Theta Applications
2007 Mu Alpha Theta National Convention
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For all questions, answer E. "NOTA" means none of the above answers is correct.

1.   Nick takes a rectangular piece of newspaper and tapes together two opposite sides at the edges, forming a hollow right cylinder. What was the length in centimeters of a diagonal of the original piece of newspaper, to the nearest centimeter, if the cylinder thus formed has a height of 15 cm and a radius of $\frac{4}{\pi}$ cm?

   A)   16   B)   17
   C)   18   D)   19   E)   NOTA

2.   If paying $5 for three hot dogs and a hamburger will get me $2.75 in change, and paying $5 for three hamburgers and a hot dog will get me $2.25 in change, how much change will I get paying $5 for a hot dog and a hamburger?

   A) $1.25   B) $2.50
   C) $3.75   D) $4.25   E)   NOTA

3.   All aces and face cards are removed from a standard 52-card deck. A card is selected at random from the remaining 36 cards, and a standard, fair six-sided die is rolled. What is the probability that the number atop the die is greater than the value of the drawn card?

   A)   5/27   B)   5/36
   C)   5/52   D)   5/54   E)   NOTA

4.   Bob's Equation o' Luck is $y = 10^{(x+2)} - 17$. Where defined, what is its inverse?

   A) $y = 83$ B) $y = 10^{(x-2)} + 17$
   C) $y = \log(x + 2) + 17$ D) $y = \log(x + 17) - 2$ E)   NOTA

5.   The windows and doorways in Liz's house are all similar rectangles. It is also known that the height of one window is three times the width of one of the doorways, and the height of that doorway is twice the width of the window. If Liz correctly measures the window's width as $\sqrt{6}$ feet, what is the area in square feet of the doorway?

   A) $\sqrt{6}$ B) $2\sqrt{6}$
   C) $4\sqrt{6}$ D) $6\sqrt{6}$ E)   NOTA

6. Skippy the Goat is outside grazing, tied to the corner of his owner's barn with a 5-yard rope. If the rectangular barn has two sides of length seven yards, two sides of length 3 yards, and some very sturdy walls (that is, Skippy must remain outside the area enclosed by the barn), what is the total area in square yards in which Skippy can graze outside? Ignore the heights of both Skippy and the rope.

   A) $\dfrac{75}{4}\pi$ B) $\dfrac{79}{4}\pi$
   C) $25\pi - 15$ D) 25π E) NOTA

7. Between the hours of 1:00 pm and 10:00 pm, people are entering Mingo's Magic Theme Park at a constant rate of 20 people per hour. Between the hours of 12:00 pm and 7:00 pm, people are leaving Mingo's Magic Theme Park at a constant rate of 22 people per hour. If there are 34 people inside the park when the gates open at 12:00 pm (they stayed overnight), and nobody enters the park from 12:00 pm to 1:00 pm, how many people are in the park at 5:30 pm?

   A) 12   B) 21
   C) 30   D) 34   E) NOTA

8. At each of the tri-monthly presidential elections held by the Matt Damon Fan Club, between 75% and 87.5% of the club's 752 members show up in person to vote. Each other member casts his or her ballot by mail, but a mail-in ballot only counts as 80% of an in-person vote. If a member requires at least 702 in-person votes or the equivalent to be elected president, with there being no winner if no candidate receives this many, what is the fewest number of mail-in ballots required to win the election?

   A) 35   B) 44
   C) 55   D) 138   E) NOTA

9. Raul has a solid metal sphere of radius 3. If he melts it down and remolds it into two identical solid cylinders of radius 3, with no loss of material, what is the height of each new cylinder formed?

   A) 1   B) 2
   C) 3   D) 4   E) NOTA

10. "If Cole plays football, then it is Alton's birthday." The converse of this statement is:

   A) "If Cole plays football, then it is not Alton's birthday."
   B) "If Cole does not play football, then it is not Alton's birthday."
   C) "If Cole does not play football, then it is Alton's birthday."
   D) "If it is not Alton's birthday, then Cole does not play football."
   E) NOTA

11. The wheel on Aneesh's unicycle makes ten complete revolutions each minute. If the radius of the wheel is six inches, how fast is the unicycle traveling in feet per hour?

