

1. 0—Using $y' = \frac{-1}{x^2}$ and $y = \frac{-4}{9}x + \frac{1}{3}$, then setting the slopes $=$, gives $x = \pm\frac{3}{2}$ & $y = \pm\frac{2}{3}$ & the sum is 0.

2. -160—a) Avg. velocity $= \frac{\Delta s}{\Delta t} = \frac{-96 - 32}{3 - 1} = -64$. b) $v = -32t \Rightarrow v(3) = -96$. Sum is -160.

3. $\frac{1+100\pi^2}{\pi}$ —a) $\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt} \Rightarrow 4 = 4\pi \frac{dr}{dt} \Rightarrow \frac{dr}{dt} = \frac{1}{\pi}$. b) $\frac{dA}{dt} = 2\pi r \frac{dr}{dt} = 20\pi \cdot 5 = 100\pi$. Sum to get ans.

4. $\frac{297}{2}$ —a) Isect pts. are $y = 2$ & -1. $A = \int_{-1}^2 [y - (y^2 - 2)] dy = \frac{9}{2}$. b) $A = 2 \int_0^3 2\sqrt{9 - x^2} dx = 144$. Sum to get ans.

5. 34—a) Approx of $f'(2) = \frac{15 - 21}{3 - 1} = -3$. b) Let $u = x^2$, $du = 2xdx \Rightarrow \frac{1}{2} \int f'(u) du = \frac{1}{2} f(u) = \frac{1}{2} f(x^2) |_0^2 = 4$.

c) Using integration by parts and $u = x$, $dv = f''(x)dx$, then $du = dx$, $v = f'(x)$

$$\Rightarrow xf'(x)|_1^3 - \int_1^3 f'(x) dx = 33. \text{ Sum to get answer.}$$

6. $\frac{3}{2} + \sqrt{122}$ —a) Speed $= \sqrt{(x'(0))^2 + (y'(0))^2} = \sqrt{(-1)^2 + 11^2} = \sqrt{122}$. b) $\lim_{t \rightarrow \infty} \frac{dy}{dx} = \lim_{t \rightarrow \infty} \frac{9e^{3t} + 2e^{-2t}}{6e^{3t} - 7e^{-7t}} = \frac{3}{2}$.

7. 0—a) The derivative is never 0 so there are not relative extrema and it is always positive, so the absolute max occurs at the right endpoint, $x = 10$. b) The derivative is never 0 so there are not relative extrema and it is always positive, so the absolute min occurs at the left endpoint, $x = -10$. c) The derivative increases from $x = -10$ to $x = 0$, indicating the function is concave up there. The derivative decreases from $x = 0$ to $x = 10$, indicating the function is concave down there. So, $x = 0$ is the point of inflection. Sum to get answer.

8. 65—a) $f'(x) = 12x^2 + 2ax + b$ and $f''(x) = 24x + 2a \Rightarrow f''(-2) = -48 + 2a = 0 \Rightarrow a = 24$.

b) $f'(-1) = 12 - 2a + b = 12 - 2(24) + b = 0 \Rightarrow b = 36$.

k) $\int_0^1 (4x^3 + 24x^2 + 36x + k) dx = 36 \Rightarrow 27 + k = 36 \Rightarrow k = 5$. Sum to get answer.

9. $\frac{130}{3}$ —a) $A = \int_0^2 (5 - (x^2 + 1)) dx = \frac{16}{3}$. b) $A = \int_1^3 (3x^2 + 2x) dx = 34$. c) $A = \int_0^{2\pi} \sin\left(\frac{x}{2}\right) dx = 4$. Sum to get ans.

10. $xe^x + 3x^2 e^{x^3} - e^{\sin x}$ —a) $\frac{d}{dx} \int_2^{e^x} \ln(t) dt = e^x \ln e^x = xe^x$. b) $\frac{d}{dx} \int_e^{x^3} e^t dt = 3x^2 e^{x^3}$

c) $\frac{d}{dx} \int_x^3 e^{\sin t} dt = -\frac{d}{dx} \int_3^x e^{\sin t} dt = -e^{\sin x}$. Sum to get answer.

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11. $\frac{64\pi}{3}$ --a) $V = \pi \int_0^2 [(4x - x^2)^2 - x^4] dx = \frac{32\pi}{3}$. b) $V = 2\pi \int_0^2 (3-x)(4x-2x^2) dx = \frac{32\pi}{3}$. Sum to get answer.

12. $\frac{163}{15}$ --a) A.V. = $\frac{1}{5} \int_4^9 \sqrt{x} dx = \frac{38}{15}$. b) A.V. = $\frac{1}{5} \int_0^5 x \sqrt{25-x^2} dx$ (use u-sub) = $\frac{125}{15}$. Sum to get answer.

13. 8-- $L = \int_0^{2\pi} \sqrt{(1-\cos t)^2 + (\sin t)^2} dt = 2 \int_0^{2\pi} \sqrt{\frac{1-\cos t}{2}} dt = 2 \int_0^{2\pi} \sin\left(\frac{t}{2}\right) dt = 8$.

14. $\frac{\pi}{2}$ -- $A = \frac{1}{2} \cdot 8 \int_0^{\frac{\pi}{4}} \cos^2 2\theta d\theta = \frac{\pi}{2}$.