Practice Round

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Practice. How many positive even integer solutions exist for the equation a + b + c + d = 100?

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# Round 1

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1. A hemispherical bowl of radius 5 units is filled with water to a height of 2 units. How much more volume is needed to completely fill the bowl?

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## Round 2

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2. Let  $f(x) = x^x$ . Find the sum of all values of x for which f(x) = f'(x).

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Round 3

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3. If  $f(x) = x^3 + ax^2 + bx + c$ , find the product *abc* given that f(x) has critical points at x = -1 and x = 5, and that f(-1) = 9.

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3. If  $f(x) = x^3 + ax^2 + bx + c$ , find the product *abc* given that f(x) has critical points at x = -1 and x = 5, and that f(-1) = 9.

## Round 4

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4. If f(x) is a differentiable and concave down quadratic polynomial on the interval [0,10], and if f(x) < 0 on [0,10] with a relative maximum at x = 2, put the letters representing these approximations in order from smallest to largest.

 $A - \int_0^{10} f(x) dx$ 

B – Left Hand Approximation of A using 10 rectangles of equal base length.

C – Right Hand Approximation of A using 10 rectangles of equal base length.

D – Midpoint Approximation of A using 10 rectangles of equal base length.

E – Trapezoid Approximation of A using 10 intervals of equal base length.

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## $A - \int_0^{10} f(x) dx$

*B* – Left Hand Approximation of *A* using 10 rectangles of equal base length.

C – Right Hand Approximation of A using 10 rectangles of equal base length.

D – Midpoint Approximation of A using 10 rectangles of equal base length.

*E* – Trapezoid Approximation of *A* using 10 intervals of equal base length.

## Round 5

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5. Let *A* be the volume of the solid formed by rotating y = 4 - x around the *x*-axis on the interval x = 0 to x = 2. Let *B* be the volume of the solid formed by the region bound by x = 2, y = 4 - x, and y = k, about the x-axis, where k > 4. For what exact value of *k* is A = B?

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## Round 6

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6. A sphere of radius  $\pi/2$  has a volume charge density of  $p(r) = \frac{\sin(r)}{r}$ . What is the total charge (Q) enclosed in the sphere? (Hint:  $dQ = p \cdot dV$ )

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Round 7

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7. An ellipse with a horizontal major axis of 6 and a vertical minor axis of 4 is revolved about its horizontal axis to form an ellipsoid. At time t = 0, a plane begins to pass through the ellipse perpendicular to its horizontal axis at a rate of 1 unit/sec. When t = 4 sec, at what rate is the cross-sectional area of the plane changing?

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## Round 8

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8. Given  $f(x) = x^3 - 2x - 4$  and  $f^{-1}(x) = g(x)$  what is g'(0)?

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Round 9

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9. Let

$$A = \int_{-\infty}^{\infty} e^{-x^2} dx$$

B = The area of a regular *n*-gon whose distance from the center to a vertex is 1 unit as  $n \to \infty$ .

What is  $A^{2B}$  (simplified)?

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Round 10

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10. Evaluate: 
$$\int_{-1}^{1} \frac{7x^{316}\sin(x^{325}) + 2x^{110} + x^{332}}{1 + x^{222}} dx$$

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Extra Round

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Extra Round

Extra. Evaluate:  $\lim_{n \to \infty} \sum_{k=1}^{n} \frac{n}{k^2 + n^2}$ 

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