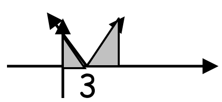
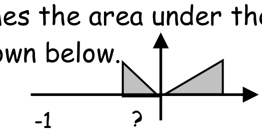
Integration Topic Test

Solutions:

1. **A**. x+c evaluated from a to b is b-a.
2. **B**. Let u=x/2 and we get the integral of $2\cos(u)du$ which is evaluated to be 2(sin$\frac{\pi}{6}$-sin0) which gives 1.
3. **C**. a length of 9 divided by 2 gives change in x is 2: 2(1+9+25+49) = 168.
4. **A**. Let u be x-1, so we get $\int 3(u+1)u^{\frac{1}{2}}du$ which is equivalent to choice A.
5. **C**. $\int_0^4 f(x)dx = 20$ by the MVT for integrals. To evaluate $\frac{1}{4}(20+\int_0^4 2dx)$ which gives 7.
6. **D**. Since g is a constant, g' is 0 at all x.
7. **B**. $\int_0^4 |x-3|\,dx$ can be evaluated with the area of two triangles: 0.5(3)(3)+0.5(1)(1) which gives 5.



8. **D**. By the FTC, and the chain rule f(x)= $3\sqrt{(3x)^2-(3x)}$ and at x=5 we get $3\sqrt{210}$.
9. **A**. For x< -1 we use the integral of 4x dx. Evaluated from -2 to -1 we get $2x^2+c = 2-8 = -6$ and and from -1 to 0 we use $5-x^2$ to get $5x-\frac{x^3}{3}+c$ which gives 14/3. -6+14/3 = -4/3.
10. **C**. $\int_0^1 (1-x^2)dx + \int_1^2 (x^2-1)dx$ which gives 2.
11. **C**.
12. **D**. $\frac{1}{6}Arc\tan\frac{3x}{2}+c$ evaluated from 0 to $\frac{2}{3}$ which gives 1/6 times Arctan1 = $\frac{1}{6} g \frac{\pi}{4} = \frac{\pi}{24}$
13. **A**. Let u be secx, to give $\int \sec^2 x(\sec x\tan x)dx$ which gives $\frac{1}{3}\sec^3 x + c$. So sec(π)+3= -1+3 = 2.
14. **A**. The radius of the semicircle is $\frac{\sqrt{x}}{2}$ so we use $\frac{\pi}{2}\int_0^9 \left(\frac{\sqrt{x}}{2}\right)^2 dx$ which gives $\frac{81}{16}\pi$.
15. **B**. $\int_a^c f(x)dx + \int_c^b f(x)dx = \int_a^b f(x)dx$. So 6=2+? gives the answer is 4.
16. **C**. Let u= g(x) and $\int_0^2 f(g(x))g'(x)dx$ = $\int_{g(0)}^{g(2)} f(u)du$ = F(g(2))-F(g(0))= F(1)-F(-1)= -1-0.5=-1.5
17. **A**. h(x)= $\ln x - \ln 1$ gives $\ln(x^3)$ which equals 3lnx or choice A.
18. **B**. $\frac{f(2)-f(0)}{2-0} = 2(\frac{1}{2-0})\int_0^2 (3x^2+2x^2+k)dx$ slope 8 equals x^3+x^2+kx+c evaluated from 0 to 2, and so k= -2.
19. **D**. Using symmetry, we integrate $e^{-x}dx$ over 0 to 2 and double the answer. $-e^{-2}-(-e^0) = -\frac{1}{e^2}+1$ times 2. Getting a common denominator gives choice D.
20. **C**. $2+\int_0^2 v(t)dt$ gives 4/3.
21. **B**. $\frac{ds}{dt} = k(2s)$ so $\frac{1}{s}ds = 2k\ dt$ and $\ln s = 2kt + c$. Using the initial condition gives c=2 and using the second gives k= -1/4. Substitute e^{-e} for s gives t=4+2e.
22. **D**. For x<0, |x|/x equals -1. So the integral on the left is equal to -1(2) which is the area of the 2 by 1 rectangle under the x-axis. So -2 equals -0.25 times the area under the absolute value graph shown below.



