Integration
2007 Mu Alpha Theta National Convention

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The abbreviation NOTA denotes "None of These Answers."

1. For real constants a and b, and $b > a > 0$ the value of $\int_a^b \frac{1}{x}\,dx$ is

A. $b - a$ B. 0
C. $\frac{1}{a^2} - \frac{1}{b^2}$ D. $\ln\left(\frac{b}{a}\right)$ E. NOTA

2. $\int_0^{\frac{\pi}{3}} \cos\left(\frac{x}{2}\right) dx =$

A. $\frac{1}{2}$ B. 1
C. $\sqrt{3}$ D. $2\sqrt{3}$ E. NOTA

3. Using a left-hand Riemann sum and four equal subdivisions, approximate $\int_1^9 x^2 dx$.

A. 90 B. 150
C. 168 D. $\frac{992}{3}$ E. NOTA

4. $\int_1^5 3x\sqrt{x-1}\ dx =$

A. $3\int_0^4 (u^{\frac{3}{2}} + u^{\frac{1}{2}})du$ B. $\frac{3}{2}\int_1^5 (u^2 + u)du$
C. $\frac{1}{2}\int_1^5 (u+1)du$ D. $3\int_0^2 u(u-1)^{\frac{3}{2}}du$
E. NOTA

5. The average value of a differentiable function f over the interval [0, 4] is 5. What is the average value of $g(x) = f(x) + 2$ over the same interval?

A. 5 B. 5.5
C. 7 D. 13 E. NOTA

6. $f'(x) = e^{4x^2-1}$ and $g(x) = \int_0^4 f'(x)dx$ for both f and g differentiable functions. What is the value of $g'(3)$?

A. $\frac{e^{35}-1}{4}$ B. $\frac{e^{35}}{4}$
C. e^{35} D. 0 E. NOTA

7. $\int_0^4 \sqrt{x^2 - 6x + 9}dx$ =

A. 4 B. 5
C. $\frac{13}{2}$ D. 20 E. NOTA

8. $f(x) = \int_1^{3x} \sqrt{t^2 - t}\ dt$ for all $x > 1$.

Give the value of $f'(5)$.

A. $6\sqrt{53}$ B. $3\sqrt{70}$
C. $\sqrt{210}$ D. $3\sqrt{210}$ E. NOTA

9. $g(x)=\begin{cases}4x & for\ |x|\geq 1\\ 5-x^2 & for\ |x|<1\end{cases}$; $\int_{-2}^{0} g(x)dx$ =

A. $-\frac{4}{3}$ B. $\frac{2}{3}$
C. $\frac{4}{3}$ D. $\frac{8}{3}$ E. NOTA

10. The total area bounded by the graphs of $y=x^2-1$, $y=0$, $x=0$ and $x=2$ is

A. $\frac{14}{3}$ B. 3
C. 2 D. $\frac{2}{3}$ E. NOTA

11. $\frac{1}{2}(0.5)(\sqrt{1}+2\sqrt{1.5}+2\sqrt{2}+2\sqrt{2.5}+\sqrt{3})$ is a trapezoidal approximation of

A. $\int_1^3 2\sqrt{x}dx$ B. $\int_1^3 2\sqrt{\frac{x}{2}}dx$
C. $\int_1^3 \sqrt{x}dx$ D. $\int_1^3 \sqrt{\frac{x}{2}}dx$ E. NOTA

12. $\int_0^{\frac{2}{3}} \frac{dx}{4+9x^2}=$

A. $\ln 2$ B. $\frac{1}{9}\ln\left(\frac{5}{2}\right)$
C. $\frac{\pi}{4}$ D. $\frac{\pi}{24}$ E. NOTA

13. $\int \sec^3 x\tan x dx = \frac{1}{n}\left(g(x)\right)^n + c$.
Give the value of $g(\pi)+n$.

A. 2 B. 3
C. 4 D. 5 E. NOTA

14. The region R is bounded by the graph of $y=\sqrt{x}$, the x-axis and the line $x=9$. A solid is formed with R as base, and cross sections perpendicular to the x-axis are semicircles. What is the volume of this solid?

A. $\frac{81}{16}\pi$ B. $\frac{81}{4}\pi$
C. $\frac{9}{8}\pi$ D. $\frac{9}{4}\pi$ E. NOTA

15. For a continuous function *f*, and $a<c<b$, if $\int_a^b f(x)dx=6$ and $\int_c^b f(x)dx=2$ then $\int_a^c f(x)dx=$

A. 8 B. 4
C. –4 D. –8 E. NOTA

16.

f(x)	*g(x)*	*f '(x)*	*g'(x)*	*F(x)*	*x*
-1.2	*-1.5*	*-2*	*-0.9*	*0.5*	*-1*
-1	*-1*	*0*	*0*	*0*	*0*
-0.75	*-0.5*	*0.5*	*1*	*-1*	*1*
-2	*1*	*-2*	*2.9*	*-4*	*2*

$F(x)=\int_0^x f(x)dx$. For continuous and differentiable functions f, F, and g, $\int_0^2 f(g(x))g'(x)dx$ has what value, given the values in the table above?

