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*For all questions, NOTA means None Of The Above answers is correct.*

1. Consider a distribution of measurements that is approximately normal. According to the empirical rule, approximately what percentage of all measurements are within the interval $\mu \pm 2\sigma$ ?

   a) 34            b) 68            c) 47.5            d) 95            e) NOTA

2. Events A and B are independent. $P(A|B) = 1/3$ and $P(B|A) = 4/5$.
   Find $P(A)P(B)$.

   a) 11/15         b) 4/5           c) 5/12            d) 1/3           e) NOTA

3. How many permutations are there of the letters in the words TAMPA BAY?

   a) 8!/3!         b) 5!            c) 8!/5!           d) 8!/(5!3!)      e) NOTA

4. The length of time required by mathletes to complete a 1-hour exam is a random variable with a density function given by
$$f(t) = \begin{cases} Ct^2 + t, & 0 \le t \le 1 \\ 0, & \text{elsewhere.} \end{cases}$$
   Find $C$.

   a) 3/2           b) 2/3           c) 2               d) 1/2           e) NOTA

5. Let X and Y have the joint probability density function given by

$$f(x, y) = \begin{cases} k(1-y), & 0 \le x \le y \le 1 \\ 0, & \text{elsewhere.} \end{cases}$$

   Find $k$.

   a) 1             b) 2             c) 3               d) 6             e) NOTA

6. The eccentricity of a rotor ($x$), due to manufacturing errors, is found to have the following probability density function:
$$p(x) = \begin{cases} kx^2, & 0 \le x \le 5 \ mm \\ 0, & \text{elsewhere} \end{cases}$$
   Find the probability that a rotor picked at random will have an eccentricity no greater than 2 *mm*.

   a) 27/125        b) 117/125       c) 8/125           d) 98/125        e) NOTA

7. What famous German mathematician is credited for inventing the normal distribution, which carries his name in many texts and is widely used in probability theory?

   a) Gauss         b) Leibniz       c) Bell            d) Einstein      e) NOTA

8. Maxwell's speed distribution law stated as

$$P(v) = 4\pi \left( \frac{M}{2\pi RT} \right)^{3/2} v^2 \exp \frac{-Mv^2}{2RT}$$

where $v$ is the speed a molecule with mass $M$ and temperature $T$. Given $M$, $R$, and $T$ are constant, identify the expression for the most probable speed. (Hint: This requires $dP/dv = 0$.)

a) $\sqrt{\dfrac{2RT}{M}}$      b) $\sqrt{\dfrac{8RT}{\pi M}}$      c) $\sqrt{\dfrac{3RT}{M}}$      d) $\sqrt[3]{\dfrac{3RT}{M}}$      e) NOTA

9. Find the number of ways that four pennies, three nickels, two dimes, and one quarter can be arranged in a line.

    a) 2,400      b) 6,300      c) 9,680      d) 12,600      e) NOTA

10. Five balls are dropped at random into three boxes. What is the probability that three balls will be in the first box, two in the second, and zero in the third, if there is no limit on the number per box.

    a) 10/27      b) 20/243      c) 10/243      d) 20/27      e) NOTA

11. Player A rolls one 6-sided dice. At the same time, player B also rolls an identical die. If B rolls a number equal to or greater than A's roll, then B wins. What is the probability that A will win?

    a) 5/6      b) 7/12      c) 5/12      d) 1/6      e) NOTA

12. Four boxes each contain 5 objects. The first box contains 1 green object and 4 gold objects, the second 2 green and 3 gold, the third 3 green and 2 gold, and the fourth 4 green and 1 gold. One of the boxes is chosen at random and 2 objects are drawn from it. What is the probability that both objects are gold?

    a) 1/16      b) 1/8      c) 1/4      d) 1/2      e) NOTA

13. Bill, Jill, Gill, and Phil sit down at a blackjack table. Hands are played in this order. The probability for each winning their hand is 1/2, 1/3, 1/4, and 1/5, respectively. What is the probability that Gill will be the first to win a hand?

    a) 1/5      b) 1/12      c) 5/24      d) 5/48      e) NOTA

14. A certain type of missile is known to hit its target 70% of the time. If two missiles are shot, what is the probability that at least one will hit its target?

    a) 9/100      b) 49/100      c) 51/100      d) 91/100      e) NOTA

15. What is the probability that in five tosses of a fair coin, exactly four tosses are heads?

    a) 27/32          b) 5/32                    c) 4/5                     d) 1/5                    e) NOTA

