

2007 MAO NATIONAL COMPETITION

MU GEMINI

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(Note: Throughout the test you may mark "E" for none of these answers.)

- How long is the shortest path from $(0, 0, 0)$ to $(10, 2, 6)$ passing through the $y=-3$ plane?
 - 10
 - $10\sqrt{2}$
 - 12
 - $12\sqrt{2}$
 - NOTA
- How many positive integers less than 10,000 are palindromes?
 - 162
 - 180
 - 198
 - 256
 - NOTA
- Evaluate $\int_0^{\pi/3} \tan^3 x dx$.
 - $3/2 - \ln 2$
 - $3/2 - \ln 3$
 - $\ln 3 + 3/2$
 - $\ln 3 + 1$
 - NOTA
- Two real numbers are between 5 and 10. What is the probability that their product is less than 50?
 - $1/2$
 - $\ln 4 - 1$
 - $(1/2)\ln 2 + 1/8$
 - $(1/2)\ln 2$
 - NOTA
- Suppose a hallway is 5ft wide and 10ft tall. It then makes a right angle turn into another hallway 5ft wide and 10ft tall. What is the longest pole, in feet, that you can carry through this turn?
 - $10\sqrt{3}$
 - $12\sqrt{2}$
 - 15
 - 18
 - NOTA
- Starting with $t=0$, water flows into a circular cylinder with a 5 m radius at a rate of $t \text{ m}^3/\text{sec}$. How fast is the water level height increasing when $t=5$ sec?
 - 5π
 - $\pi/5$
 - $5/\pi$
 - $1/(5\pi)$
 - NOTA

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7. What is the number of solutions to $\int_0^x 0.9 - \sin^2(t^2) dt = 0$?
- A. 0
 - B. 1
 - C. 3
 - D. Infinitely many
 - E. NOTA
8. What is the radius of the circumcircle of a triangle with side lengths 5, 5, 8?
- A. $25/6$
 - B. $16/3$
 - C. $49/16$
 - D. $64/25$
 - E. NOTA
9. It took me four times longer to walk up a down-escalator than to walk down the same escalator. What is the ratio of my speed to the escalator's speed?
- A. 7:4
 - B. 11:5
 - C. 11:7
 - D. 8:5
 - E. NOTA
10. Two circles of radius R overlap, such that the lines from each center to the intersection points form a square. What is the area of the overlapped region?
- A. $(\pi - 2) R^2$
 - B. $(\pi/2 - 1) R^2$
 - C. $(\pi/2) R^2$
 - D. $2 R^2$
 - E. NOTA
11. A roll of paper towels is a cylinder with a 3 inch radius. Each sheet is 0.1 inches thick (assume there is no space between the sheets). I am going to unroll it at 10 inches/second. How many revolutions per second is it making when I have unrolled half of the length of it?
- A. 5π
 - B. $5\pi/3$
 - C. $5\sqrt{2}/(3\pi)$
 - D. $\sqrt{2}\pi$
 - E. NOTA

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12. $y = (x^2)^{(x^2)}$. Find dy/dx .

- A. $2xy(2\ln x + 1)$
- B. $2x(2\ln y + 1)$
- C. $2x(2\ln x + 1)$
- D. $xy(\ln x + 1)$
- E. NOTA

13. A tree starts at height 0, and grows $(3 - h)$ units a day, where h is its height. How tall does it grow?

- A. $\ln 3$
- B. 1
- C. 3
- D. 9
- E. NOTA

14. Suppose that f is twice differentiable over all real numbers, and $f(0) = f'(0) = f''(0) = 0$. If

$f'(x) \leq 1, f''(x) \leq 1$ for all x , what is the maximum possible value of f on $[0, 5]$?

- A. 3.5
- B. 5
- C. 5.5
- D. 7
- E. NOTA

15. $\log_{16} x + \log_4 x = 6$. If you add the sum of the digits of x , which of the following is a possible result?

