2007 Mu Alpha Theta National Convention
Mu Limits and Derivatives

Directions: For all questions, NOTA means none of the above answers is correct. Have fun!

1.) $f(x) = x^3 + x - 108$. Find $f'(3)$.
a.) -80
b.) 28
c.) 30
d.) 82
e.) NOTA

2.) $f(x) = x^3 + x - 108$. Find $\frac{df}{df'}$ at $x = 2$.
a.) $-\frac{49}{6}$
b.) $-\frac{98}{13}$
c.) $\frac{13}{12}$
d.) $\frac{13}{6}$
e.) NOTA

3.) If $f(x) = \dfrac{x^3 - 3x^2 + x - 3}{x^3 - 3x^2 + 2x - 6}$, find $22 \cdot (\lim\limits_{t \to -\infty} f(t) + \lim\limits_{t \to 3} f(t) + \lim\limits_{t \to \infty} f(t))$.
a.) 20
b.) 31
c.) 42
d.) 64
e.) NOTA

4.) If $f(x) = x^3 + 1$, evaluate $\lim\limits_{h \to 0} \dfrac{f(x+h)^2 - f(x)^2}{2h}$ at $x = 1$.
a.) $-\infty$
b.) -3
c.) 0
d.) 6
e.) NOTA

5.) Let $m(x)$ denote the measure of angle x. Consider a triangle ABC with $m(a) = \frac{\pi}{4}$ and $m(b) = \frac{\pi}{6}$. The length of side A equals $4\sqrt{2}$, and the length of side B equals 4. Using the formula Area$=\frac{1}{2}AB \sin c$, use differentials to approximate the change in the area of the triangle if the measure of angle a is decreased by $\frac{\pi}{6}$, while the measure of angle b is increased by $\frac{\pi}{12}$. Note that $\sin \frac{\pi}{12} = \frac{\sqrt{6}-\sqrt{2}}{4}$.
a.) $\frac{-4\sqrt{2}\pi}{3}$
b.) $\frac{(1-\sqrt{3})\pi}{3}$
c.) $2 - 2\sqrt{3}$
d.) $2 + 4\sqrt{2} - 2\sqrt{3} + 2\sqrt{6}$
e.) NOTA

6.) What is the slope of the tangent line to the graph of $y = \ln x$ at $x = -4$?
a.) $-\frac{1}{4}$
b.) 0
c.) $\frac{1}{4}$
d.) There is no such tangent line.
e.) NOTA

7.) Evaluate $\lim_{x\to 3^+} \dfrac{\sqrt{x^2-9}-\sqrt{6x-18}}{x-3}$.
a.) 0
b.) $\frac{1}{4}$
c.) 6
d.) ∞
e.) NOTA

8.) A particle is moving along the curve $y = x^2 \sin(x\pi) + 9\log x$. If its velocity in the x-direction is constant at $2\frac{units}{sec}$, which of the following is closest to its velocity in the y-direction the last time its y-coordinate is 18?
a.) -10000π
b.) 0
c.) 10000
d.) 20000π
e.) NOTA

9.) The infield of a baseball field is formed by a square of side length r, and a circle with radius r, centered at one of the vertices of the square. Let A equal the area of the circle. At the moment when $A = 4\pi$ and $\frac{dA}{dt} = 2\pi$, what is the rate of change of the area inside the square, but outside of the circle?
a.) $4-\pi$
b.) $2-\frac{\pi}{2}$
c.) $\frac{3}{2}\pi$
d.) 3π
e.) NOTA

10.) If $x = y^2 + 2y + 2$, what is $\frac{dx}{dy}$ at the point where $x = 1$?
a.) undefined
b.) 0
c.) $\frac{1}{4}$
d.) 4
e.) NOTA

11.) Beatriz is playing a game which consists of rolling an n-sided die n times. Which of the following is the closest to her probability of rolling a 1 at least once as n approaches infinity?
a.) 0
b.) $\frac{1}{2}$
c.) 1
d.) 2
e.) NOTA