A. –4 B. –1
C. –1.5 D. 2 E. NOTA

17. If $h(x) = \int_1^x \frac{1}{u}\,du$ for all $x \geq 1$ then $h(x^3) =$

A. $3gh(x)$ B. $h(3x)$
C. $\left(h(x)\right)^3$ D. $h(x+3)$ E. NOTA

18. The average rate of change of $f(x) = 3x^2 + 2x + k$ over the interval $[0, 2]$ is equal to twice the average value of $f(x)$ over the same interval. Give the value of k.

A. -4 B. -2
C. 2 D. 4 E. NOTA

19. $\int_{-2}^{2} e^{-|x|}dx =$

A. $2e$ B. $\frac{2}{e} - 2$
C. 0 D. $2(\frac{e^2-1}{e^2})$ E. NOTA

20. A particle travels along the x-axis with velocity given by $v(t) = 4 - 3t - t^2$. If the particle is at position 2 on the x-axis at $t = 0$, then where is the particle at $t = 2$?

A. $-\frac{2}{3}$ B. $\frac{2}{3}$
C. $\frac{4}{3}$ D. $\frac{13}{3}$ E. NOTA

21. The rate at which the amount of a substance changes is directly proportional to twice the amount of the substance at time t seconds. When $t = 0$, the amount of the substance is e^2. At $t = 3$ seconds, the amount of the substance is $\sqrt{e}$. At what time t seconds will the amount of the substance be $\frac{1}{e^e}$?

A. $2 + e$ B. $4 + 2e$
C. $2 + 4e$ D. $4 + 4e$ E. NOTA

22. $\int_{-3}^{-1} \frac{|x|}{x}dx = -\frac{1}{4}\int_{-1}^{m} |x|\,dx$. If $m > 0$ then give the value of m^2.

A. 23 B. 17
C. 16 D. 15 E. NOTA

23. For $[x]$ denoting the greatest integer less than or equal to x, $\int_{-\frac{1}{3}}^{-\frac{1}{5}} [x]dx =$

A. $-\frac{1}{8}$ B. $-\frac{2}{15}$
C. 0 D. $\frac{1}{15}$ E. NOTA

24. For $x > 0$, the function $\frac{dy}{dx} = ke^{-kx} - k$ is directly proportional to the function g such that $g(x) = \int_0^x ke^{-kt}dt$. Find the constant of proportionality.

A. $-k^2$ B. $-\frac{1}{k}$
C. 1 D. $-k$ E. NOTA

25. For a continuous function $v(t)$ where $v(t)$ represents the velocity of a particle moving along the x-axis at time t, and $|v(t)| \neq v(t)$ at some value of t over the interval [0, 10], then $\int_0^{10} v(t)dt$ represents what quantity?

A. total distance traveled by the particle over [0, 10]
B. position of the particle at $t = 10$
C. change in velocity of the particle over [0, 10]
D. average rate of change of the particle over [0, 10].
E. NOTA

26. $\int_{-1}^{2\sqrt{3}-1} \frac{dx}{x^2 + 2x + 5} = \frac{\pi}{k}$. Give the value of k.

A. 3 B. 4
C. 6 D. 12 E. NOTA

27. The region in quadrant I bounded by the graphs of $y = x^2$, $y = x + 2$ and the y-axis is rotated about the line $x = -2$. What is the volume of the resultant solid ?

A. 24π B. $\frac{56}{3}\pi$
C. $\frac{22}{3}\pi$ D. 8π E. NOTA

28. $\int 9^{3x}dx = \frac{3^{g(x)}}{n \ln 3} + c$. Give the value of $n + g(4)$.

A. 25 B. 26
C. 30 D. 32 E. NOTA

29. $\int_{\frac{1}{2}}^{1} \frac{dx}{2\sqrt{x - x^2}} =$

A. $\frac{1}{2}$ B. $\frac{\sqrt{3}}{3}$
C. $\frac{\pi}{2}$ D. $\frac{\pi}{4}$ E. NOTA

30. $\int_0^{\frac{\pi}{3}} \cos^2(2x)dx = D + F$ g $Arc\sin\left(\frac{1}{2}\right)$.
Give the value of D g F.

A. $-\frac{\sqrt{3}}{16}$ B. $\sqrt{3}$
C. 1 D. $\frac{\sqrt{3}}{2}$ E. NOTA