16. If the probability of an event happening 4 times in 5 trials is 10/243, find the probability of the event happening in a single trial?

    a) 1/2            b) 1/3                     c) 1/4                    d) 1/6                    e) NOTA

17. Seven people enter a room with four available chairs. Those without a chair are asked to leave. How many seating arrangements are possible?

    a) 210            b) 144                     c) 35                     d) 24                     e) NOTA

18. Six fair coins are tossed and the number of heads is recorded. Let M be the most probable number of heads in a trial. What is the probability that M heads are observed in a given trial.

    a) 13/16          b) 11/16                   c) 5/16                   d) 3/16                   e) NOTA

**For Problems 19 – 20, let *x* be a continuous random variable with probability density function:**

$$f(x) = \begin{cases} \exp(-x), & x \geq 0 \\ 0, & else \end{cases}$$

19. Find the expected value of *x*.

    a) 2              b) 1/2                     c) exp(-1)                d) 0                      e) NOTA

20. Identify the distribution function, *F(x)*?

    a) $F(x) = \begin{cases} \exp(-x), & x \geq 0 \\ 0, & else \end{cases}$

    b) $F(x) = \begin{cases} \exp(-x) - 1, & x \geq 0 \\ 0, & else \end{cases}$

    c) $F(x) = \begin{cases} 1 - \exp(-x), & x \geq 0 \\ 0, & else \end{cases}$

    d) $F(x) = \begin{cases} -\exp(-x), & x \geq 0 \\ 0, & else \end{cases}$

    e) NOTA

21. From a group of 10 Ph.D. engineers, 5 are selected at random for employment. What is the probability that 3 of the 5 five best engineers are selected from the pool of 10?

    a) 3/5            b) 2/5                     c) $\dfrac{(5!)^4}{(12^2)10!}$                d) $1 - \dfrac{(5!)^4}{(12^2)10!}$                e) NOTA

22. Two events *A* and *B* are such that P(*A*) = 0.2, P(*B*) = 0.3, and P(*A* ∩ *B*) = 0.1. Find P($A^C \cap B^C$). Note: $A^C$ is the complement of set *A*.

    a) 0.1            b) 0.6                     c) 0.4                    d) 0.9                    e) NOTA

23. Two integers, X and Y, are chosen from the interval [1,100] with replacement. What is the probability that X/Y is greater than unity?

    a) 99/200          b) 101/200          c) 49/100          d) 51/100          e) NOTA

24. Which of the following vectors is a probability vector?

    a) $\mathbf{j} - \mathbf{k}$          b) $\mathbf{j}$          c) $\mathbf{i} + \mathbf{j} + \mathbf{k}$          d) $\sqrt{3}/3(\mathbf{i} + \mathbf{j} - \mathbf{k})$          e) NOTA

25. A multiple choice test consists of 10 questions, 5 answer options per question. One point is awarded for each correct answer. The first two incorrect answers are without penalty, whereas every incorrect answer beyond the first two is scored as negative 0.25 points. Unanswered questions are neither correct nor incorrect. What is the expected score if a random decision is made to answer A, B, C, D, E or not answer each problem?

    a) 1/12          b) 1/3          c) 1/2          d) 2          e) NOTA

26. Two standard six-sided die are cast. Which of the following pairs of sets are mutually exclusive?

    a) doubles; at least one 6                          b) sum less than 4; at least one 4

    c) product greater than 16, sum less than 10          d) at least one 2; sum of 8          |

    e) NOTA

27. Anxiety Prone Exams (APEs) are scored 1 through 5, each score occurring with identical relative frequency. Suppose five exams are chosen at random. What is the probability that among the selected exams there will be exactly one 1, one 2, one 3, one 4, and one 5?

    a) 1/5          b) 24/625          c) 601/625          d) 4/5          e) NOTA

28. Two real numbers are chosen at random from the interval [-2,8]. What is the probability that their sum is less than 4?

    a) 45/121          b) 17/25          c) 15/121          d) 36/121          e) NOTA

29. A bag contains six blue balls and two orange balls. Three balls are chosen at random and without replacement. What is the probability that at least one orange ball is chosen?

    a) 5/14          b) 1/2          c) 9/14          d) 11/14          e) NOTA

30. The numeric answer to this problem has been assigned to one of the five letters A thru E. If you randomly choose a letter to answer this problem, what is the probability that you will answer it correctly?

    a) 1/6          b) 1/5          c) 1/2          d) 1          e) NOTA