Mu Individual Test Answers 2007 Mu Alpha Theta National Convention

1. A
2. C
3. C
4. D
5. C
6. E
7. D
8. C
9. A
10. D
11. C
12. A
13. E
14. A
15. A
16. B
17. D
18. D
19. B
20. A
21. B
22. D
23. A
24. A
25. B
26. A
27. B
28. B
29. D
30. D

1. For $y=\ln x$, $y'=\frac{1}{x}$; slope of tangent = 1. For $y=\arctan x$, $y'=\frac{1}{1+x^2}=1$ at $x=0$. Point is (0,0). **A**

2. $\int_2^9 \frac{e^{4x}}{e^{4x-1}}dx=\int_2^9 e\,dx=ex]_2^9=9e-2e=7e^1$. $A+B=7+1=8$. **C**

3. $f'(x)=\frac{-x^2-2x+1}{\left(x^2+1\right)^2}=0$ at $x=-1\pm\sqrt{2}$ $\Rightarrow$ $y=\frac{1\pm\sqrt{2}}{2}$. **C**

4. $R'(x)=F'(G(x))\cdot G'(x)=30(G(x)+1)^5\cdot G'(x)$. $R'(4)=30(3)^5\cdot\frac{1}{9}=\frac{30(3)^5}{3^2}=30(3)^3=810$. **D**

5. $V=e^3;\frac{dV}{dt}=3e^2\frac{de}{dt}$. $A=6e^2;\frac{dA}{dt}=12e\frac{de}{dt}$. Ratio is $\frac{3e^2\frac{de}{dt}}{12e\frac{de}{dt}}=\frac{e}{4}$. When $A=216; e=6$.
$\frac{6}{4}=\frac{3}{2}$. **C**

6. $V=\frac{4}{3}\pi r^3$; $dV=4\pi r^2dr$; $dV=4\pi(36)^2\left(\frac{1}{16}\right)=324\pi$. **E**

7. If $f(x)=(x-4)^3$, then $\lim_{x\to 4}h(x)=0$. But if $f(x)=(x-4)$, then $\lim_{x\to 4}h(x)$ does not exist. **D**

8. $\frac{dy}{dx}=\frac{\sqrt{(1+y^2)^5}}{6y}$ $\Rightarrow$ $\int\left(1+y^2\right)^{-5/2}y\,dy=\frac{1}{6}\int dx$ $\Rightarrow$ $-\frac{1}{3}\left(1+y^2\right)^{-3/2}=\frac{1}{6}x+C$.
$-\frac{1}{3}(1+3)^{-3/2}=\frac{1}{6}(-1)+C\Rightarrow C=\frac{1}{8}$. $-8\left(1+y^2\right)^{-3/2}=4x+3\Rightarrow x=-2\left(1+y^2\right)^{-1.5}-.75$ **C**

9. Original area = $\int_0^2\left[x-\left(x^2-x\right)\right]dx=\frac{4}{3}$. $\frac{1}{2}\cdot\frac{4}{3}=\frac{2}{3}=\int_0^k\left(2x-x^2\right)dx$ $\Rightarrow$ $k=1$.
$\int_2^3\left(x^2-x\right)dx=\frac{23}{6}$. **A**

10. $f'(x)=3\cos x-\sin x+e^{-x}+\frac{3}{x}+4$. $f''(x)=-3\sin x-\cos x-e^{-x}-\frac{3}{x^2}$.
$f'''(x)=-3\cos x+\sin x+e^{-x}+\frac{6}{x^3}$. Sum is: $\ln x^3+4x+7+\frac{3}{x}+4-\frac{3}{x^2}+\frac{6}{x^3}$.
Evaluating at $x=1$ $\Rightarrow$ $0+4+7+3+4-3+6=21$. **D**

11. $v(t)=5t^4-8t^3+9t^2-12t+5$. $a(t)=20t^3-24t^2+18t-12$. $a'(t)=60t^2-48t+18$.
$a'_{avg}=\frac{a'(5)-a'(0)}{5-0}=\frac{1278-18}{5}=252$. **C**

12. $\int_a^b h(2x)dx=\frac{1}{2}g(2x)]_a^b=\frac{1}{2}g(2b)-\frac{1}{2}g(2a)$. **A**

13. $\frac{dy}{dx}=\frac{1}{x}-6x^2-10x$. When $x=-2$; $\frac{dy}{dx}=-\frac{1}{2}-24+20=-\frac{9}{2}$. $\frac{d\left(\frac{1}{x^2+2}\right)}{dx}=\frac{-2x}{(x^2+2)^2}$.

