

**Mu Applications**  
**2007 Mu Alpha Theta National Convention**

---

For all questions, answer E. "NOTA" means none of the above answers is correct.

- Big Hungry Joe is throwing a rock in an attempt to get an apple out of a tree for lunch. The position of the rock can be described by the parametric equations  $x(t) = 50t \text{ m/s}$  and  $y(t) = 20(t^2 + 2t - 3) \text{ m/s}$ . Find the velocity of the rock in meters per second at  $t = 2$ .  
A. 120      B. 130      C. 150      D. 170      E. NOTA
- The rate at which Thomas views Wikipedia pages online can be given by the function  $r(t) = 4x^2 + 5x + 2$  where  $t$  is the time in minutes. What is the average rate of pages viewed per minute from  $0 \leq t \leq 5$ ?  
A. 25      B.  $\frac{287}{6}$       C. 125      D.  $\frac{1435}{6}$       E. NOTA
- Find the volume of the solid formed when the area bounded by the curve  $f(x) = x^2 + 2$ , the  $x$  and  $y$  axes and the line  $x = 5$  is revolved about the line  $y = -1$ .  
A.  $\frac{2795\pi}{12}$       B.  $\frac{2795\pi}{6}$       C.  $915\pi$       D.  $1840\pi$       E. NOTA
- Find  $\frac{d^2y}{dx^2}$  for  $x = \sec \theta$  and  $y = \tan \theta$ .  
A.  $-\cot^3 \theta$       B.  $-\csc \theta \cot \theta$       C.  $2 \sec \theta \tan \theta$       D.  $\tan^3 \theta$       E. NOTA
- Keith is building a new addition on his house and he wants to have a unique roof for the room. He wants each cross-section of the roof to be an isosceles triangle whose height is half of its width. The base of the roof is in the shape of an ellipse with a major axis of length 120 ft and minor axis 60 ft. and the cross-sections of the roof will perpendicular to major axis. What is the volume of the roof that Keith wants to construct?  
A. 36000      B. 64000      C. 72000      D. 144000      E. NOTA
- If  $f$  and  $g$  are inverse functions and  $f(x) = e^{2x} + 2e^x + 1$ , where  $x \geq 0$ , find the slope of the tangent line to the graph of  $g(x)$  at  $(4,0)$ .  
A. -4      B.  $-\frac{1}{4}$       C.  $\frac{1}{4}$       D. 4      E. NOTA

**Mu Applications**  
**2007 Mu Alpha Theta National Convention**

---

7. Approximate the left-hand area bounded by the curve  $f(x) = \frac{1}{x+1}$ , the x-axis,  $x = 0$  and  $x = 2$  using four rectangular sub-intervals of equal width.
- A.  $\frac{47}{60}$       B.  $\frac{57}{60}$       C.  $\frac{67}{60}$       D.  $\frac{77}{60}$       E. NOTA
8. Find the maximum error in volume of a baseball that has a diameter of 3 inches with a possible error of  $\pm \frac{1}{8}$  in.
- A.  $\frac{9\pi}{16}$       B.  $\frac{9\pi}{8}$       C.  $\frac{9\pi}{4}$       D.  $\frac{9\pi}{2}$       E. NOTA
9. A particle's position is given by the function  $s(t) = t^3 - 12t^2 + 21t + 5$ . Find the interval(s) in which the speed of the particle is increasing when  $t \geq 0$ .
- A.  $[0,1) \cup (7,\infty)$       B.  $(7,\infty)$       C.  $(1,4)$       D.  $(1,4) \cup (7,\infty)$       E. NOTA
10. A liquid is flowing into a vertical cylindrical tank of radius 8 feet at the rate of 6 cubic feet per minute. How fast, in feet per minute, is the surface of the water rising?
- A.  $\frac{5}{64\pi}$       B.  $\frac{3}{32\pi}$       C.  $\frac{3}{8\pi}$       D.  $\frac{5}{8\pi}$       E. NOTA
11. The Acme Silo Company is constructing a new model cylindrical silo that must have a volume of  $81\pi$  cubic ft. It costs Acme \$3 per square foot for the bases and \$2 per square foot for the lateral area. What is the minimum cost in dollars to produce the silo?
- A.  $112\pi$       B.  $130\pi$       C.  $162\pi$       D.  $188\pi$       E. NOTA
12. Keith is playing with his favorite rock while hang-gliding with constant horizontal motion. When he is at a height of 128 ft above the ground, he accidentally drops his rock. How much longer will it take the rock to fall the first 64 feet than the second 64 feet? Acceleration due to gravity is 32 ft/sec.
- A.  $4 - 2\sqrt{2}$       B.  $\sqrt{2}$       C.  $\sqrt{2} - 1$       D.  $2\sqrt{2} - 2$       E. NOTA

