Question #1 Calculus STATE Bowl
2007 Mu Alpha Theta National Convention

a) The tangent lines to the parabola $y = ax^2 + b$ at the point (2, 3) and (-2, 3) contain the origin. Find the area of the region enclosed by the parabola and the two tangent lines.

b) Find the total area between the curves $y = x^3$ and $y = x$.

c) Let R be the region in the 4th quadrant enclosed by the x-axis and the curve $y = x^2 - 2kx$, where $k > 0$. If the area of the region R is 36, find k.

d) Let R be the region in the 1st quadrant under the curve $y = \frac{1}{x}$ and bounded by x = 2 and x = k. If the area of the region R is ln 4, find k.

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Question #2 Calculus STATE Bowl
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a) The base of a solid is the region in the 1st quadrant bounded by the line $x + 2y = 4$ and the coordinate axes. What is the volume of the solid if every cross-section perpendicular to the x-axis is a semicircle?
b) A solid has a circular base, centered at the origin, of radius 3. What is the volume of the solid if every cross-section perpendicular to the x-axis is an equilateral triangle?
c) The base of a solid is the region enclosed by the ellipse $4x^2 + y^2 = 1$. What is the volume of the solid if every cross-section perpendicular to the x-axis are semicircles?
d) The base of a solid is the region enclosed by the graph of $y = 3(x-2)^2$ and the coordinate axes. What is the volume of the solid if every cross-section perpendicular to the x-axis is a square?

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Question #3 Calculus STATE Bowl
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a) Find the maximum value of $f(x) = 2x^3 - 9x^2 + 12x - 1$ on $[-1, 2]$.
b) At what value(s) of x does $f(x) = x^4 - 8x^2$ have a relative minimum?
c) At what value(s) of x does $f(x) = 4x^6 - 8x^3 + 18$ have a relative minimum?
d) Find the minimum value of $f(x) = e^x - 2x$.

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Question #4 Calculus STATE Bowl
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a) If $\int_1^7 \ln x dx$ is approximated by 3 circumscribed rectangles of equal width on the x-axis, then, terms of ln, what is that approximation?

b) If $\int_0^4 (x^2 - 6x + 9) dx$ is approximated by 4 inscribed rectangles of equal width on the x-axis, then what is that approximation?

c) Use the following table and the Trapezoidal Rule with 4 subdivisions to approximate $\int_1^5 f(x) dx$.

x	1	2	3	4	5
$f(x)$	0	1.1	1.4	1.2	1.5

d) If $\int_0^6 (x^2 - 2x + 2) dx$ is approximated by 3 inscribed rectangles of equal width on the x-axis, then what is that approximation?

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Question #5 Calculus STATE Bowl
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a) A searchlight is located at a point A, 40 feet from a straight wall. The light revolves counterclockwise at a rate of $\frac{\pi}{30}$ radians/second. At any point on the wall, the strength of the light L (in lumens) of the light is inversely proportional to the square of the distance d from point A. At the closest point from the light to the wall, L = 10,000 lumens. If θ is defined to be the angle that the straight line from the wall to point A makes with where the line of light is at the present time, then how fast (in lumens/second) is the strength of the light changing when $\theta = \frac{\pi}{4}$?

b) If the radius of a sphere is increasing at the rate of 2 inches/second, how fast, in cubic inches/second, is the volume increasing when the radius is 10 inches?

c) Let A(w) be the area, in sq. centimeters, of the region in the 1st quadrant enclosed by the x-axis and the graph of $f(x) = 24x^2 - 12x^3$ between the origin and a vertical line x = w, where 0 < w < 2. If w is moving to the right at a constant rate of 0.05 cent/sec, how fast is A(w) changing when w = 1?

d) The volume of an expanding sphere is increasing at a rate of 12 cu.ft/sec. When the volume of the sphere is 36π cu ft, how fast, in sq.ft/sec, is the surface area of the sphere increasing?

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Question #6 Calculus STATE Bowl
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a) A particle moves along the x-axis so that at any time t ($t \geq 0$) its position is given by $x(t) = (t+1)(t-3)^3$. For what values of t is the velocity of the particle increasing?
b) A particle moves along the x-axis so that at any time t ($t \geq 0$) its position is given by $x(t) = 3t^3 - 18t^2 + 24t$. At what time(s) t is its average velocity zero?
c) A particle moves along the x-axis so that at any time t ($t \geq 0$) its position is given by $x(t) = \ln t$. Find the average velocity of the particle for $1 \leq t \leq e$.
d) A particle moves along the x-axis so that at any time t ($t \geq 0$) its position is given by $x(t) = \frac{1}{2}\sin t + \cos(2t)$. What is the acceleration of the particle at $t = \frac{\pi}{2}$?

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Question #7 Calculus STATE Bowl
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a) Let $f(x) = (\sin x)^x$. Find the value of $f'\left(\frac{\pi}{2}\right)$.

b) If $x^2 + y^2 = 1$, find, in terms of x & y, an expression for $\frac{d^2y}{dx^2}$.

c) Suppose that $w = u \circ v$ and $u(1) = 0, v(0) = 2, u'(0) = 3, u'(2) = 4, v'(0) = 5,$ and $v'(2) = 6$. Find $w'(0)$.

d) Let $y = x^4 + x^2 + 1$, $x = 1$, and $dx = 1$. Find the value of the differential dy.

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Question #8 Calculus STATE Bowl
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a) Find the interval of convergence for $\sum_{n=1}^{\infty}\frac{(x-2)^n}{n\cdot 3^n}$.

b) Find the radius of convergence of the Taylor Series for f about x = 0 if the nth derivative of f at x = 0 is given by $f^{(n)}(0)=\frac{(-1)^{n+1}(n+1)!}{5^n(n-1)^2}, n\geq 2.$

c) Find the coefficient of x^{22} in the Maclaurin Series for $f(x)=\sin(5x+\frac{\pi}{4})$.

d) If $f(x)=\frac{1}{3}+\frac{2x}{3^2}+\frac{3x^2}{3^3}+\ldots+\frac{(n+1)x^n}{3^{n+1}}+\ldots$ then find $\lim_{x\to 0}\frac{f(x)-\frac{1}{3}}{x}$.

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Question #9 Calculus STATE Bowl
2007 Mu Alpha Theta National Convention

a) Evaluate $\int_0^1 x^{-1/3} dx.$

b) Evaluate $\int_0^\infty \frac{dx}{\sqrt{x}(x+1)}.$

c) Evaluate $\int_{-\infty}^0 \frac{e^x}{e^x+1} dx.$

d) Evaluate $\int_{-\infty}^\infty \frac{e^x}{e^x+1} dx.$

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d) Evaluate $\int_{-\infty}^\infty \frac{e^x}{e^x+1} dx.$

Question #10 Calculus STATE Bowl
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a) Solve the differential equation $y' = \frac{\ln x}{xy}$, subject to the condition $y(1) = 2$, and from your solution, find $y(e)$.

b) Solve the differential equation $y' = y$, subject to the initial condition $y(0) = 1$, and from your solution, find $y(e)$.

c) Solve the differential equation $y' = x^2$, subject to the initial condition $y(0) = 3$, and from your solution, find $y(1)$.

d) Solve the differential equation $(\ln y)y' = \frac{y}{x}$, subject to the initial condition $y(1) = e$, and from your solution, find $y(e^4)$.

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c) Solve the differential equation $y' = x^2$, subject to the initial condition $y(0) = 3$, and from your solution, find $y(1)$.

d) Solve the differential equation $(\ln y)y' = \frac{y}{x}$, subject to the initial condition $y(1) = e$, and from your solution, find $y(e^4)$.