When $x=-2$; $\frac{d\left(\frac{1}{x^2+2}\right)}{dx}=\frac{4}{36}=\frac{1}{9}$. $\frac{-9/2}{1/9}=-\frac{81}{2}$. **E**

14. $y'(x)=6x^5+4x^3-4x-3$. $y'(1)=6+4-4-3=3$. $m_{norm}=-\frac{1}{3}$. Equation of normal line: $y+2=-\frac{1}{3}(x-1) \Rightarrow -3y-6=x-1 \Rightarrow x+3y+5=0$. **A**

15. First ball: $s_1(t)=-16t^2+144=0$ when $t=3$.
Second ball: $s_2(t)=-16t^2+v_0t+144=0$ when $t=3-1=2$.
$s_2(2)=-64+2v_0+144=0 \Rightarrow v_0=-40$. **A**

16. $\frac{dy}{dt}=yt \Rightarrow \int\frac{dy}{y}=\int t\,dt \Rightarrow \ln y=\frac{1}{2}t^2+C_1 \Rightarrow y=C_2e^{t^2/2}$. $y=10$ when $t=0$ implies $C_2=10$. $y(4)=10e^8$. **B**

17. $A=\int_0^{2\pi/3}(\cos x-\cos 2x)dx+\int_{2\pi/3}^{\pi}(\cos 2x-\cos x)dx=$
$(\sin x-\frac{1}{2}\sin 2x)]_0^{2\pi/3}+(\frac{1}{2}\sin 2x-\sin x)]_{2\pi/3}^{\pi}=\left(\frac{\sqrt{3}}{2}+\frac{\sqrt{3}}{4}\right)-(0)+(0)-\left(-\frac{\sqrt{3}}{4}-\frac{\sqrt{3}}{2}\right)=\frac{3\sqrt{3}}{2}$.
D

18. $\int\left[\sin\left(\frac{x}{5}-1\right)\right]^{-1}\cos\left(\frac{x}{5}-1\right)dx=5\ln\left|\sin\left(\frac{x}{5}-1\right)\right|+C=-5\ln\left|\csc\left(\frac{x}{5}-1\right)\right|+C$. **D**

19. Since $\lim_{x\to\infty}\left(\sqrt{ax^2+bx}-cx\right)$ is finite, c must be positive. After multiplying the numerator and the denominator by the conjugate,
$\lim_{x\to\infty}\left(\sqrt{ax^2+bx}-cx\right)=\lim_{x\to\infty}\left(\frac{(a-c^2)x^2+bx}{\sqrt{ax^2+bx}+cx}\right)$. The limit is finite so $a-c^2=0$ and since $c^2+a=18$, $a=9$ and $c=3$. Since $\frac{b}{\sqrt{a}+c}=-2$, then $\frac{b}{6}=-2$ so $b=-12$.
$a+b+5c=9+(-12)+5(3)=12$. **B**

20. $A=\int_0^4(2x^2-3x+6)dx=\frac{2}{3}x^3-\frac{3}{2}x^2+6x]_0^4=\frac{128}{3}-24+24=\frac{128}{3}$.
$T(4)=\frac{4-0}{2(4)}(6+2(5)+2(8)+2(15)+26)=\frac{1}{2}(88)=44$. $44-\frac{128}{3}=\frac{4}{3}$. **A**

21. $f'(x)=3x^2+1$; $f'(2)=13$. $f(1)=2 \Rightarrow g(2)=1$. $g'(2)=\frac{1}{f'(1)}=\frac{1}{4}$.
$f(g(2))=g(f(2))=2$ since f and g are inverse functions.
$10-13+3(2)-2(2)+5(1)-4\left(\frac{1}{4}\right)=3$. **B**

