - xC(x) = 3x^2 + 600 - x\left(\frac{x^2-30x}{10} + \frac{200}{x}\right)$

$= 3x^2 + 600 - \frac{x^3}{10} + 3x^2 - 200 = 6x^2 - \frac{x^3}{10} + 400 \quad P'(x) = 12x - \frac{3x^2}{10} = 0 \rightarrow x = 0, 40$

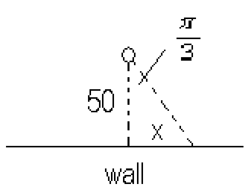
x = 0 is min, x = 40 is maximum

21. **<u>A</u>** – Area = $\frac{1}{2}\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} (2\cos 3\theta)^2 d\theta = \cancel{2}\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \left(\frac{1+\cos 6\theta}{\cancel{2}}\right) = \int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} (1+\cos 6\theta) = \theta + \frac{\sin 6\theta}{6}\Big|_{-\frac{\pi}{6}}^{\frac{\pi}{6}} = \frac{\pi}{3}$

22. **<u>B</u>** - $x\cdot\sqrt[3]{x}\cdot\sqrt[3]{\sqrt[3]{x}}\cdot\sqrt[3]{\sqrt[3]{\sqrt[3]{x}}}\ldots = x^1\cdot x^{1/3}\cdot x^{1/9}\cdot x^{1/27}\ldots = x^{1+1/3+1/9+1/27\ldots} \quad S_n = \frac{1}{1-1/3} = \frac{1}{2/3} = \frac{3}{2}$

$x^{3/2} \rightarrow 9^{3/2} = 27$

23. **<u>A</u>** - $\frac{dx}{dt} = 4 \quad \tan\theta = \frac{x}{50} \rightarrow \sec^2\theta\frac{d\theta}{dt} = \frac{1}{50}\frac{dx}{dt} \quad \sec^2\left(\frac{\pi}{3}\right)\frac{d\theta}{dt} = \frac{1}{50}(4) \quad \frac{d\theta}{dt} = \frac{1}{50}\cdot 4\cdot\frac{1}{4} = \frac{1}{50}$



24. **<u>A</u>** - $x + y = 25 \rightarrow x = 25 - y \;\; x + y = 25 \rightarrow x = 25 - y$

Maximize $x^2 \cdot y^3 = (25-y)^2 \cdot y^3 = y^5 - 50y^4 + 625y^3$

$5y^4 - 200y^3 + 1875y^2 = 5y^2\left(y^2 - 40y + 375\right) = 0 \quad 5y^2\left(y-15\right)\left(y-25\right) = 0$ y = 0, 15, 25

0 and 25 yield minimums so y = 15 and x =10. 15-10 = 5

25.**C**- Use $f(x) + f'(x)\Delta x$ to approximate. $\sqrt[3]{x} + \frac{1}{3\sqrt[3]{x^2}}(\Delta x) = \sqrt[3]{125} + \frac{1}{3\sqrt[3]{125^2}}(1) = 5 + \frac{1}{75} = \frac{376}{75}$

26. **B** - $2x\frac{dy}{dx} - \ln x^2 = 0 \rightarrow 2x\frac{dy}{dx} - 2\ln x = 0 \rightarrow \frac{dy}{dx} = \frac{\ln x}{x}$. Separate variables and integrate:

$\int dy = \int \frac{\ln x}{x} dx \rightarrow y = \frac{(\ln x)^2}{2} + C$ Given $y\left(e^2\right) = 5 \quad 5 = \frac{\left(\ln e^2\right)^2}{2} + C \rightarrow 5 = \frac{2^2}{2} + C \rightarrow C = 3$

Particular solution: $y = \frac{(\ln x)^2}{2} + 3$

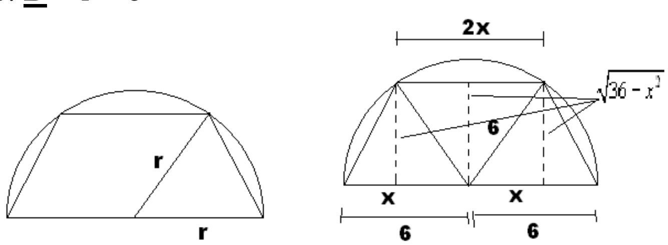
27. **B** - $\bar{x} = \frac{1}{A}\int_a^b x\left(f(x) - g(x)\right)$ A= $\int_0^9 \left(\sqrt{x}+1\right) - \left(\frac{1}{3}x + 1\right) = \int_0^9 \sqrt{x} - \frac{x}{3} = \left[\frac{2x^{3/2}}{3} - \frac{x^2}{6}\right]_0^9 = \frac{9}{2}$

$\bar{x} = \frac{2}{9}\int_0^9 x\left(\sqrt{x} + 1 - \left(\frac{x}{3}+1\right)\right) = \frac{2}{9}\int_0^9 x\left(\sqrt{x} - \frac{x}{3}\right) = \frac{2}{9}\int_0^9 x^{3/2} - \frac{x^2}{3} = \frac{2}{9}\left[\frac{2x^{5/2}}{5} - \frac{x^3}{9}\right]_0^9 = \frac{18}{5}$

28. **C** - The center of the ellipse is (-9,5) and is 9 units from the y-axis. a = 5 and b = 4. The area of the ellipse is $A = ab\pi \rightarrow A = 5\cdot 4\cdot \pi = 20\pi$. Use Pappus:
$V = A\cdot 2\pi r = 20\pi \cdot 2\pi \cdot 9 = 360\pi^2$

29. **E** - f(x)=g(x) Curves intersect at 0,1,2
Area = $\int_0^1 \left[(x-1)^3 - (x-1)\right] + \int_1^2 \left[(x-1) - (x-1)^3\right] = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

30. **D** - r = 6



The area of the trapezoid equals the sum of the area of the 3 triangles.

A(x), $Area = 2\left(\frac{1}{2}\cdot 6\cdot\sqrt{36-x^2}\right) + \frac{1}{2}\cdot 2x\cdot\sqrt{36-x^2} = 6\sqrt{36-x^2} + x\sqrt{36-x^2} = (6+x)\sqrt{36-x^2}$

A'(x) = $\sqrt{36-x^2}-\dfrac{x^2+6x}{\sqrt{36-x^2}}$,   A'(x) = $\sqrt{36-x^2}-\dfrac{x^2+6x}{\sqrt{36-x^2}}=0$

$36-x^2-x^2-6x=-2x^2-6x-36=0$ solve for x, x = 3.

Area of trap = $\frac{1}{2}\cdot(b_1+b_2)\cdot h=\frac{1}{2}\cdot(12+6)\cdot 3\sqrt{3}=27\sqrt{3}$