   A) 60π B) 300π
   C) 600π D) 1800π E) NOTA

12. Tyler and his friends are looking for seats in the Infinite Theatre. They find that each row has a composite number of seats, such that the first ten rows have 4, 6, 8, 9, 10, 12, 14, 15, 16, and 18 seats, respectively. None of the seats are currently occupied, but each time Tyler advances a row to look for seats (Tyler and his friends all want to sit in the same row as one another), two more of his friends get bored and leave the theatre to go back home. If Tyler had 49 friends with him at Row 1 (and thus 47 at Row 2, etc.), in which row will Tyler find enough seats to fit himself and his remaining friends? Assume that the pattern regarding seat numbers remains constant.

    A)   Row 14   B)   Row 24
    C)   Row 30   D)   Row 36   E)   NOTA

13. Tyler's friend James is not a big fan of operas, and he feels that the Infinite Theatre's production of "Mobius!" is taking forever. Bored, he takes some measurements with his stopwatch and determines that, sure enough, the lengths of the musical numbers form an infinite geometric sequence, of common ratio $\frac{4}{5}$. If the opening number was 3 minutes and 14 seconds in length, and there are no breaks between successive numbers, how much time will have elapsed since the show's beginning, when it finally ends?

    A)   Four minutes, two and a half seconds   B)   Nine minutes, forty-two seconds
    C)   Fifteen minutes, seven seconds   D)   Sixteen minutes, ten seconds
    E)   NOTA

14. Duncan and Steve are playing Calvinball Scrabble, in which the letters put down do not have to form actual, English words. For example, the sequence of letters "ORLY," while nonsensical in English, is still a legal move in Calvinball Scrabble. It is Duncan's turn to go first, and he has drawn the letters N, O, N, O, Y, E, and S. Using only these letters, how many distinct seven-letter "words" can Duncan spell this turn?

    A)   720   B)   1260
    C)   1337   D)   5040   E)   NOTA

15. Marco's swimming pool is in the shape of two semicircles of radius five feet on opposite ends of a rectangle of side lengths ten and twenty feet (the semicircles are on the rectangle's shorter sides). For some reason, Marco has decided to plant a flower garden around the outside of his pool, two feet wide at every point. How much land, in square feet, will his new garden fill?

    A) $40 + 16\pi$ B) $80 + 24\pi$
    C) $200 + 25\pi$ D) $280 + 49\pi$ E)   NOTA

16. Sammy the snail crawls N inches North, E inches East, S inches South, and W inches West. If N, E, S, and W are all positive, relatively prime integers, with no two of the four values sharing a common factor greater than 1, what is the smallest possible distance, in inches, between Sammy's starting and ending positions?

    A)   1   B) $\sqrt{5}$
    C) $2\sqrt{2}$ D)   5   E)   NOTA

17.   Every day, Jon flips a standard coin. If and only if the coin lands on heads, Jon draws a card at random from a standard 52-card deck. If and only if the card drawn is not a face card (king, queen, or jack), he rolls a standard 6-sided die. If and only if Jon rolls the die and it lands with a prime number on its top face, Jon will wear a raincoat that day. Today, there is a fifty percent chance of rain. What is the probability that Jon will get wet today? That is, what is the probability that it will rain and Jon will not be wearing a raincoat?

A) $\dfrac{3}{104}$ B) $\dfrac{5}{52}$

C) $\dfrac{21}{52}$ D) $\dfrac{49}{104}$ E)   NOTA

18.   Todd finds three dollars on the ground on Day 1. The next day (which happens to be Day 2), he finds another five dollars on the ground. If he finds seven dollars on the ground on Day 3, and so on in this arithmetic progression for the next 500 days, on which day will he be able to pay off his massive $10,000 gambling debt? Keep in mind that Todd had $1 in his possession before finding the first three dollars on Day 1, and that he neither gains nor loses money through any means other than his daily findings on the ground.

A)   Day 16   B)   Day 81
C)   Day 99   D)   Day 148   E)   NOTA

19.   You have three brothers: Mu, Alpha, and Theta. You know that Mu always tells the truth, Alpha always lies, and Theta is honest some of the time, lying or telling the truth as it suits him. Unfortunately, you've never learned to tell your brothers apart, and two of them are standing before you now. The first tells you he's Alpha, but the second says that this is not true. Which brother is not before you right now?