22. The two numbers are x and $(20-x)$. $P(x)=(20-x)^2x^3=400x^3-40x^4+x^5$. $P'(x)=5x^2\left(x^2-32x+240\right)=0$ at $x=0$ or $x=12$ or $x=20$. Maximum product occurs at $x=12$, so the two numbers are 8 and 12. Their difference is 4. D

23. $\int_3^4 f(x)g(x)dx=\int_3^4[g(x)]^1\cdot g'(x)dx=\frac{[g(x)]^2}{2}\Big]_3^4=\frac{g^2(4)}{2}-\frac{g^2(3)}{2}=\frac{81}{2}-\frac{49}{2}=16$. A

24. $\int_0^5\left|x^3-x\right|dx=\int_1^5\left(x^3-x\right)dx-\int_0^1\left(x^3-x\right)dx=\left(\frac{x^4}{4}-\frac{x^2}{2}\right)\Big]_1^5-\left(\frac{x^4}{4}-\frac{x^2}{2}\right)\Big]_0^1=$
$\left(\frac{5^4}{4}-\frac{5^2}{2}\right)-\left(\frac{1}{4}-\frac{1}{2}\right)-\left(\frac{1}{4}-\frac{1}{2}\right)=\frac{625}{4}-\frac{50}{4}+\frac{1}{4}+\frac{1}{4}=\frac{577}{4}$. A

25. Slope is given by $\frac{dy}{dx}=\frac{dy/dt}{dx/dt}=\frac{3t^2-1}{2t+1}$ evaluated at $t=1$ which is the only value of t for which $x=3$ and $y=-1$. Slope $=\frac{2}{3}$ and so equation of tangent line is $y+1=\frac{2}{3}(x-3)$ or $y=\frac{2}{3}x-3$. x-intercept is $\frac{9}{2}$. B

26. $V=2\pi\int_0^5(6-x)(2x^2)dx=2\pi\int_0^5\left(12x^2-2x^3\right)dx=2\pi\left(4x^3-\frac{1}{2}x^4\right)\Big]_0^5=375\pi$. A

27. $r^2+h^2=k^2 \Rightarrow r^2=k^2-h^2$. $V=\frac{\pi}{3}r^2h=\frac{\pi}{3}\left(k^2-h^2\right)h=\frac{\pi}{3}\left(k^2h-h^3\right)$.
$\frac{dV}{dh}=\frac{\pi}{3}\left(k^2-3h^2\right)=0$ when $h^2=\frac{k^2}{3}$ or $h=\frac{k\sqrt{3}}{3}$. $r^2=k^2-\frac{k^2}{3}=\frac{2k^2}{3}$ so $r=\frac{k\sqrt{6}}{3}$.
B

28. $f'(x)=x^2-1=0$ at $x=1$ or $x=-1$. $f''(x)=2x=0$ at $x=0 \Rightarrow$ inflection point at (0,3).
$f(-2)=\frac{7}{3} \Rightarrow$ relative/absolute minimum at $\left(-2,\frac{7}{3}\right)$.
$f(-1)=\frac{10}{3} \Rightarrow$ relative maximum at $\left(-1,\frac{10}{3}\right)$.
$f(1)=\frac{7}{3} \Rightarrow$ relative/absolute minimum at $\left(1,\frac{7}{3}\right)$.
$f(3)=9 \Rightarrow$ absolute maximum at (3,9).
Only statement II is true. B

29. $F(x)=t^2\Big]_x^{\sin x}=\sin^2x-x^2$. $F'(x)=2\sin x\cos x-2x=\sin 2x-2x$. D

30. $2xy\frac{dy}{dx}+y^2+5\cos x+\frac{dy}{dx}=\frac{y^2-2xy\frac{dy}{dx}}{y^4}$. $0+1+5+\frac{dy}{dx}=1 \Rightarrow \frac{dy}{dx}=-5$. D