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The area of this is 0.5(1)(1)+0.5(m)(m). So

$$-2=-\frac{1}{4}\left(\frac{1}{2}+\frac{1}{2}(m^2)\right)$$ which solves

to 16=1+ m^2 or $m=\sqrt{15}$.

23. **B**. The y-values for the intervals (-1,0] is -1. The area under the curve is thus $\left(\frac{1}{3}-\frac{1}{5}\right)$ and the value is negative. Answer is B: $\frac{-2}{15}$.

24. **D**. $ke^{-kx}-k=p(-\frac{1}{k})(ke^{-kx}+k)$

$k(e^{-kx}-1)=p(-1)(e^{-kx}-1)$ so $p=-k$

25. **E**. Displacement over [0, 10].

26. Answer **C**.

$$\int_{-1}^{2\sqrt{3}-1}\frac{dx}{x^2+2x+5}=\int_{-1}^{2\sqrt{3}-1}\frac{dx}{x^2+2x+1+4}$$

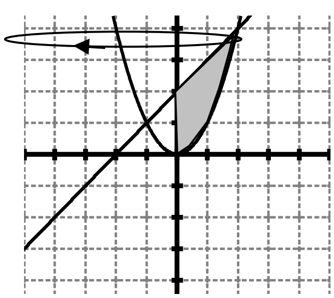
by completing the square.

$$\int_{-1}^{2\sqrt{3}-1}\frac{dx}{(x+1)^2+4}=$$

$$\frac{1}{2}Arc\tan\frac{2\sqrt{3}-1+1}{2}-\frac{1}{2}Arc\tan 0$$

So $\frac{1}{2}g\frac{\pi}{3}$ is the result which gives k=6.

27.



Answer **B**. Either use y-variables

$$\pi\int_0^2((2+\sqrt{y})^2-4)dy+$$

$$\pi\left[\int_2^4(2+\sqrt{y})^2dy-\int_2^4(2+(y-2))^2dy\right]$$

or x-variables

$$2\pi\int_0^2(2+x)(x+2-x^2)dx$$. Neither is nice, and requires multiplying to get a polynomial expression and then integrating. The answer is $\frac{56}{3}\pi$. Work for the second integral:

$$2\pi\int_0^2(2+x)(x+2-x^2)dx=$$

$$2\pi\int(-x^3-x^2+4x+4)dx=$$

$$2\pi(-\frac{x^4}{4}-\frac{x^3}{3}+2x^2+4x+c)$$ evaluated

to get $2\pi(-4-\frac{8}{3}+8+8)=\frac{56}{3}\pi$.

28. **C**. $\int 9^{3x}dx=\frac{9^{3x}}{\ln 9}g\frac{1}{3}+c$ = $\frac{3^{6x}}{2\ln 3}g\frac{1}{3}$+c

so n=6 and g=6x. So 6+24=30

29. **D**. $\int_{0.5}^{1}\frac{dx}{2\sqrt{x-x^2}}=\int\frac{dx}{2\sqrt{x}\sqrt{1-x}}$. Let u= $\sqrt{x}$

and $du=\frac{1}{2\sqrt{x}}$ so we have $\int\frac{1}{\sqrt{1-u^2}}du$

which gives $Arc\sin\sqrt{x}+c$ evaluated

from x=0.5 to x=1. Arcsin1-Arcsin$\frac{\sqrt{2}}{2}$

gives $\frac{\pi}{2}-\frac{\pi}{4}=\frac{\pi}{4}$.

30. $\int_0^{\frac{\pi}{3}}\cos^2(2x)dx=\int_0^{\frac{\pi}{3}}\left(\frac{1}{2}\cos(4x)+\frac{1}{2}\right)dx$

since $\cos(4x)=2\cos^2(2x)-1$ by the cosine double angle formula.

Integrate to get $\frac{1}{8}\sin(4x)+\frac{x}{2}+c$

which is evaluated to get $\frac{1}{8}\sin\frac{4\pi}{3}+\frac{\pi}{6}$

$\frac{\pi}{6}-\frac{1}{8}g\frac{\sqrt{3}}{2}=\frac{\pi}{6}-\frac{\sqrt{3}}{16}$ to give choice **A**