12.) Simply due to hype and Disney branding, Pirates of the Caribbean brings in $f(x) = 5000 + x - x^2$ dollars of profit per movie theatre, x days after its release. This might seem somewhat meager; however, the addition of Orlando Bloom and Johnny Depp causes hoardes of cheering fangirls to add $g(x) = 6000 - x^3$ dollars to this amount, also x days after the movie's release. If marginal profit is defined as the derivative of profit, what is the absolute value of the difference between the marginal profit of the movie with the stars and the marginal profit of the movie without the stars, 20 days after the movie's release? Consider $f(x)$ and $g(x)$ to be differentiable.
a.) 41
b.) 1159
c.) 1200
d.) 1241
e.) NOTA

13.) Two tangent lines are drawn to the curve $y = x^3 - 7x$, one at $x = 2$ and one at $x = a$, such that the acute angle of their intersection is 45 degrees. Which of the following could be $9 \cdot a$?

a.) -23
b.) $\frac{27\sqrt{23}}{23}$
c.) $3\sqrt{69}$
d.) 23
e.) NOTA

14.) Find $\lim_{n\to\infty} \sum_{i=0}^{n} \frac{ni - i^2}{n^3}$.

a.) $-\frac{1}{3}$
b.) $\frac{1}{6}$
c.) $\frac{1}{2}$
d.) $\frac{5}{6}$
e.) NOTA

15.) If the position function of a particle, $p(t)$, is continuous for all $t > 0$ and the velocity function is given by $v(t) = t^2$ on the interval $[0, 2)$ and $v(t) = kt^2 + 3$ on the interval $[2, \infty)$, for what value of k is $p(t)$ differentiable for all values of t, $t > 0$?

a.) $\frac{1}{4}$
b.) 1
c.) All values of k, such that $k > 0$.
d.) No such value of k.
e.) NOTA

16.) What is the maximum volume of the shape that is created when the area bounded by the curve $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is rotated about the line $x = c$, where $c > b > a > 0$?

a.) $2abc\pi^2$
b.) $abc^2\pi^2$
c.) $2a^2b^2c\pi^3$
d.) $2(a + b)c^2\pi^2$
e.) NOTA

For problems 17-19, consider the smiley face drawn by: $f(x) = -x$ on the interval $[-2, -1)$, $g(x) = \sqrt{1 - x^2}$ on the interval $[-1, 1]$, and $h(x) = x$ on the interval $(1, 2]$.

17.) What value of x satisfies the Mean Value Theorem for Derivatives on the interval $[0, 1]$ on $g(x)$? Note that the Mean Value of Derivatives is satified by an x-value c, on a function $f(x)$, on an interval $[a, b]$, if $f'(c) = \frac{f(b)-f(a)}{b-a}$, if $f(x)$ is differentiable on the interval (a, b), and if $c \neq a$ and $c \neq b$.

a.) $\frac{1}{2}$
b.) $\frac{\sqrt{2}}{2}$
c.) $\frac{\sqrt{3}}{2}$
d.) 1
e.) NOTA

18.) Rolle's Theorem is a special case of the Mean Value Theorem for Derivatives, adding the necessary condition that $f(a) = f(b)$. Considering $g(x)$, there is only one possible x-value that will satisfy Rolle's Theorem for any two correctly chosen endpoints. What is this x-value?
a.) -1
b.) $\frac{-\sqrt{2}}{2}$
c.) 0
d.) $\frac{\sqrt{2}}{2}$
e.) NOTA

19.) If the inverses of the functions that compose the smiley face are graphed, two tangent lines can be drawn at $x = -\frac{1}{2}$ and two more tangent lines can be drawn at $x = \frac{3}{2}$. What is the sum of the absolute values of the slopes of these four tangent lines?
a.) $-\sqrt{3}$
b.) $3 - \sqrt{3}$
c.) 3
d.) $3 + \sqrt{3}$
e.) NOTA

20.) Two tangent lines can be drawn from the point $(-2, -2)$ to the function $f(x) = x^2 + 2x + 3$. What is the sum of the x-coordinates of the points where those lines are tangent to $f(x)$?
a.) -4
b.) -2
c.) 0
d.) $2 + 2\sqrt{5}$
e.) NOTA