A)   Mu   B)   Alpha
C)   Theta   D)   Not enough information   E)   NOTA

20.   Mark rounds numbers by first rounding to the nearest tens place, then rounding the rounded number to the nearest hundreds place, and then finally rounding that number to the nearest thousands place. What is the smallest number that Mark would round to 2000?

A)   1435   B)   1445
C)   1455   D)   1500   E)   NOTA

21.   On Sunday, Brad gets 133_{10} in allowance from his parents. On Monday he receives 133_9, and so on through Saturday, when he receives 133_4. In base-11, how much total money has Brad received this week?

A) 450_{11} B) 539_{11}
C) 777_{11} D) 931_{11} E)   NOTA

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22. The tangent of a given angle is $\frac{3}{4}$. If the angle is in a right triangle of semiperimeter 12, what is the triangle's area?

A) 6   B) 12
C) 18   D) 24   E) NOTA

23. Chelsea's lemonade is 50% lemon juice, 25% sugar, 20% water, and 5% hydrochloric acid (for that extra bite). On reflection, however, she decides that that much sugar may be harmful to her customers. How many ounces of hydrochloric acid will be in the final product, if Chelsea adds enough pure hydrochloric acid to the twenty ounces of lemonade she has already prepared to bring the sugar level down to 20%?

A) 1   B) 5
C) 6   D) 8   E) NOTA

24. Arielle by herself can eat eleven scoops of ice cream in just three minutes. Her friend Lissa can eat ten scoops by herself in the same three minutes. How long would it take the two of them, eating out of the same bowl, to finish the UltraMegaHurriQuakeNado, an ice cream treat consisting of thirty-five scoops?

A) Three minutes   B) Four minutes
C) Five minutes   D) Six minutes   E) NOTA

25. Doug has a rectangular piece of tin foil measuring 7 x 10 inches. If Doug wants to cut away four identical squares from the corners, so as to be able to fold up the resulting flaps to form a lidless box of base 28 square inches, what total area of material, in square inches, should he cut away?

A) 1.5   B) 3
C) 6   D) 9   E) NOTA

26. Dr. Lyons has always refused to conform to the ice cube standards of his closest colleagues. Instead, he freezes his ice into right circular cones of radius 3 cm and height 5 cm. For reasons known only to him, he is now filling a hollow spherical pumpkin of inner radius 15 cm with the cones, which melt into liquid water as soon as they enter the (nonporous) pumpkin. How many ice-cones will it take to completely fill the pumpkin?

A) 15   B) 150
C) 200   D) 300   E) NOTA

27. Admission prices for tickets to see yesterday's game were $30 for students and $40 for everyone else. If 231 tickets were sold for a total of $8050 in sales, how many non-students paid to see the game?

A) 112   B) 115
C) 119   D) 132   E) NOTA

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28. Two pegs are securely fastened ten feet apart from each other, and tied together with a 20-foot rope. Lassie's collar is attached to this rope, in such a way that it can slide freely along the rope. What is the area, in square feet, of the region in which Lassie can travel?

A) $50\pi\sqrt{3}$ B) $75\pi\sqrt{3}$

C) $150\pi\sqrt{3}$ D) $400\pi\sqrt{3}$ E)   NOTA

29. Horace bet you $\sqrt{30+\sqrt{30+\sqrt{30+\sqrt{30+...}}}}$ dollars that you would get this question wrong. So, how much money is he going to lose (assuming that you answer correctly, of course)?

A) $5   B) $6

C) $11   D) $30   E)   NOTA

30. Right now, there are 99 red balloons floating in the summer sky, and there are 71 bottles of ginger ale on the wall. But 6 hours earlier, when there were 99 bottles of ginger ale on the wall, there were only 15 red balloons floating in the summer sky.If red balloons are being set free at a constant rate, and bottles of ginger ale are being taken down and passed around at a constant rate, how many hours ago did the number of bottles of ginger ale on the wall equal the number of red balloons floating in the summer sky?

A)   1.5   B)   3.0

C)   3.5   D)   4.5   E)   NOTA