**Mu Applications**  
**2007 Mu Alpha Theta National Convention**

---

13. Hooke's law states that the force  $F$  required to stretch or compress a spring is proportional to the distance  $d$  that the spring is stretched or compressed from its original length. A mass of 600 grams stretches the spring 3 inches. Find the work required to stretch the spring an additional 6 inches.
- A. 3600      B. 7200      C. 7600      D. 9600      E. NOTA
14. The number of students in the cafeteria during lunch can be given by the function  $s(t) = t^2 + 3t$  where  $t$  is the time in minutes. Find the average rate of students entering the cafeteria during the first five minutes of the lunch period.
- A. 4      B. 8      C.  $\frac{35}{6}$       D.  $\frac{95}{6}$       E. NOTA
15. Find the volume of the solid formed when the area bounded by the graphs of  $f(x) = x^2$  and  $f(x) = \sqrt[3]{x}$  is rotated about the y-axis.
- A.  $\frac{2\pi}{7}$       B.  $\frac{5\pi}{28}$       C.  $\frac{5\pi}{14}$       D.  $\frac{2\pi}{5}$       E. NOTA
16. The rate of growth of a population of Lilliputians is directly proportional to the population at that time. If the initial population of the Lilliputians is 2, and after 2 years the population is 50, find the population after 4 years.
- A. 625      B. 785      C. 950      D. 1250      E. NOTA
17. Find the cosine of the acute angle between the lines tangent to the curves  $f(x) = -\ln \cos x$  and  $g(x) = \ln \sin x$  at  $x = \frac{\pi}{3}$ .
- A.  $\frac{1}{2}$       B.  $\frac{\sqrt{2}}{2}$       C.  $\frac{\sqrt{3}}{2}$       D. 1      E. NOTA
18. Find the surface area formed when the curve  $f(x) = 2\sqrt{x}$  on the interval  $[4, 9]$  is rotated about the x-axis.
- A.  $42\pi$       B.  $\frac{136\pi}{3}$       C.  $\frac{152\pi}{3}$       D.  $\frac{175\pi}{4}$       E. NOTA
19. Ben and Alton are playing basketball one night at Ben's house. There is a light mounted on the roof of the garage at a height of 15 ft. Ben is standing at a point 6 feet from the base of the garage when he drops the ball from above his head. Find the speed at which the shadow of the ball is moving when the ball is 6 feet off the ground and is falling at a rate of 2 ft/sec.
- A.  $\frac{2}{3}$       B.  $\frac{10}{9}$       C.  $\frac{20}{9}$       D.  $\frac{10}{3}$       E. NOTA

**Mu Applications**  
**2007 Mu Alpha Theta National Convention**

---

20. The factory that makes miniature models of factories wants to maximize its profit. The factory has a revenue function of  $R(x) = 3x^2 + 600$ , where  $x$  is the number of units sold and the cost per unit is  $C(x) = \frac{x^2 - 30x}{10} + \frac{200}{x}$ . How many miniature models of factories must the factory produce and sell to maximize profit?
- A. 40      B. 45      C. 60      D. 120      E. NOTA
21. Find the area of one petal of the graph given by  $r = 2 \cos 3\theta$ .
- A.  $\frac{\pi}{3}$       B.  $\frac{2\pi}{3}$       C.  $\frac{7\pi}{6}$       D.  $\frac{4\pi}{3}$       E. NOTA
22. Find the value of the expression  $x \cdot \sqrt[3]{x} \cdot \sqrt[3]{\sqrt[3]{x}} \cdot \sqrt[3]{\sqrt[3]{\sqrt[3]{x}}} \dots$  when  $x = 9$ .
- A.  $9\sqrt[3]{9}$       B. 27      C.  $27\sqrt[3]{9}$       D. 81      E. NOTA
23. Mr. Ahrens is taking a tour of the Great Wall of China and is standing 50 feet from the wall at its nearest point. Mr. Ahrens wants to film each point of the wall for the same amount of time so the scanning beam will be moving at a constant rate of 4ft/sec. Find the positive rate at which the angle of the camera is changing when it is making an angle of  $60^\circ$  with the perpendicular in  $\frac{rad}{sec}$ .
- A.  $\frac{1}{50}$       B.  $\frac{1}{25}$       C.  $\frac{2}{25}$       D.  $\frac{4}{25}$       E. NOTA
24. The sum of two positive numbers is 25. Find the positive difference between these numbers if the product of the square of one of the numbers and the cube of the other is a maximum.
- A. 5      B. 10      C. 15      D. 20      E. NOTA
25. Use differentials to approximate  $\sqrt[3]{126}$ .
- A.  $\frac{188}{75}$       B.  $\frac{76}{15}$       C.  $\frac{376}{75}$       D.  $\frac{426}{15}$       E. NOTA
26. Find the particular solution to the differential equation  $2xy' - \ln x^2 = 0$ , given  $y(e^2) = 5$ .
- A.  $y = \frac{\ln x}{2} + 3$       B.  $y = \frac{(\ln x)^2}{2} + 3$       C.  $y = \ln x + 3$       D.  $y = \frac{\ln x^2}{2} + 3$       E. NOTA

**Mu Applications**  
**2007 Mu Alpha Theta National Convention**

---

27. Find the x-coordinate of the centroid of the region bounded by the graphs  $f(x) = \sqrt{x} + 1$  and  $g(x) = \frac{1}{3}x + 1$ .

- A.  $\frac{5}{2}$       B.  $\frac{18}{5}$       C.  $\frac{9}{2}$       D.  $\frac{15}{2}$       E. NOTA

28. Find the volume of the torus formed when the graph of  $\frac{(x+9)^2}{25} + \frac{(y-5)^2}{16} = 1$  is revolved about the y-axis.

- A.  $200\pi^2$       B.  $270\pi^2$       C.  $360\pi^2$       D.  $400\pi^2$       E. NOTA

29. Find the area bounded by the curves  $f(x) = (x-1)^3$  and  $g(x) = x-1$ .

- A. 0      B.  $\frac{1}{4}$       C.  $\frac{3}{4}$       D. 1      E. NOTA

30. Find the maximum area of a trapezoid inscribed in a semicircle of radius 6.

- A.  $9\sqrt{3}$       B. 27      C.  $24\sqrt{3}$       D.  $27\sqrt{3}$       E. NOTA