0) -4

$$\sum_{i=1}^{\infty}\log(\frac{i}{i+1}) = \log(\frac{1}{2})+\log(\frac{2}{3})+\log(\frac{3}{4})+...$$
$$= \log[(\frac{1}{2})(\frac{2}{3})(\frac{3}{4})...]$$
$$= \log(\frac{1}{10000})$$
$$= -4$$

1) 53
Synthetic divison leads to $s = 1, r = -5, t = -11$. $3(1) - 5(-11) + (-5) = 17$

2) 10
We can use the same idea that we use when we work with continued fractions.

$$5x\log(5x\log(5x\log(...))) = 100$$
$$5x\log(100) = 100$$
$$10x = 100$$
$$x = 10$$

3) $\frac{1}{20}$

$$(\frac{23}{5}+\frac{44}{5}i)(1-3i) = 31-5i$$
$$\frac{(31-5i)(2+5i)}{(2-5i)(2+5i} = 3+5i\ \frac{3!}{5!} = \frac{1}{20}$$

4) $52+48\sqrt{2}$

We can think of this octagon as square with side length $6+4\sqrt{2}$ and 4 isosceles right triangles cut from the corner with length $2\sqrt{2}$. So area of square is equal to $(6+4\sqrt{2})^2 = 68+48\sqrt{2}$. Each corner triangle has area equal to $(\frac{1}{2})(2\sqrt{2})^2 = 4$. So the total area of the octagon is $68+48\sqrt{2}-(4)(4) = 62+48\sqrt{2}$.

5) 32

$1512 = (2^3)(3^3)(7^1)$. Now to find the number of positive integral divisors, we add one to the exponents and multiply them together. $\rightarrow (2+1)(3+1)(1+1) = 32$

6) 11

$$\begin{aligned}
\frac{1}{p} &= .d_1d_2d_1d_2d_1d_2d_1d_2... \\
\frac{100}{p} &= d_1d_2.d_1d_2d_1d_2d_1d_2... \\
\frac{100}{p} - \frac{1}{p} &= d_1d_2 \\
\frac{99}{p} &= d_1d_2
\end{aligned}$$

From this we can see that p must divide 99. The only prime numbers that divide 99 are 3 and 11, and since $p > 3$, we can conclude $p = 11$.

7) 68

By definition, the radius of the circle will be perpendicular to the tangent line. Therefore, we can use the distance from a point to a line formula to calculate the length of the radius.

$$\frac{|4(5) + 3(16) - 5|}{\sqrt{4^2 + 3^2}} = \frac{63}{5}$$

So $a = 63, b = 5$. Therefore, $a + b = 68$.

8) 1225

It is not that hard to show that for a matrix A, $|A^2| = |A|^2$

$$\begin{aligned}
|A| &= 1(10+4) - 3(6+1) + 6(12-5) \\
&= 1(14) - 3(7) + 6(7) \\
&= 35 \\
|A|^2 &= 1225
\end{aligned}$$

9) $\sqrt{5}$

Multiplying by the factors yeild:

$$3x + 7 = a(x - 1) + b(x + 4)$$

Plugging in $x = 1$ leads to: $b = 2$
Plugging in $x = -4$ leads to: $a = 1$
Therefore, $\sqrt{(1)^2 + (-4)^2} = \sqrt{5}$

10) 7

The ratio of the volume of the water to the volume of the cup is $\frac{27}{1000}$. Therefore, the ratio of the height of the water to the height of the cup is $\frac{3}{10}$. This yeilds to the height of the water being 3 cm. So the distance from the top if the cup to the top of the water is $10 - 3 = 7$.