21.) Consider the following proof:
1. $x^2 = x + x + x + x + ...$ (since x squared equals x, added x times)
2. $2x = 1 + 1 + 1 + 1 + ...$ (taking the derivative of both sides)
3. $2x = x$ (since the right side has x ones)
4. $2 = 1$. (dividing both sides by x).
Which is the first step with an error?
a.) 1
b.) 2
c.) 3
d.) 4
e.) NOTA

22.) Evaluate $\frac{d}{dx} \displaystyle\int_{\cos x}^{\sin x} t \, dt$
a.) $\sin 2x$
b.) $\cos 2x$
c.) 0
d.) 1
e.) NOTA

23.) Evaluate $\frac{d}{dx}\int_{\sinh^2 x}^{\cosh^2 x} t\,dt$. Note that $\sinh x = \frac{e^x - e^{-x}}{2}$ and $\cosh x = \frac{e^x + e^{-x}}{2}$.
a.) $\sinh 2x$
b.) $\cosh 2x$
c.) $\frac{\sinh 4x}{2}$
d.) 0
e.) NOTA

24.) The sum of the elements of the x^{th} row of Pascal's Triangle, counting the top row as the zeroth row, is $f(x) = a^x$ for some positive integer a. If we consider consider $f(x)$ to be differentiable, what is its derivative at $x = c$, if the elements of the c^{th} row of Pascal's Triangle sum to 64?
a.) 5
b.) $32 \ln 2$
c.) $64 \ln 2$
d.) $128 \ln 2$
e.) NOTA

25.) If $y = x^{x^{x^{\cdots}}}$, evaluate $\lim_{x \to 0^+} x^y$.
a.) -1
b.) 0
c.) 1
d.) ∞
e.) NOTA

26.) If $f(t) = t + c$, where c is an integer, find $\lim_{x \to \infty} \sum_{n=0}^{\infty} \frac{f(\frac{x}{2})^n}{f(x)^n}$.
a.) 0
b.) 1
c.) 2
d.) ∞
e.) NOTA

27.) $\lim_{x \to -\infty} \frac{x + x^2 + x^3}{\sqrt{9x^6 + 4x^4 + x^2}} =$
a.) $-\infty$
b.) $-\frac{1}{9}$
c.) $\frac{1}{3}$
d.) $\frac{1}{9}$
e.) NOTA

28.) Gabor is traveling from Rock Springs, WY to Jackson, WY. Interestingly, Wyoming roads, WYO-n, are numbered so that n is exactly ten times their speed limit in miles per hour. The only exceptions to this rule are Interstate-80 and Interstate-90, on which the speed limit is 90 miles per hour. All roads numbered WYO-n intersect I-80 exactly n miles from the Eastern state border, are perfectly straight, and also run perpendicular to both I-80 and I-90. I-80 and WYO-200 run through Rock Springs, while I-90 and WYO-380 run through Jackson. Unfortunately, Gabor took I-90, then WYO-380 to Jackson, a trip that took him $\frac{88}{19}$ hours. If Gabor follows the speed limit exactly, and he is trying to minimize his travel time to I-80, which road should he have taken from I-90 to I-80?

a.) WYO-200
b.) WYO-300
c.) WYO-320
d.) WYO-380
e.) NOTA

29.) If Gabor's average speed, in miles per hour, in the preceeding problem was k, and the speed of light, also in miles per hour, is c, which of the following is closest to $\frac{c}{k}$?

a.) 0
b.) 1
c.) 10
d.) ∞
e.) NOTA

30.) Paul, Gus, Praveen, and Gabor are playing a game in which each is assigned a function. If a player's function is even, that player can be said to be right. If a player's function is odd, that player can be said to be wrong. Functions are assigned in the following manner: Gabor's function is less than or equal to the absolute value of any other function at any value of x. Gus's function is always of the form $y = ax^2 - bx + c$, while Praveen's function is always of the form $y = ax^2 + bx - c$, where a, b, and c are nonzero real numbers. Paul's function is the derivative of the sum of Praveen's, Gabor's, and Gus's functions. Which of the following statements is false?

a.) Paul is always wrong.
b.) Gus and Praveen are either both wrong, or both right.
c.) Gabor is always wrong.
d.) Gabor is always right.
e.) NOTA