- A. 7
- B. 13
- C. 14
- D. 19
- E. NOTA

16. Let
$$y = \frac{1}{1 + \sqrt{1 + \frac{1}{1 + \sqrt{1 + \frac{1}{1 + \sqrt{1 + \frac{1}{1 + \dots}}}}}}}}$$

Evaluate $y^3 + 2y + 5$

- A. 4.5
- B. 5
- C. 5.5
- D. 6
- E. NOTA

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17. I take a liter of milk and a liter of water. I put 20% of the water into the milk, and 20% of the resulting mixture into the water. What is the final amount of water in the original water container?
- A. 0.8 L
 - B. 0.82 L
 - C. 0.84 L
 - D. 0.88 L
 - E. NOTA
18. For a given x , what is the y for which the function $\sin(x+y)\cos(y)$ is maximized?
- A. $\pi/4 - x/2$
 - B. $\pi/2 - x/4$
 - C. $\pi/2 - x/2$
 - D. $\pi/4 - x/4$
 - E. NOTA
19. How many positive integer solutions are there to $a+b+c = 8$?
- A. 18
 - B. 21
 - C. 24
 - D. 36
 - E. NOTA
20. A 30-question test gives 4 points for a correct answer and -1 points for a wrong answer (a question can be left blank to score 0 points on that answer). How many scores can be achieved in exactly 4 distinct sets of (# of correct answers, # of wrong answers)?
- A. 1
 - B. 2
 - C. 4
 - D. 6
 - E. NOTA
21. The area in between three cotangent circles of radius 1 is
- A. $\pi - \sqrt{3}$
 - B. $\sqrt{3} - \pi/2$
 - C. $\sqrt{3}$
 - D. $\pi/2$
 - E. NOTA
22. An equilateral triangle has each side expand at the rate of 1 unit/sec (it remains an equilateral triangle as it expands). What is the rate of change of the area when a side is 10 units long?
- A. $5\sqrt{2}$
 - B. $6\sqrt{2}$
 - C. $10\sqrt{2}$
 - D. $5\sqrt{3}$
 - E. NOTA

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23. The hyperbolic sine function is defined as $\sinh(x) = \frac{e^x - e^{-x}}{2}$. Find the derivative of $\operatorname{arcsinh}(x)$ at $x = 0$,

where $\operatorname{arcsinh}(x)$ is the inverse of $\sinh(x)$.

A. $1/2$

B. $\frac{\sqrt{2}}{2}$

C. 1

D. $\sqrt{2}$

E. NOTA

24. $\int_{\pi/6}^{\pi/3} \frac{1}{1 - \sin x} dx =$

A. $\pi - 1$

B. $\pi/2 + 1/2$

C. $\pi/3 + 1$

D. 2

E. NOTA

25. Let n and k be positive integers, $k < n$. Given a semicircle defined by $f(x) = \sqrt{1 - x^2}$, what is its arc

length located in the region $\frac{k}{n} < f(x) < \frac{k+1}{n}$?

A. π/n

B. $\pi/n + k\pi(\frac{n}{2} - k)$

C. $2(\arccos(\frac{k}{n}) - \arccos(\frac{k+1}{n}))$

D. $2(\arcsin(\frac{k+1}{n}) - \arcsin(\frac{k}{n}))$

E. NOTA

26. I cut a slice out of a circle, and roll up the rest, forming a conical cup. For what angle (in degrees) of the slice cut out is the volume of the cup maximized?

A. 300

B. 240

C. $360 - \frac{240}{\sqrt{\pi}}$

D. $360 - \frac{240}{\sqrt{\pi}}\sqrt{3}$

E. NOTA

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27. Evaluate $\int_0^{\pi/6} \sec x \tan^2 x dx$.

- A. $1/3 + \frac{\ln 3}{4}$
- B. $\ln(2 + \sqrt{3})$
- C. $\ln(2 + 2\sqrt{3})$
- D. $2\sqrt{3}$
- E. NOTA

28. Five red chairs and five blue chairs are put spaced evenly in some random order around a round table. Five red plates and five blue plates are placed in some random order front of the chairs. What is the probability that all the red chairs are in front of the red plates?

- A. $\frac{5!}{10!}$
- B. $\frac{5!2!}{10!}$
- C. $(\frac{5!}{10!})^2$
- D. $\frac{(5!)^2}{10!}$
- E. NOTA

29. Evaluate $\int_{\pi/3}^{\pi/2} \frac{1}{\sin x + \sqrt{3} \cos x} dx$.

- A. $\frac{1}{2} \ln \frac{3 + 2\sqrt{3}}{3}$
- B. $\ln \frac{2\sqrt{3}}{3}$
- C. $2 \ln \frac{2\sqrt{3}}{3}$
- D. $\frac{1}{2} \ln \frac{2\sqrt{3}}{3}$
- E. NOTA

30. $4x + 5y^2 + 6xy = 5$. Find dy/dx at the point $(0, 1)$

- A. -1
- B. 0
- C. 1
- D. 2
- E. NOTA